Simple adaptive synchronization of chaotic systems with random components

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Practical systems usually possess random components. Random components often affect the robustness of synchronism and must be taken into consideration in the design of synchronization. In the present study, we assume that the system satisfies the Lipschitz condition, and the random component is uniformly bounded. By the partial stability theory, we are able to prove that two simple adaptive variable structure controllers achieve synchronization of chaotic systems. Moreover, we discuss how the controllers can be modified to eliminate the undesired phenomenon of chattering. The Duffing two-well system and the Chua circuit system are simulated to illustrate the theoretical analysis. © *2006 American Institute of Physics*. [DOI: 10.1063/1.2211607]

Chaos is a useful property of nonlinear systems and synchronization of chaos has many practical applications. However, there are random components in real chaotic systems. It is therefore important to synchronize a chaotic system with random components. In this work, we propose an adaptive variable structure control method to achieve synchronization of chaotic systems with random components. Here the original system is extended to include new equations for two system parameters: Lipschitz constant and boundedness constant for random components. As the error state vector becomes a part of the extended dynamical variable, the partial stability theory is particularly useful as a tool in verifying the asymptotic stability of the zero error state. Based upon this theory, we derive two criteria regarding synchronization of chaotic systems with random components. The Duffing two-well and the Chua circuit system are simulated to illustrate the validity of the theoretical analysis.

I. INTRODUCTION

Chaotic systems were thought difficult to be synchronized or controlled in the past as chaotic systems exhibit sensitive dependence on initial conditions. Since the 1980s, researchers have realized that chaotic motions can be synchronized through a feedback mechanism[1,2] or linking two systems by common signals.[3] In the past decade, we have seen a rapid growth of theoretical and experimental studies for chaos synchronization. This is partly because chaos synchronization has potential applications in secure communication,[5,6] information processing,[7] pattern formation,[8] etc.

Synchronization means that the state of the response system eventually approaches that of the driving system. Two kinds of chaos synchronization are most often discussed. (1) The master-slave scheme, introduced by Pecora and Carroll,[3] consists of two identical systems. The master system evolves into a chaotic orbit and some state variables of the slave system are replaced by the corresponding state variables of the master system. Synchronization occurs if and only if all the (conditional) Lyapunov exponents of the unreplaced state variables are negative.[3,4] (2) The coupling scheme is the second kind of synchronization, which deals with two identical chaotic systems except that the coupling term can be either unidirectional or bidirectional. Under certain conditions the response system may eventually evolve into the same orbit of the driving system.

The synchronization discussed previously is called complete synchronization or simply synchronization. There are other types of synchronization such as generalized synchronization, phase synchronization, lag, and anticipated synchronization. Generalized synchronization means that there is a functional relation between the state variables of the driving and the response systems as time evolves.[9–11] This function is not necessarily defined on the whole phase space but on the attractor only. Phase synchronization is that the phases of two systems come closely[12–14] as time evolves though amplitudes remain almost uncorrelated. Lag and anticipated synchronization means that the state of the response system eventually approach that of the driving system with a time delay[14,15] or a time lead,[15,16] respectively.

In actual situations, random components usually exist in systems and they often induce complicated dynamics. Sometimes random components are added to increase security in communication. In the past, some efforts were devoted to synchronization of coupled systems without uncertainty.[17,18]

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Recently, there are some control methods to synchronize chaotic systems with unstructured uncertainty such as adaptive control technique,[19] observer-based control,[20] and adaptive variable structure method.[21]

This article is aimed at the investigation of synchronization for chaotic systems with random components. The systems studied are assumed to satisfy the Lipschitz condition. Random components in the system often affect the robustness of synchronization and it must be addressed in the design of synchronism. Here, random components are required to satisfy a boundedness condition, but we do not assume precise knowledge about the boundedness constant nor the Lipschitz constant. In the present study, an adaptive variable structure method is proposed to achieve synchronization where the variable structure control is to deal with the random component, whereas the adaptive technique makes the control convenient to be implemented.

The adaptive control design introduces new equations for the system parameters: the Lipschitz constant and the boundedness constant. Therefore, we have to deal with an extended state vector, which consists of the original state vector, the error state, and the Lipschitz and the boundedness constants. However, in performing the Lyapunov analysis, the estimates of the Lipschitz and the boundedness constants cancel out in evaluating the time derivative of the Lyapunov function. This implies that only the stability but not the asymptotic stability of the zero error state can be verified by the traditional Lyapunov method. On the other hand, the partial stability theory is a stability theory for a partial state that can overcome this shortcoming and facilitates the asymptotic analysis of the zero error state. This theory was first applied to synchronization by Ge and Chen[22] and then was applied to adaptive synchronization of chaotic systems without uncertainty.[23] A brief review of this theory can be found in Ref. 22 and in the appendix of Ref. 24. In the present study, we use the partial stability theory to prove criteria for two adaptive variable structure controllers that ensure synchronization of the chaotic systems. Under the criteria, synchronization is robust even when we do not know much about the system uncertainty, nor the Lipschitz constant. The Duffing two-well oscillator and the Chua circuit system are simulated to illustrate the theoretical analysis.

II. THEORETICAL ANALYSIS

Consider a system with random component

$$\dot{\mathbf{x}} = \mathbf{f}(t,\mathbf{x}) + \mathbf{g}(t,\mathbf{x}), \tag{1}$$

and a controlled system

$$\dot{\hat{\mathbf{x}}} = \mathbf{f}(t,\hat{\mathbf{x}}) + \mathbf{u}, \tag{2}$$

where $\mathbf{x},\hat{\mathbf{x}} \in \mathbb{R}^N$ denote the state vectors, and $\mathbf{\Omega}$ is a domain containing the origin. The function $\mathbf{f}:\mathbf{\Omega} \subset \mathbb{R}\times\mathbb{R}^N \to \mathbb{R}^N$ satisfies the Lipschitz condition $\|\mathbf{f}(t,\mathbf{x}_1)-\mathbf{f}(t,\mathbf{x}_2)\| \le L\|\mathbf{x}_1-\mathbf{x}_2\|$ for all $(t,\mathbf{x}_1)$ and $(t,\mathbf{x}_2)$ in Ω with a Lipschitz constant L, $\mathbf{g}(t,\mathbf{x}):\mathbb{R}\times\mathbb{R}^N \to \mathbb{R}^N$ is the random component which may come from outer disturbance or are added to the systems, and $\mathbf{u}$ is a controller to be determined. The constant L is not unique as any number greater than L is also a Lipschitz constant. *In fact, we actually do not need to know* L, which is estimated by an adaptation in this article.

Let the state error be $\mathbf{e}=\hat{\mathbf{x}}-\mathbf{x}$, then systems (1) and (2) can be recast into

$$\dot{\mathbf{x}} = \mathbf{f}(t,\mathbf{x}) + \mathbf{g}(t,\mathbf{x}), \tag{3}$$

$$\dot{\mathbf{e}} = \mathbf{f}(t,\mathbf{x}+\mathbf{e}) - \mathbf{f}(t,\mathbf{x}) - \mathbf{g}(t,\mathbf{x}) + \mathbf{u}. \tag{4}$$

The purpose is to choose an appropriate controller $\mathbf{u}$ such that the partial state $\mathbf{e}=\mathbf{0}$ is asymptotically stable, i.e., system (2) synchronizes to system (1). In order to handle the random component, one must have some knowledge about $\mathbf{g}$, which we assume to meet the following boundedness condition:

$$\|\mathbf{g}(t,\mathbf{x}(t))\| \le K < \infty \quad \text{for all } t, \tag{5}$$

where $K>0$ *is not necessarily known a priori*. Random components satisfying (5) need not vanish while synchronization occurs. If this is the case it is not easy to design $\mathbf{u}$ to achieve synchronization.

In the present work, we do not assume precise knowledge about the Lipschitz and boundedness constants K and L, they are estimated through the adaptive control process. Let $\hat{K}$ and $\hat{L}$ be two estimates of K and L, respectively. Their estimated errors are defined by $\tilde{K}=\hat{K}-K$ and $\tilde{L}=\hat{L}-L$. There are two unknown constants K and L to be estimated, and therefore chaotic systems (3) and (4) are now extended by appending two new equations for the system parameter errors $\tilde{K}$ and $\tilde{L}$. The full dynamical variable is now $[\mathbf{x}^T\ \mathbf{e}^T\ \tilde{K}\ \tilde{L}]^T$, in which $\mathbf{e}$ is considered a partial state. A criterion of chaos synchronization for systems (1) and (2) with random components satisfying (5) is provided by the following theorem.

Theorem 1: *The partial state* $\mathbf{e}=\mathbf{0}$ *is uniformly asymptotically stable if we choose* $\mathbf{u}=-(\delta+\hat{L})\mathbf{e}-\hat{K}\mathbf{e}/\|\mathbf{e}\| -\gamma\,\mathrm{sgn}(\mathbf{e})\|\mathbf{e}\|$, *with* $\mathrm{sgn}(\mathbf{e})=[\mathrm{sgn}(e_1)\cdots\mathrm{sgn}(e_n)]^T$, $\delta,\gamma>0$, *and the estimates* $\hat{K}$ *and* $\hat{L}$ *obey differential equations:*

$$\dot{\hat{K}} = \dot{\tilde{K}} = \|\mathbf{e}\|, \tag{6a}$$

$$\dot{\hat{L}} = \dot{\tilde{L}} = \|\mathbf{e}\|^2. \tag{6b}$$

Proof: Choose a function as

$$V = \tfrac{1}{2}\mathbf{e}^T\mathbf{e} + \tfrac{1}{2}\tilde{K}^2 + \tfrac{1}{2}\tilde{L}^2,$$

which is positive definite with respect to $\mathbf{e}$ and possesses an infinitesimal upper bound.[22,24] Differentiating V with respect to t, we have the estimate

$$\begin{aligned}\dot{V} &= \mathbf{e}^T\dot{\mathbf{e}} + \tilde{K}\dot{\tilde{K}} + \tilde{L}\dot{\tilde{L}}\\ &= \mathbf{e}^T[\mathbf{f}(t,\mathbf{x}+\mathbf{e}) - \mathbf{f}(t,\mathbf{x}) - \mathbf{g}(t,\mathbf{x},\hat{\mathbf{x}}) + \mathbf{u}]\\ &\quad + (\hat{K}-K)\|\mathbf{e}\| + (\hat{L}-L)\|\mathbf{e}\|^2\\ &\le L\|\mathbf{e}\|^2 + K\|\mathbf{e}\| + \mathbf{e}^T\mathbf{u} + (\hat{K}-K)\|\mathbf{e}\| + (\hat{L}-L)\|\mathbf{e}\|^2\\ &= \hat{K}\|\mathbf{e}\| + \hat{L}\|\mathbf{e}\|^2 + \mathbf{e}^T\mathbf{u},\end{aligned}$$

where we have used the Cauchy-Schwarz inequality, the Lipschitz condition for $\mathbf{f}$ and boundedness condition (5) for the random component $\mathbf{g}$. Substituting $\mathbf{u}=-(\delta+\hat{L})\mathbf{e}-\hat{K}\mathbf{e}/\|\mathbf{e}\|-\gamma\,\mathrm{sgn}(\mathbf{e})/\|\mathbf{e}\|$ with $\delta,\gamma>0$ yields

$$\dot{V}\le-\delta\|\mathbf{e}\|^2-\gamma\le-\delta\|\mathbf{e}\|^2. \tag{7}$$

Here, it must be noted that (7) implies boundedness of $\dot{V}$ by $-\delta\|\mathbf{e}\|^2$, but not by $-V$ multiplied by a positive constant. In principle, we can only assure the stability of $\mathbf{e}=\mathbf{0}$ but not the asymptotic stability using the traditional Lyapunov theory. However, that V is positive definite with respect to $\mathbf{e}$ and condition (7) suffice to guarantee the uniformly asymptotic stability of $\mathbf{e}=\mathbf{0}$ by the partial stability theory. □

Remark 1: As the adaptive law is $\dot{\hat{L}}=\|\mathbf{e}\|^2\ge 0$. Therefore, $\hat{L}$ and $\tilde{L}$ are increasing functions of t and so is $\hat{L}+\delta$. If the initial value $\hat{L}_0$ of $\hat{L}$ or δ is large, then the feedback gain $\hat{L}+\delta$ is always large. Hence, the larger $\hat{L}_0$ or δ the faster $\|\mathbf{e}\|$ converges to 0. The situation of $\hat{K}$ is similar. Even so, the constant δ in the control $\mathbf{u}$ cannot be chosen arbitrarily as it is restricted by the power of control actuator in practice.

In fact, the inequality $\dot{V}<-\delta\|\mathbf{e}\|^2-\gamma$ holds when $\|\mathbf{e}\|\ne 0$, and $\dot{V}=0$ while $\|\mathbf{e}\|=0$. When $\|\mathbf{e}\|$ equals to zero, the state slides into the sliding surface and $\mathbf{u}=\mathbf{0}$. But this is the ideal situation as an infinite switching frequency cannot be implemented actually. Further, although the designed $\mathbf{u}$ can achieve synchronization mathematically, it may induce chattering due to the delay of control switching in practice.

Remark 2: A sliding layer (or boundary layer)[25] is commonly used to eliminate the chattering phenomenon. This method replaces the term $\mathbf{e}/\|\mathbf{e}\|$ by a saturation function

$$\mathbf{sat}(\mathbf{e},\beta)=\begin{cases}\mathbf{e}/\|\mathbf{e}\| & \|\mathbf{e}\|>\beta\\ \mathbf{e}/\beta & \|\mathbf{e}\|\le\beta.\end{cases}$$

If the trajectory enters into the sliding surface, the controller is forced to be continuous. The state error stays in a layer of thickness 2β.

Remark 3: A pseudosliding is another method to eliminate the chattering problem. It employs a continuous control $\mathbf{u}'=-(\delta+\hat{L})\mathbf{e}-\hat{K}\mathbf{e}/(\|\mathbf{e}\|+\alpha)-\gamma\,\mathrm{sgn}(\mathbf{e})/\|\mathbf{e}\|$ to replace $\mathbf{u}$, where $\alpha>0$ is a small constant. The two modifications introduce little loss of precision. However, we have $\mathbf{u}'\to\mathbf{u}$ as $\alpha\to 0$.

Remark 4: Notice that the first term of $\mathbf{u}$ is dominant as $\|\mathbf{e}\|$ is large; the first and second terms dominate as $\|\mathbf{e}\|$ is close to unity. The third term dominates when $\|\mathbf{e}\|$ is small. Once $\|\mathbf{e}\|$ is near zero, the effect of $-\gamma\,\mathrm{sgn}(\mathbf{e})/\|\mathbf{e}\|$ may induce a busting in the state error and the control because of the reciprocal of $\|\mathbf{e}\|$. Thus $-\gamma\,\mathrm{sgn}(\mathbf{e})/\|\mathbf{e}\|$ should be dropped if the state slides into the sliding layer.

There are many other possible designs of $\mathbf{u}$. Below, we provide an alternative $\mathbf{u}$ to achieve synchronization.

Theorem 2: *The partial state* $\mathbf{e}=\mathbf{0}$ *is uniformly asymptotically stable if we choose* $\mathbf{u}=-(\delta+\hat{L})\mathbf{e}-\hat{K}\,\mathrm{sgn}(\mathbf{e})-\gamma\,\mathrm{sgn}(\mathbf{e})/\|\mathbf{e}\|$ *with* $\delta,\gamma>0$, *and the estimates* $\hat{K}$ *and* $\hat{L}$ *obey Eq. (6).*

Proof: Choose a function V to be the same as that in the proof in Theorem 1, then we also have an estimate of $\dot{V}$ as

$$\dot{V}\le\hat{K}\|\mathbf{e}\|+\hat{L}\|\mathbf{e}\|^2+\mathbf{e}^T\mathbf{u}.$$

Substituting $\mathbf{u}=-(\delta+\hat{L})\mathbf{e}-\hat{K}\,\mathrm{sgn}(\mathbf{e})-\gamma\,\mathrm{sgn}(\mathbf{e})/\|\mathbf{e}\|$ with δ, $\gamma>0$ yields (7). By the partial stability theory, the partial state $\mathbf{e}=\mathbf{0}$ is uniformly asymptotically stable. □

Remark 5: To eliminate chattering, the controller $\mathbf{u}$ can be modified to be

$$\mathbf{u}'=-(\delta+\hat{L})\mathbf{e}-\gamma\,\mathrm{sgn}(\mathbf{e})/\|\mathbf{e}\| -\hat{K}[e_1/(|e_1|+\alpha)\ \cdots\ e_n/(|e_n|+\alpha)]^T$$

or

$$\mathbf{u}''=-(\delta+\hat{L})\mathbf{e}-\gamma\,\mathrm{sgn}(\mathbf{e})/\|\mathbf{e}\| -\hat{K}[\mathrm{sat}(e_1,\beta)\ \cdots\ \mathrm{sat}(e_n,\beta)]^T,$$

where α, β, and γ are constants.

Integrating the inequality $\dot{V}\le-\delta\|\mathbf{e}\|^2-\gamma\le-\gamma$, we obtain $\|\mathbf{z}(t)\|\le\sqrt{\|\mathbf{z}(t_0)\|^2-2\gamma(t-t_0)}$, where $\mathbf{z}\triangleq[\mathbf{e}^T\ \tilde{K}\ \tilde{L}]^T$. Hence $\mathbf{z}(t)$ becomes $\mathbf{0}$ no later than the time $T=[\|\mathbf{z}(t_0)\|^2+2\gamma t_0]/2\gamma$. The state will enter into the sliding surface within a finite time T and stays there forever. This reaching time is somewhat over estimated as the effect of $\mathbf{u}$ excluding γ is not considered in the estimation of T. As mentioned in Remark 4, the controlling term $-\gamma\,\mathrm{sgn}(\mathbf{e})/\|\mathbf{e}\|$ in Theorems 1 and 2 may induce a busting in the state error and it should be dropped when the state enters into the sliding layer. On the other hand, synchronization still occurs if the controller $\mathbf{u}$ does not include $-\gamma\,\mathrm{sgn}(\mathbf{e})/\|\mathbf{e}\|$. Hence we do not employ it in practical applications though it is easy to estimate the finite reaching time by including $-\gamma\,\mathrm{sgn}(\mathbf{e})/\|\mathbf{e}\|$ in $\mathbf{u}$.

III. NUMERICAL ILLUSTRATIONS

Although based on nonautonomous system, the criteria developed in the previous section also apply to autonomous system. The Duffing two-well and Chua circuit system, which are nonautonomous and autonomous systems respectively, will be simulated to demonstrate the theoretical analysis.

(A) The Duffing two-well oscillator. The equations are given by

$$\dot{x}=y+g_1,\quad \dot{y}=-x^3+x-by+A\sin\Omega t+g_2,$$

$$\dot{\hat{x}}=\hat{y}+u_1,\quad \dot{\hat{y}}=-\hat{x}^3+\hat{x}-b\hat{y}+A\sin\Omega t+u_2,$$

where the parameters $b=0.25$, $A=0.4$, and $\Omega=1$ with the initial conditions $\mathbf{x}_0=[\mathbf{x}_0^T\ \hat{\mathbf{x}}_0^T]^T=[0.2\ 0\ 1\ 1.5]^T$ ensure the existence of the chaotic attractor.

In this example, the random components of $\mathbf{g}$ are taken to be $g_1=g_2=0.001\Theta(r(t),10)$, where $r(t)$ is the normally distributed random number with mean 0 and variance 1, and Θ is an indicator function defined as

$$\Theta(\nu,\sigma)=\begin{cases}\nu, & |\nu|\le\sigma\\ 0 & \text{otherwise}.\end{cases}$$

Choose the controller $\mathbf{u}=-(0.1+\hat{L})\mathbf{e}-\hat{K}\mathbf{e}/\|\mathbf{e}\|$ with $\hat{L}_0=\hat{K}_0=1$ according to Theorem 1. The simulated results are shown

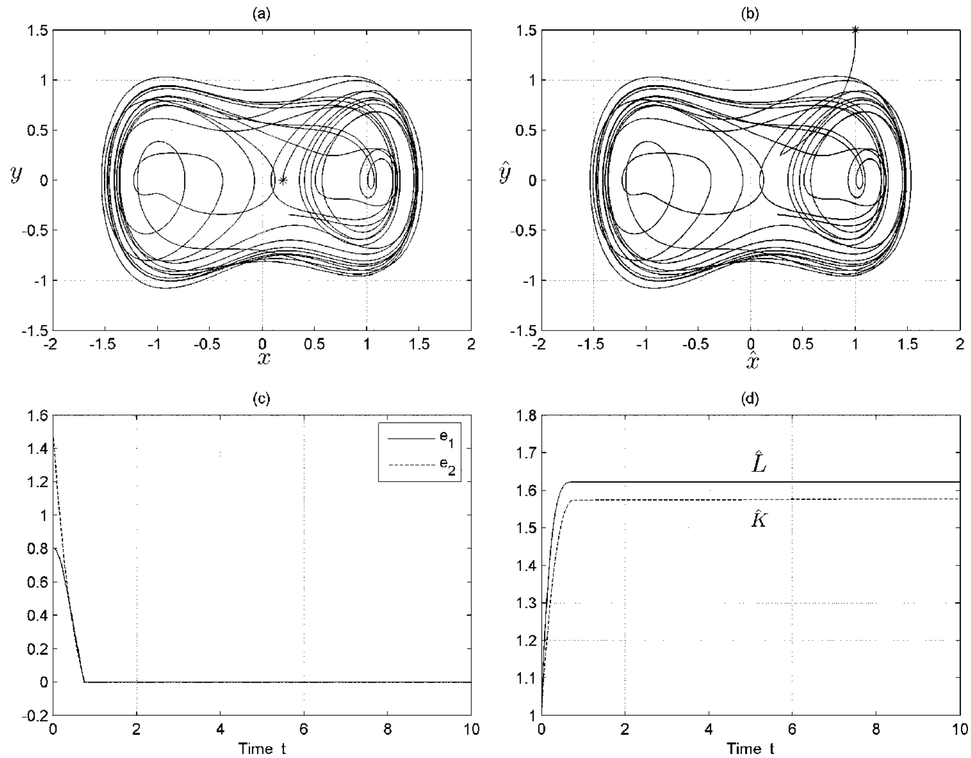

FIG. 1. Simulated results for the Duffing two-well oscillator with the random components $g_1 = g_2 = 0.001\Theta(r(t),10)$. The asterisk symbol denotes the initial point. The chaotic attractor of (a) the driving system and (b) the response system. Time histories of (c) the components of **e** and (d) the estimated $\hat{L}$ (solid line) and the estimated $\hat{K}$ (dashed line).

in Figs. 1 and 2. The chaotic attractors of the driving and response systems are shown in Figs. 1(a) and 1(b), respectively. Figure 1(c) shows that the components of **e** approach zero and the reaching time is about 0.75. Figure 1(d) shows the time histories of the estimated $\hat{L}$ and the estimated $\hat{K}$, and they converge to some K and L in the same reaching time of the components of **e**. The left-hand column in Fig. 2 reveals that the time histories of **u** oscillate in higher frequencies. To overcome this shortcoming, we change **u** to be a continuous $\mathbf{u}' = -(0.1+\hat{L})\mathbf{e} - \hat{K}\mathbf{e}/(\|\mathbf{e}\|+0.01)$. The simulated results of **e**, $\hat{L}$, and $\hat{K}$ are similar to those in Fig. 2 and are not shown. The time histories of $\mathbf{u}'$ in the right column of Fig. 2 are smooth in contrast to those in the left-hand column. The controller $\mathbf{u}'' = -(\delta+\hat{L})\mathbf{e} - \hat{K}\mathbf{sat}(\mathbf{e},\beta)$ can also eliminate the chattering phenomenon. The modified $\mathbf{u}'$ and $\mathbf{u}''$ just discussed are the ones mentioned in the remarks following Theorem 1. The modified controllers $\mathbf{u}'$ and $\mathbf{u}''$ in the remark following Theo-

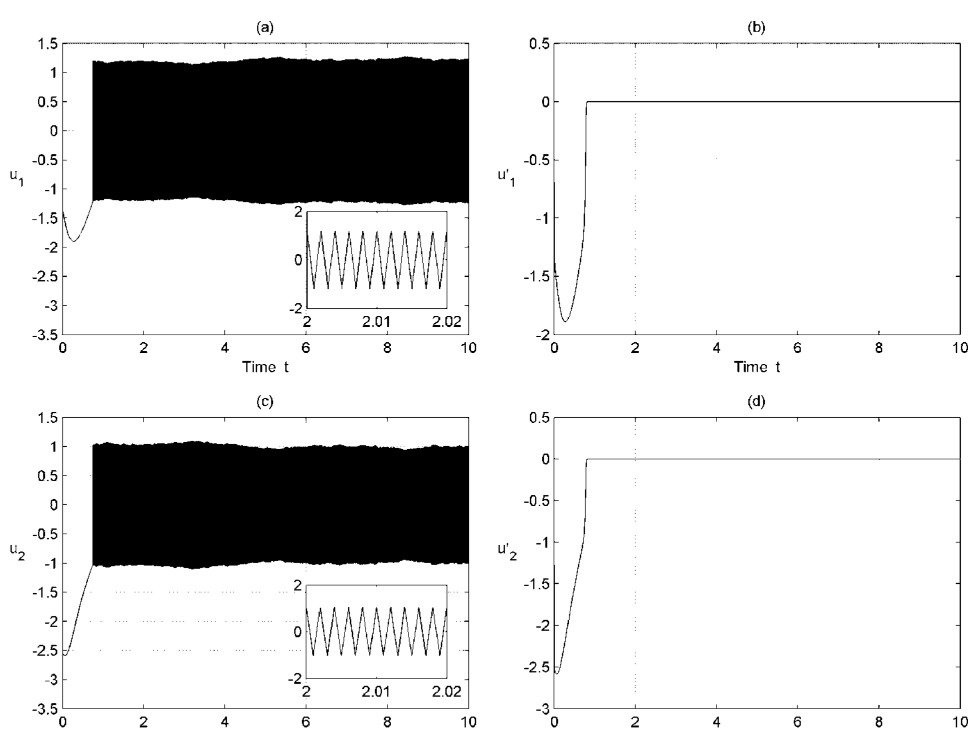

FIG. 2. Two different controllers are applied to the Duffing two-well oscillator with the same random components as in Fig. 1. (a) and (c) Two components of **u** oscillate in higher frequencies. (b) and (d) Two components of the modified controller $\mathbf{u}'$ are smooth.

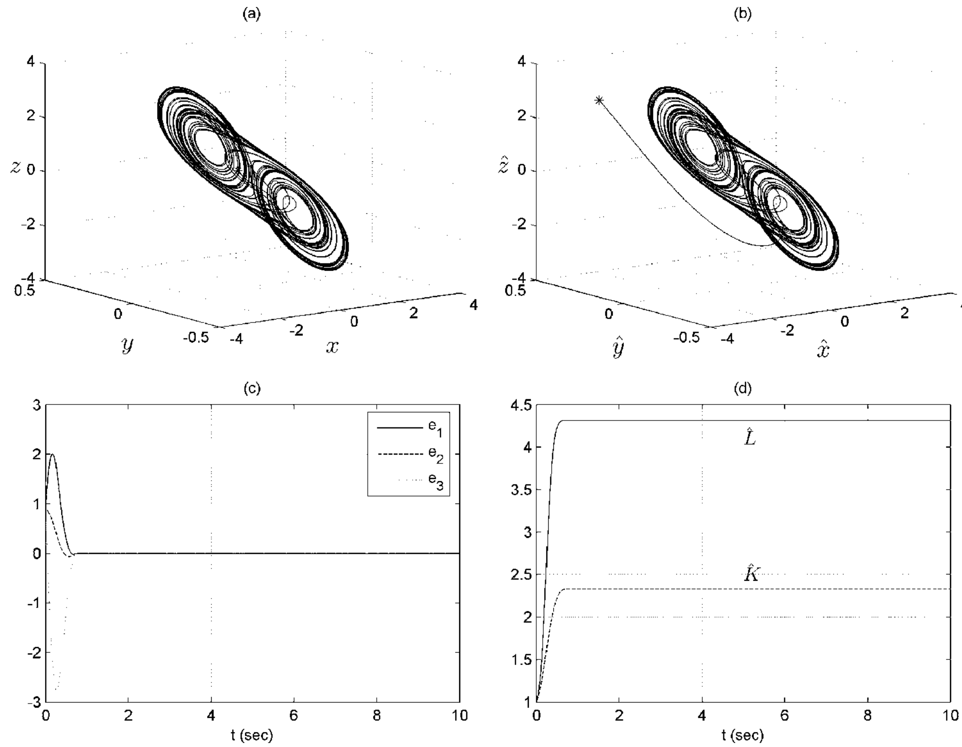

FIG. 3. Simulated results for the Chua circuit system with the random components $g_1=g_2=g_3=0.001\Theta(r(t),10)$. The asterisk symbol denotes the initial point. The chaotic attractor of (a) the driving system and (b) the response system. Time histories of (c) the three components of $\mathbf{e}$ and (d) the estimated $\hat{L}$ (solid line) and the estimated $\hat{K}$ (dashed line).

rem 2 apply equally well to eliminate the phenomenon of chattering.

(B) The Chua circuit system. The differential equations are

$$\dot{x}=c_1(y-x-h(x))+g_1,\quad \dot{y}=c_2(x-y+z)+g_2,$$

$$\dot{z}=-c_3y+g_3,\quad \dot{\hat{x}}=c_1(\hat{y}-\hat{x}-h(\hat{x}))+u_1,$$

$$\dot{\hat{y}}=c_2(\hat{x}-\hat{y}+\hat{z})+u_2,\quad \dot{\hat{z}}=-c_3\hat{y}+u_3,$$

where the piecewise linear function

$$h(x)=m_1x+0.5(m_0-m_1)(|x+1|-|x-1|)$$

represents three different voltage-current regimes of the diode. The parameters $c_1=15.6$, $c_2=1$, $c_3=25.58$, $m_0=-8/7$, and $m_1=-5/7$ with the initial conditions $\mathbf{x}_0=[\mathbf{x}_0^T\ \hat{\mathbf{x}}_0^T]^T=[0.2\ 0.2\ 0.2\ 1\ 1\ 1]^T$ ensure the existence of the chaotic attractor.

In this example, the random components $g_1=g_2=g_3=0.001\Theta(r(t),10)$ are added to the system. We choose the controller $\mathbf{u}'=-(0.1+\hat{L})\mathbf{e}-\hat{K}\mathbf{e}/(\|\mathbf{e}\|+0.01)$ with $\hat{L}_0=\hat{K}_0=1$ according to Theorem 1. The chaotic attractors of the driving and the response systems are shown in Figs. 3(a) and 3(b), respectively. Figure 3(c) shows that the components of $\mathbf{e}$ approach zero and the reaching time is about 0.71. Figure 3(d) shows the time histories of the estimated $\hat{L}$ and the estimated $\hat{K}$, and they converge to steady states in the same reaching time of the components of $\mathbf{e}$.

IV. CONCLUDING REMARKS

Synchronization of chaotic systems with random components was studied. The systems are assumed to satisfy a Lipschitz condition and the random components are uniformly bounded by a constant. However, the Lipschitz and the boundedness constants are not necessarily known *a priori*; they are estimated through an adaptive control process. The controlled system is recast to an equation for the error state, which is defined to be the difference between the state vectors of the system and the controlled system. Synchronization of the chaotic system is therefore equivalent to the achieving asymptotic stability of the zero error state.

In the present study, we propose two adaptive variable structure controllers to achieve synchronization by employing the partial stability theory to prove the asymptotic stability of the zero error state. In particular, it is shown that synchronization ensured by these controllers will occur within a finite time. Moreover, we have discussed how to modify the controller to eliminate the undesired phenomenon of chattering by introducing a sliding layer or a pseudosliding technique. Finally, one nonautonomous system—the Duffing's two-well oscillator and one autonomous system—the Chua circuit system were simulated to illustrate the effectiveness of the proposed controllers.

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