

# Dispersion of laser generated surface waves in an epoxy-bonded layered medium

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## Abstract

In this paper, the dispersion of laser generated surface wave in an epoxy bonded copper–aluminum layered specimen is studied. A laser ultrasonic experiment based on the point-source/point-receiver (PS/PR) technique was conducted to measure the surface wave signals in the layered specimen. The received wave signals were then processed in the frequency domain to obtain the dispersion relation of the fundamental surface wave mode. Theoretical calculations of the dispersion relations of the fundamental surface wave modes in two-layered and three-layered specimens were conducted to explore the influence of the bonding layer thickness on the dispersion relation. The experimental dispersion relation for the epoxy bonded copper–aluminum layered specimen is in good agreement with the calculated dispersion relation. The influence of the bonding layer thickness on the dispersion relation is studied and the potential application of the present results to the NDE of bonded layered media based on laser ultrasonics is also addressed.

**Keywords:** Laser ultrasonics; Layered medium; Bonding layer

## 1. Introduction

In recent years, laser ultrasonics has demonstrated its great potential in the Non-Destructive Evaluation (NDE) applications due to its noncontact feature in the measuring process and the ability of broadband signal generation [1]. An overview of state-of-the-art in laser ultrasonics can be found in a recent book by Scruby and Drain [2]. The laser generated ultrasonic waves has been applied to the investigation of Lamb wave propagation in thin plates [3–5]. In the studies, the measurements and discussions of the lowest-order symmetric and anti-symmetric Lamb waves generated by laser point source were given. In a paper by Veidt and Sachse [6], the laser point source was utilized to obtain the scan image of thin graphite/epoxy laminates and silicon wafer and further to recover the elastic constants of the materials. Since, the rise-time of the laser generated elastic wave can be made shorter enough to achieve the wave-front measurement to a certain accuracy, there are also applications of the laser ultrasonics to the

nondestructive evaluations of the elastic properties of materials. An early example of this application was demonstrated by Ledbetter and Moulder [7]. They measured the longitudinal, transverse and Rayleigh wave velocities in an isotropic material (aluminum). The laser generated ultrasonic bulk waves has also been applied to the determination of elastic constants of anisotropic materials by Castagnede et al. [8] with good accuracy. Recently, a study on the propagation of laser generated surface waves in anisotropic solids was given by Wu and Chai [9] while its application to the determination of anisotropic elastic constants from the group velocity measurement was given by Chai and Wu [10]. In the above wave velocity or elastic constant measurements, piezoelectric detectors were used for the investigations.

The propagation of elastic waves in a layered medium has been a problem of interest in both the fields of NDE of materials and geophysics. Hirao et al. [11] studied the propagation of Rayleigh surface wave in a solid with cold-worked surface layer. Kim and Achenbach [12] utilized the line focus acoustic microscopy to measure the dispersion of surface waves in thin film coated materials and determined inversely their elastic properties.

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In the field of geophysics, propagation of elastic wave in a stratified layer has been utilized to either map out the interior structure of the earth [13] or determine the medium properties of the earth layer [14].

In this paper, we studied the dispersion of laser generated surface waves in an epoxy bonded copper–aluminum layered specimen. Laser ultrasonic experiment based on the point-source/point-receiver [14] (PS/PR) technique was conducted to measure surface wave signals in the layered specimen. A Nd:YAG laser was utilized as a point source and the elastic wave signals were received through the utilization of a PZT transducer with a small acting area. The received wave signals were then processed in the frequency domain to obtain the dispersion relation of the fundamental surface wave mode. The influence of thickness of the bonding layer on the dispersion relation is studied and the potential application of the present results to the NDE of layered media based on laser ultrasonics is also addressed.

## 2. Measurement of laser generated dispersive surface waves

The ultrasonic point-source/point-receiver (PS/PR) technique has demonstrated as a convenient method for material characterization applications [6,10,14]. The system and operational characteristics of the PS/PR technique have been reviewed in a paper by Sachse [15]. The elastic waves generated by a point source consist of longitudinal, transverse and surface waves (if the receiver is on the same surface as the point source) which propagate in all directions. The elastic wave signals were detected by a small aperture sensor (or array of sensors) located on the surface of the specimen. It is obvious that the wave signals generated by a point source and those recorded are much more complicated than those of the conventional plane wave assumed source. However, under proper signal processing, the former wave signals (point source) may extract more information about the structure under test.

In the present work, an experimental study was made of propagation of generalized Rayleigh surface waves in an aluminum block with a copper surface layer. The dimensions of the aluminum block is  $12.5 \times 14.7 \times 9.8$  cm, while the thickness of the copper layer is 0.78 mm. The density of the aluminum specimen is  $2698 \text{ kg/m}^3$  and that of the copper is  $8500 \text{ kg/m}^3$ . The copper layer was bonded carefully on the aluminum block with epoxy glue. The longitudinal and shear wave velocities of the copper and aluminum specimens are measured by the ultrasonic pulse-echo method and the results are

aluminum:  $C_L = 6399.7 \text{ m/s}$ ,  $C_T = 3130.4 \text{ m/s}$ ,

copper:  $C_L = 4899.6 \text{ m/s}$ ,  $C_T = 2344.1 \text{ m/s}$ .

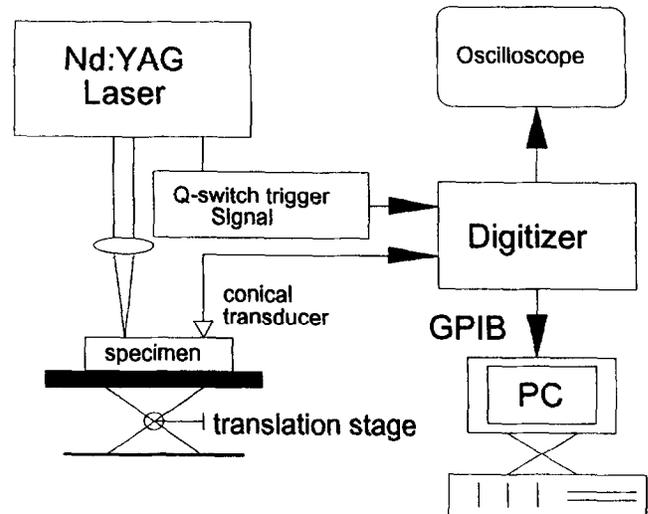


Fig. 1. Experimental set-up of the laser ultrasonic experiment.

Shown in Fig. 1 is the experimental set-up utilized in the present study. A Nd:YAG pulsed laser (Quanta-Ray, GCR-130) (wavelength 532 nm) was utilized to generate elastic waves in the layered specimen. The duration of the laser pulse utilized was 10 ns and the energy carried was about 100 mJ. As shown in Fig. 1, the layered specimen was rest on a precision translation stage to accurately control the distance between the source and the receiver. The generated elastic wave signals from the laser sources were measured by a NBS conical transducer [16]. The received voltage signals from the conical transducer were then pre-amplified by a preamplifier and recorded by a 100 MHz digital oscilloscope (LeCroy 9314L). A trigger signal synchronized with the laser source was utilized to trigger the digital oscilloscope. The recorded signals were then sent to a personal computer via GPIB.

For a point source acting on a layered half space, it is known that the wave signals received at two different receivers ( $R_1$  and  $R_2$  in Fig. 2) are different in both the amplitudes and phases due to the wave dispersion in the layered structure. Since the pulsed laser utilized in this study generates stable point sources, instead of using one point source and two receivers, one point receiver and two point sources were utilized. The advantage of the aforementioned alternative is that only one receiver is needed in the experiment, further, the precision of the change of the source to receiver distance can be controlled by a precise translation stage.

Shown in Fig. 3 is a typical wave signal (vertical displacement) received on the surface of the copper–aluminum specimen. The distance between the laser point source and receiver is 50 mm and the vertical axis represents the relative amplitude of the displacement signal. Fig. 4 shows a similar wave signal with source to receiver distance equal to 55 mm. From Figs. 3 and 4, we found that, in spite of the small source to receiver

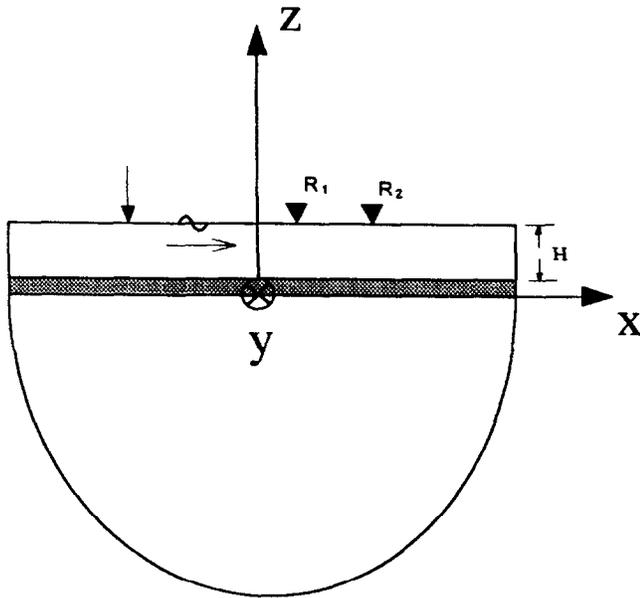


Fig. 2. Coordinates and geometry of the layered half space.

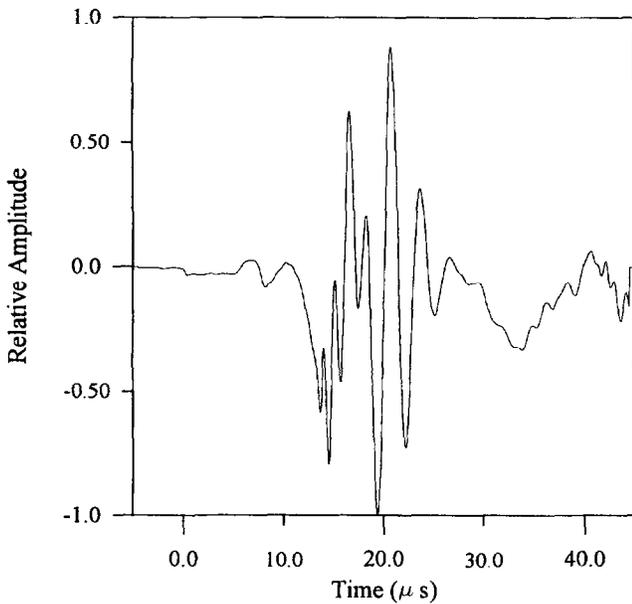


Fig. 3. The time domain wave signal recorded on the surface of a copper-aluminum specimen. The source to receiver distance is 50 mm.

distance change 5 mm, the phases of these two wave signals can be observed easily.

### 3. Spectral analysis of surface waves

The characteristics of elastic waves generated by a point source at the surface of a layered medium is complex. From the time domain signals it is difficult (if possible) to extract information regarding to the medium properties or layer structures. Further, the propagation of elastic waves in a layered medium is dispersive, i.e.

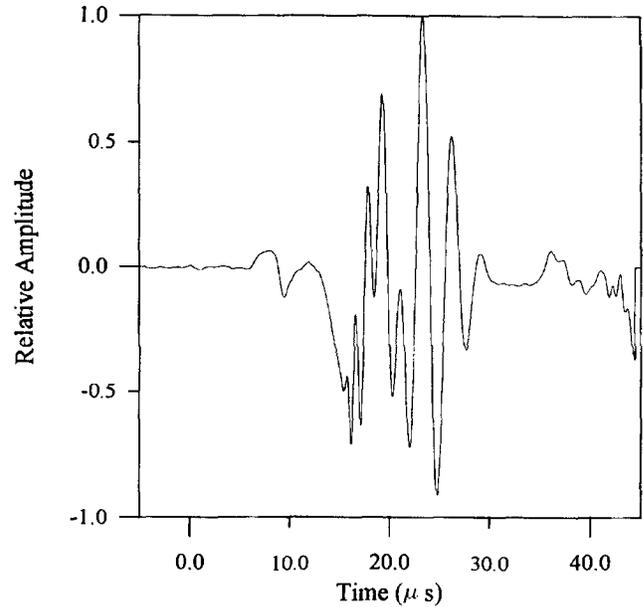


Fig. 4. The time domain wave signal recorded on the surface of a copper-aluminum specimen. The source to receiver distance is 55 mm.

harmonic waves with different frequency or wavelength propagate with different velocity. Therefore, an analysis or measurement of the dispersion curve of a layered medium is necessary to probe more information about a layered medium. In the field of geotechnical engineering, the spectral analysis of surface waves has been utilized to determine the velocity profiles of soil sites and stiffness profiles of pavement systems [14]. In the study, it is realized that for the case of a point source applied at the surface of a layered medium, the energy of the signals received at the same surface is mostly associated with the fundamental mode of the generalized surface wave.

In this paper, we adopted the spectral analysis method utilized in Ref. [14] to obtain the experimental dispersion curve of the fundamental mode of the generalized surface waves in the epoxy bonded layered specimen. Consider the fundamental mode of a generalized surface wave in a multi-layered medium, the displacement  $u$  can be expressed in terms of a combination of infinite numbers of harmonic waves as

$$u(x, t) = \sum_{j=1}^{\infty} C_j \exp[i(\omega_j t - k_j x)], \quad (1)$$

where  $\omega_j$  is the circular frequency,  $k_j$  is the wave number and  $C_j$  is the amplitude of the waves.

On utilizing the orthogonality condition, the Fourier transform of  $u(x, t)$  evaluated at a particular frequency  $\omega_j$  can be expressed as

$$\hat{u}(x, \omega_j) = C_j \exp(-ik_j x). \quad (2)$$

The Fourier transform of the wave responses recorded at two different positions  $x = x_1$  and  $x = x_2$  evaluated at

frequency  $\omega_j$  have the forms

$$\begin{aligned} \hat{u}_1 &= \hat{u}(x_1, \omega_j) = C_j \exp(-ikx_1), \\ \hat{u}_2 &= \hat{u}(x_2, \omega_j) = C_j \exp(-ikx_2). \end{aligned} \quad (3)$$

From Eqs. (3), it is obvious that the phase difference  $\phi$  for the waves with frequency  $\omega_j$ , received at positions  $x_1$  and  $x_2$  is

$$\phi = k(x_1 - x_2). \quad (4)$$

The phase difference of two wave signals received at two different positions is a function of frequency and can be obtained from the frequency spectrums of the wave signals received at  $x_1$  and  $x_2$  according to the following operation

$$S_{x_1, x_2}(f) = \frac{1}{n} \sum_{i=1}^n \{ [R_1(f)]_i \cdot [R_2^*(f)]_i \}, \quad (5)$$

where  $f$  is the frequency of the harmonic wave and  $R_2^*$  denotes the complex conjugate of  $R_2$ , while  $n$  is the number of times the experiments are repeated. The phase difference  $\phi$  as a function of frequency is thus equal to the phase angle of the complex function  $S_{x_1, x_2}$ . The reliability of the measured signals is related to the coherence function  $r^2(f)$  defined as

$$r^2(f) = \frac{|S_{x_1, x_2}|^2}{A_{x_1}(f) \cdot A_{x_2}(f)}, \quad (6)$$

where

$$A_{x_1}(f) = \frac{1}{n} \sum_{i=1}^n \{ R_1(f) \cdot R_1^*(f) \} \quad (7)$$

and  $A_{x_2}$  has a similar expression. If the recorded signals are reliable, the value  $r^2$  will approach unity.

On knowing the phase difference  $\phi$  as a function of frequency and the distance  $x_1 - x_2$ , the phase velocity  $v$  can be obtained from the relation

$$v = 2\pi f \frac{x_1 - x_2}{\phi}. \quad (8)$$

#### 4. Calculations of surface wave dispersions in layered media

The propagation of surface waves in an isotropic layered half space have been studied by many researchers and the associated references can be found in the book by Ewing et al. [17]. In this paper, a general purpose computer program [18] for the calculations of the dispersion curves of isotropic as well as anisotropic multi-layered media was utilized to calculate the dispersion relations of the epoxy bonded layered specimen. The coordinates and geometry of the layered specimen are shown in Fig. 2.

#### 5. Experimental results

From the formulae shown in the section of spectral analysis of surface waves, the time domain wave signals shown in Figs. 3 and 4 can further be processed to obtain the experimental dispersion curve of surface wave propagating in the layered specimen. Shown in Figs. 5 ( $R_1(f)$ ) and 6 ( $R_2(f)$ ) are the Fourier spectrum of the time domain wave signals shown in Figs. 3 and 4. The highest frequency of the recorded surface waves signals is approximately 3 MHz. In the present study, experiments with the same configuration were repeated three times and therefore the  $n$  in Eq. (5) is equal to 3. With

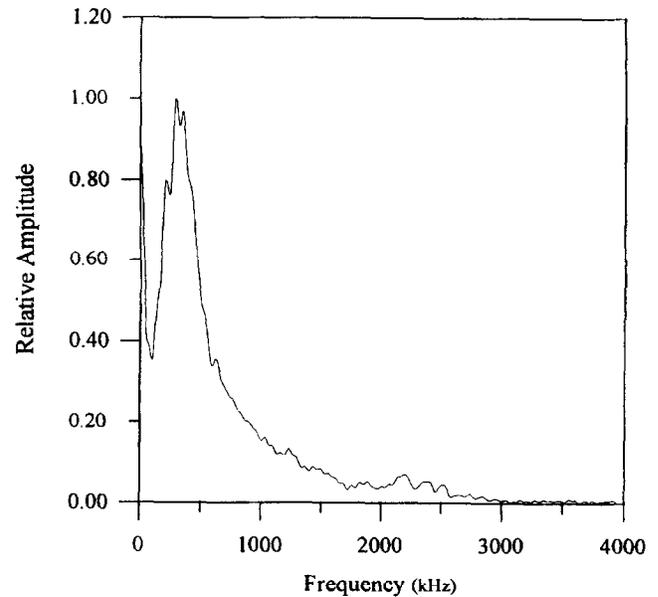


Fig. 5. The Fourier spectrum of the wave signal shown in Fig. 3.

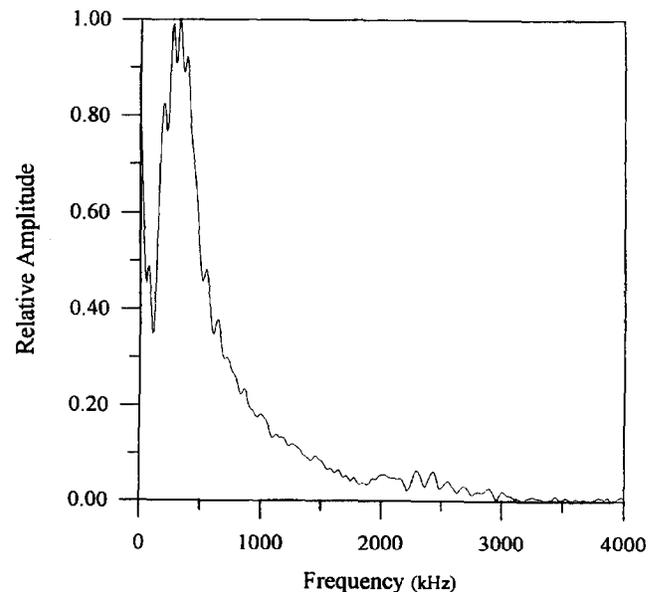


Fig. 6. The Fourier spectrum of the wave signal shown in Fig. 4.

$R_1(f)$  and the complex conjugate of  $R_2(f)$  known, the function  $S_{x_1, x_2}(f)$  in Eq. (5) can be calculated and the phase angle of  $S_{x_1, x_2}(f)$  is shown in Fig. 7.

Fig. 8 shows the coherence function calculated through the utilization of Eq. (6). As can be found from Fig. 8, the wave signals utilized in the present study are reliable with frequency up to 2.5 MHz, since the coherence values in this region are close to unity. Fig. 9 shows the integration result of Fig. 7 for the continuation of the phase difference. With the phase difference  $\phi$  as a function of frequency known, the dependence of the phase velocity of the surface waves on the nondimensionalized wave number  $k_x H$  can be obtained easily from Eq. (8).

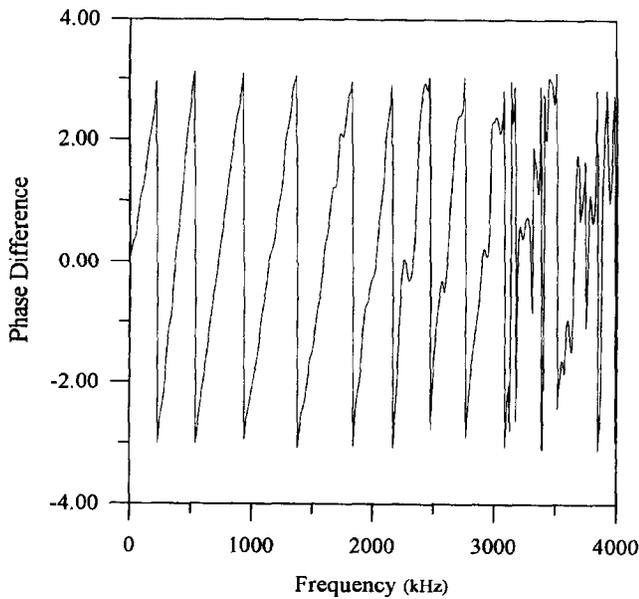


Fig. 7. The phase angle of the complex function  $S_{x_1, x_2}$ .

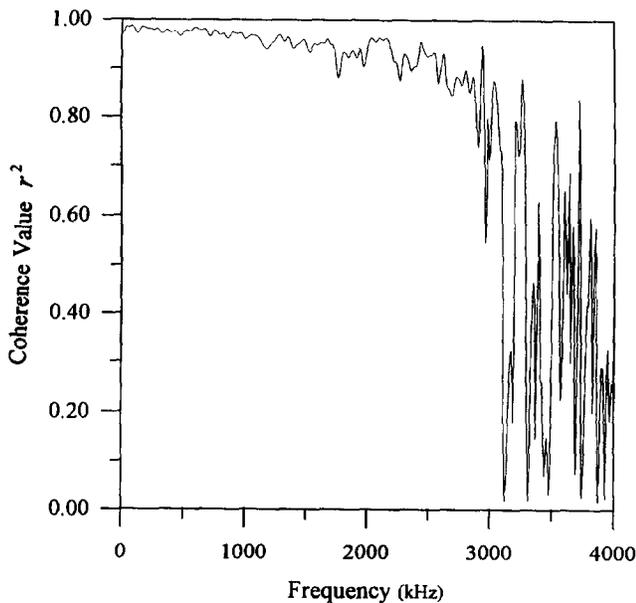


Fig. 8. The coherence function of the waves recorded.

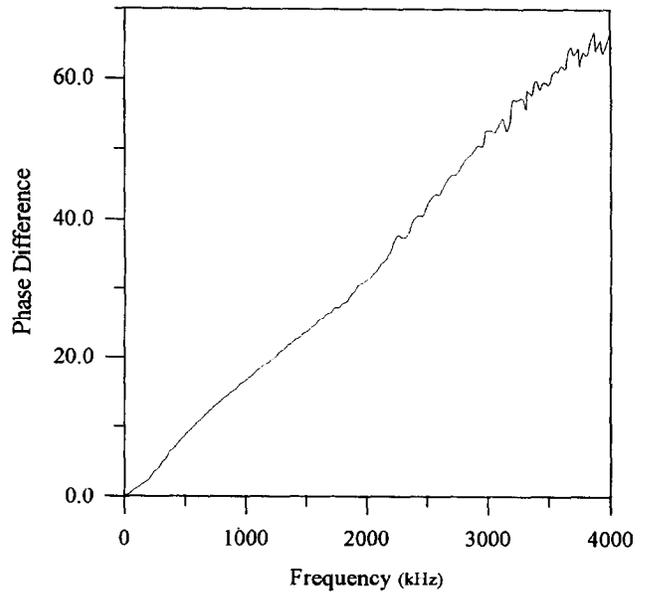


Fig. 9. The continue phase difference as a function of frequency.

The open circles shown in Fig. 10 are the dispersion relation obtained from the above mentioned procedures. The  $H$  in the horizontal axis of Fig. 10 is the thickness of the copper layer (0.78 mm).

We note that the measured phase velocities for  $k_x H$  lower than about 0.3 are scattered especially for those with  $k_x H$  approaching zero. It is easy to understand that the accuracy of the phase velocities of the low frequency signals is dependent on the spacing of the receivers. For example, in the present case, the spacing of the receivers (laser sources) is 5 mm, while the associated wavelength of  $k_x H = 0.3$  is about 16 mm. It

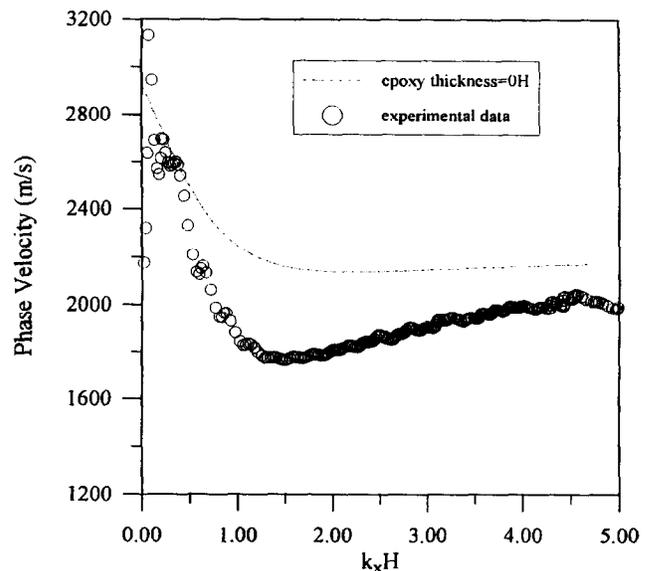


Fig. 10. The dispersion of the phase velocity of the surface waves (a) experimental results - open circles (b) calculated results (neglecting bonding layer) - dotted line.

is obvious that the spacing of the receiver spacing is too small to probe the wave signal with wavelength three times larger than the spacing (5 mm). To increase the accuracy of the phase velocity measurement at low frequency, the spacing of the receivers should be increased.

The dotted line in Fig. 10 is the theoretical dispersion relation of the fundamental surface waves in a copper–aluminum half space without considering the thickness of the bonding epoxy layer. The calculation was based on the general purpose computer program for surface wave dispersion in a multi-layered medium described in the previous section. As shown in the curve, the phase velocity of the fundamental surface wave mode in the copper–aluminum layered half space approaches to the Rayleigh surface wave velocity of aluminum as the wavenumber approaches zero. On the other hand, the phase velocity of the fundamental surface wave mode approaches to the Rayleigh surface wave velocity of copper as the wave number is large.

From Fig. 10, it is noted that there is a considerable difference between the calculated and the measured dispersion curves. As will be demonstrated later, the main reason for the inconsistency of the measured and the calculated dispersion curves is solely due to the existence of a finite bonding layer.

## 6. Discussion of finite bonding layer thickness

In an epoxy-bonded layered medium, the thickness of the bonding layer is often neglected provided that the bonding thickness is small compared with the thickness of the layers. Further, since the elastic properties of the epoxy is quite different from metallic materials, such as aluminum, copper, etc., it is believed that the phase and therefore the group velocities of the generalized surface waves are also dependent on the thickness of the epoxy bonding layer. To demonstrate the influence of the bonding thickness on the dispersion relation of a layered specimen, a series of theoretical dispersion relations were calculated for cases with different bonding thickness.

Shown in Fig. 11 is the calculated dispersion relations of the fundamental surface waves in a three-layered half space (copper–epoxy–aluminum half space). The five curves in Fig. 11 represent the dispersion relations with the thickness of the epoxy layer varied from 0 to  $0.3H$ . From the calculated results, it is confirmed that the thickness of the epoxy layer play an important role on the dispersion of the fundamental surface waves in such a bonded layered half space. We found that the thicker the epoxy bonding layer, the deeper the dip in the dispersion curve, i.e. the phase velocity of the fundamental surface wave mode decreases from the Rayleigh surface wave velocity of aluminum to a local minimum and then increases to that of copper. Furthermore, it is found that

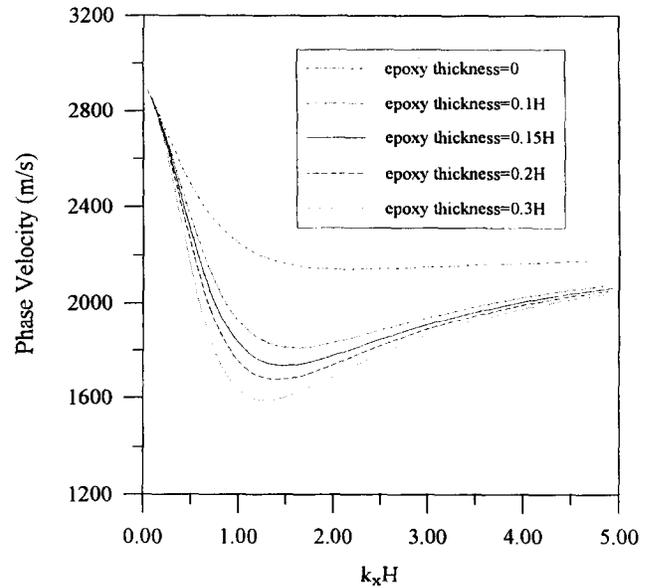


Fig. 11. The dispersion of the phase velocity of the surface waves with bonding layer thickness from 0 to  $0.3H$ .

even for a small epoxy thickness change (for example, from  $0.15H$  to  $0.2H$  in Fig. 11), there is a considerable change in the dispersion curve of the fundamental mode of surface wave. The phase velocity difference can be as large as about 200 m/s in this case. It is worth noting that the dip in the surface wave dispersion of a bonded layered half space provides a qualitative (and quantitative, if an inversion algorithm is implemented) indication of the thickness of the bonding layer.

From the theoretical calculations of the dispersion relations of layered media with finite bonding thickness shown in Fig. 11 and the measured dispersion relation shown in Fig. 10, it is found that the bonding thickness of the current example is approximately equal to  $0.15H$ , where  $H$  is the thickness of the copper layer, i.e. 0.78 mm. Shown in Fig. 12 is the comparisons between the calculated (with  $0.15H$  bonding thickness) and the measured dispersion curves. The results show that they are in very good agreements for  $k_x H$  lies between 0.4 and 4.

To further verify the present conclusion, the thickness of the epoxy layer in the copper–aluminum layered half space was examined using an optical microscope from the side-view of the specimen. The thickness was determined to be about 0.12 mm which is about 0.15 times of the thickness of the copper layer (0.78 mm).

## 7. Conclusions

The dispersion of laser generated surface waves in a epoxy bonded copper–aluminum layered half space has been investigated experimentally. Laser ultrasonic experiment based on the PS/PR technique was conducted to measure the surface wave signals in a layered

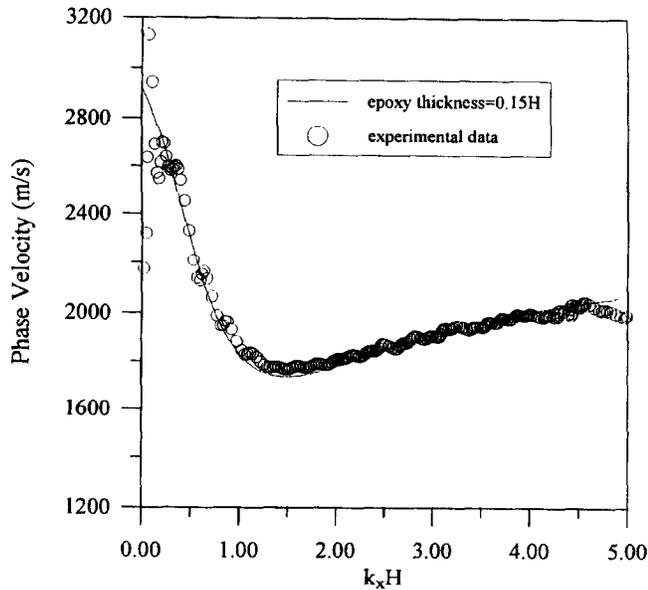


Fig. 12. Comparisons between the measured (open circles) and calculated (solid line) dispersion of the surface wave phase velocity.

specimen. Spectral analyses of the measured surface waves were performed to obtain the dispersion relation of the fundamental surface wave mode. From the calculated and the measured dispersion of surface waves in the copper–aluminum layered specimen, it is found that the thickness of the epoxy layer can not be neglected. On considering the epoxy layer, the calculated and the measured dispersion relations of the fundamental surface wave mode of the copper–aluminum layered specimen are in good agreement. The present study shows that the existence of a soft layer in a layered medium do give a significant influence on the dispersion relation of the generalized surface wave. Finally, we note that the present finding show the feasibility of evaluating the

thickness of a bonding layer or the elastic properties of the layers, if a proper inverse algorithm is invoked [10].

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