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半導體供應往路決策促成技術研究子計畫三 – 半導體生產網路中之需求規劃策略（2/2）

Demand Planning Strategies for Semiconductor Manufacturing Networks

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中文摘要

半導體代工製造網路是由 IC 設計公司或 IDM 公司、代工晶圓廠、電訊測試、組裝廠及測試廠所組成，需求規劃在整個製造網路的規劃中為一極關鍵的步驟，其規劃結果為每一個網路中規劃活動的基礎且決定整個網路規劃決策的品質。然而，需求資訊卻是在整個規劃過程中最不可靠的資訊，尤其是經過供應鏈層層傳遞後，需求資訊時常因此變得更不穩定，這種不確定的需求資訊為傷害製造網路規劃決策品質的主要因素。在本研究的第一年，我們已建構一完整的需求規劃組織架構，此架構包含下列四種必要的基礎功能:(1)多元需求規劃策略(2)統計需求預測(3)需求規劃與預測之複合與解析(4)製造網路間的需求協調規劃。在本研究的第二年中，我們發展出提高需求規劃正確性與效率之方法。將需求從不同的觀點來做策略規劃，並從幾種不同的觀點中做必要的複合與解析，我們亦從更多的考慮面發展出適合用於供應鏈規劃的統計需求預測方法。

關鍵詞：需求規劃、需求預測、供應鏈規劃

Abstract

Semiconductor manufacturing network consists of IC design houses/IDM, foundry fabs, probing, assembly, and final test processes. Demand planning is the very first critical task for the planning of the entire manufacturing network. Its result serves as the basis of every planning activity and ultimately determines the quality of the planning decisions and thus the efficiency of operations in the network. Nevertheless, the demand information propagated through the network is the most uncertain information that plagues the planning quality. In the first year of this research, we have constructed a complete framework of demand planning. The framework consists of multidimensional demand planning, statistical demand forecast, demands aggregation/granularity, and synchronization of demand signals in the network. In the second year, multidimensional planning strategies are proposed to better handle the complicated supply-demand relationships in the network. Statistical forecast techniques with aggregation/granularity considerations are also developed accordiingly.

Keywords：demand planning, demand forecast, supply chain planning (SCP)

1. Introduction

Demand planning is the very first step of supply chain planning. Its results affect the quality of its subsequent planning activities. Yet, the demand signal is known to be the most unreliable information in supply chain planning. The demand uncertainty is then propagated and further magnified (Lee et al., 1997) in the supply chain. That is, the further down the supply chain level, the worse the planning quality. To improve the quality of supply chain planning, demand planning becomes one of greatest challenges facing modern manufacturers.

It is known that demand uncertainty can be effectively reduced through appropriate demand aggregation (Simchi-Levi et al., 2000) and forecasting. An On-Line Analytical Processing (OLAP) tool is thus useful for analysis of multi-perspective (multi-dimensional) demand aggregation and forecasting. Demand planners can use the tool to quickly roll up demands to an aggregated level for a total demand or drill down a total demand to detailed demands from different perspectives. For example, a semiconductor demand planner can roll up (or aggregate) the detailed demand to calculate the total demand for logic IC in North America and Europe during the first during the last two quarters of the year. The demand planner may find such an aggregated demand is less fluctuated and more suitable for demand forecasting and supply chain planning. The demand planner can also drill down (or disaggregate) the total demand to see, for example, the proportion of the North American market. There are three perspectives (dimensions) of demands: time, product type, and region. To better understand the natures of certain demands, users of OLAP tools can choose desired perspectives to perform the roll-up and/or drill-down analyses. Such analyses are also referred to as “slice-and-dice” analyses. However, the demand planners have to rely on their own understanding of the market or simply their intuitive, subjective judgment to perform the aggregation analysis. Following demand aggregation/disaggregation, demand forecasting is the next step of demand planning also noted as an important means to improve the accuracy of demand plans. However, the effect of statistical forecasting is obscure and planners are hesitant to use the pre-determined statistical models because the flawed models often incur more errors and cause poorer forecasts.

This paper will use the bivariate vector autoregression (VAR(1)) time series model as a study vehicle to investigate the effects of aggregating two interrelated demands. Performance of corresponding forecasting approaches will be then derived and evaluated. The goal of this paper is to use certain statistical properties of the demands to develop principles that can assist the demand planners to determine whether demand aggregation and/or statistical forecasting are needed. This paper is organized into five sections. Following the introduction section, we first briefly describe the VAR(1) demand model and five demand planning approaches. The performance of the five approaches is then analytical derived in Section 3. Section 4 will use eight scenarios to evaluate and compare the performances among different approaches. Finally, principles and guidelines will be provided to practitioners for adopting appropriate aggregation/forecasting approaches.

2. VAR(1) Demand Model

In practice, most time-variant demands are observed to follow autoregression time-series models. Particularly, the first order autoregression, AR(1), model is widely applied in both practice and literature (Lee et al., 1997). Since the interrelation of demands is the focal point of our research, the first order bivariate vector autoregression, VAR(1), time series model is chosen as a study vehicle. Bivariate VAR(1) demands can be denoted as a vector: $\underline{X}_t = [X_{1t}, X_{2t}]'$ and the VAR(1) model can be expressed as:

$$\underline{X}_t = \underline{u} + \underline{\Phi}\underline{X}_{t-1} + \underline{a}_t \qquad (1)$$

where

$\underline{u} = [u_{x1}, u_{x2}]'$ is the constant vector;

$\underline{\Phi} = \begin{bmatrix} \phi_{11} & \phi_{12} \\ \phi_{21} & \phi_{22} \end{bmatrix}$ is the autoregression parameter matrix;

and $\underline{a}_t = [a_{1t}, a_{2t}]'$ is the white noise vector following iid bivariate normal distribution:

$$N_2 = (\underline{0}, \Sigma) \quad \text{with} \quad \underline{0} = [0,0]' \ \cdot \ \Sigma = \begin{bmatrix} \sigma_{11} & 0 \\ 0 & \sigma_{22} \end{bmatrix}.$$

In the VAR(1) model, ϕ_{11} and ϕ_{22} represent the "auto-correlation elements" that dictate how much a demand depends on its own earlier demands; ϕ_{12} and ϕ_{21} represent the "inter-correlation elements" that determine how the two demands correlate to each other.

It can be seen that the bivariate VAR(1) can clearly describe the interrelation of two autoregressive time series:

(1) When both the two interrelation elements of $\underline{\Phi}$, ϕ_{12} and ϕ_{21}, equal to zero, the two time series are independent and can be in effect expressed as two separate AR(1) time series models with autoregression parameters ϕ_{11} and ϕ_{22}, respectively;

(2) When only one of ϕ_{12} and ϕ_{21} is zero, the relation will be uni-directional. That is, if ϕ_{12}=0 and $\phi_{21} \neq 0$, then X_{1t} is a univariate AR(1) while X_{2t} will be affected by X_{1t-1};

(3) When both ϕ_{12} and ϕ_{21} are not zero, the two time series are interrelated; and

(4) When all the elements of $\underline{\Phi}$, ϕ_{11}, ϕ_{12}, ϕ_{21} and ϕ_{22}, are statistically insignificant, the two time series will appear as two time-invariant data sequences.

The demand plans serve as basis of inventory planning and/or capacity planning. Safety stock and/or auxiliary capacity are prepared to minimize the effect of demand uncertainty. For instance, under the (s, S) inventory policy, the reorder point s is set based on a safety stock level that meets a predetermined service level (Caplin, 1985, and Silver et al., 1998):

$$s = L \times AVG + z \times STD \times \sqrt{L}$$

where

AVG is the average unit time demand;

STD is the standard deviation of unit time demand;

L is the replenishment lead time; and

$z \times STD \times \sqrt{L}$ is the safety stock prepared to meet a desired service level under demand uncertainty.

It can be seen that the inventory cost under the (s, S) policy increases as the demand uncertainty, i.e. STD, grows. Similarly, the capacity cost rises owing to the preparation of auxiliary capacity for uncertain demand. However, demands are not aggregated without an additional planning cost. In the inventory problem, the replenishment lead-time is often prolonged because the inventory planned for the aggregated demand is stocked in a centralized warehouse and the transportation distance to the demand sources is thus extended. For the capacity planning problem, aggregating demands often means a higher product mix on the shop floor and causes possibly lower yield, more machine change-over time and higher machine breakdown rate.

3. Demand Planning Approaches

In this research, we investigate five possible demand planning approaches in response to the bivariate VAR(1) demands:

(1) Approach 1: The manufacturer lacks the technology of statistical forecasting. Demands are handled as simple time-invariant data sequences. The demand variability is measured by the standard deviation. The safety stock (or production capacity) is planned separately for each demand based on a multiple of its standard deviation.

(2) Approach 2: The manufacturer aggregates the two demands together. The aggregated demand, denoted by Y_t (=X_{1t}+X_{2t}), is handled as a time-invariant data sequence. The safety stock (or production capacity) is planned for two demands together based on a multiple of Y_t's standard deviation.

(3) Approach 3: The manufacturer owns the statistical forecasting technology but lacks knowledge of multivariate time series. Demands are handled as two independent time series. AR(1) time series models are used as the statistical forecasting models. Statistical forecasting is carried out separately based on the

estimated AR(1) time series model for each demand. The safety stock (or production capacity) is planned separately for each demand based on a multiple of its forecast standard error.

(4) Approach 4: The manufacturer aggregates the two demands together. The aggregated demand is handled as an AR(1) time series. The safety stock (or production capacity) is planned for two demands together based on a multiple of the Y_t's forecast standard error.

(5) Approach 5: The manufacturer owns the technology of forecasting multivariate time series. Statistical forecasting is based on the VAR(1) model. Safety stock (and/or production capacity) is planned separately for each demand based on a multiple of its forecast standard error.

4. Performance Analysis of Demand Planning Approaches

To analyze the performance of Approaches 1 and 2, the variances of individual demands, denoted as $_{x1x1}$ and $_{x2x2}$ respectively for variances of demands X_{1t} and X_{2t}, and the covariance, $_{x1x2}$, has to be first derived under the VAR(1) time series model. Without loss of generality, we assume that the constants are zero, i.e. $\underline{u} = \underline{0}$. Model (1) becomes:

$$\underline{X}_t = \underline{\Phi}\underline{X}_{t-1} + \underline{a}_t . \tag{2}$$

It is then straightforward to derive $_{x1x1}$, $_{x2x2}$ and $_{x1x2}$. Therefore, under Approach 1, the safety stock (or production capacity) will be prepared for each demand separately. The total safety stock will be then based on a multiple of $_{x1}+{}_{x2}$ ($=\sqrt{\pm_{x1x1}}+\sqrt{\pm_{x2x2}}$). Similarly, under Approach 2, the safety stock (or production capacity) will be prepared based on a multiple of the aggregated standard deviation, denoted by $_{y}$:

$$\pm_y = \sqrt{\pm_{x1x1} + \pm_{x2x2} + 2\pm_{x1x2}} \tag{3}$$

To derive the forecast standard error under Approach 3, we first derive the following theorem.

Theorem 1:
If X_{1t} and X_{2t} follow VAR(1) model in (2), then X_{1t} can be expressed as $V_{1t}+V_{2t}$ where V_{1t} and V_{2t} are two AR(1) time series:

$$V_{1t} = {}^{a}{}_{1} V_{1t-1} + a^{v}_{1t} \qquad and$$

$$a^{v}_{1t} = \frac{e_{11}e_{22}}{e_{11}e_{22} - e_{12}e_{21}} a_{1t} - \frac{e_{11}e_{21}}{e_{11}e_{22} - e_{12}e_{21}} a_{2t}$$

$$V_{2t} = {}^{a}{}_{2} V_{2t-1} + a^{v}_{2t} \qquad and$$

$$a^{v}_{2t} = \frac{-e_{21}e_{12}}{e_{11}e_{22} - e_{12}e_{21}} a_{1t} + \frac{e_{21}e_{11}}{e_{11}e_{22} - e_{12}e_{21}} a_{2t}$$

Similarly, X_{2t} can be expressed as $W_{1t}+W_{2t}$ where W_{1t} ▪ W_{2t} are two AR(1) time series:

$$W_{1t} = {}^{a}{}_{1} W_{1t-1} + a^{w}_{1t} \qquad and$$

$$a^{w}_{1t} = \frac{e_{12}e_{22}}{e_{11}e_{22} - e_{12}e_{21}} a_{1t} - \frac{e_{12}e_{21}}{e_{11}e_{22} - e_{12}e_{21}} a_{2t}$$

$$W_{2t} = {}^{a}{}_{2} W_{2t-1} + a^{w}_{2t} \qquad and$$

$$a^{w}_{2t} = \frac{-e_{22}e_{12}}{e_{11}e_{22} - e_{12}e_{21}} a_{1t} + \frac{e_{22}e_{11}}{e_{11}e_{22} - e_{12}e_{21}} a_{2t}$$

In above Equations, $_1$ and $_2$ are two eigenvalues of $\underline{\Phi}$ with corresponding eigenvectors:

$$\underline{e}_1 = \begin{bmatrix} e_{11} \\ e_{12} \end{bmatrix} \quad and \quad \underline{e}_2 = \begin{bmatrix} e_{21} \\ e_{22} \end{bmatrix}$$

The result of Theorem 1 simplifies the bivariate time series into two individual time series. In addition, each time series can be expressed as a sum of two AR(1) time series. Granger and Morris (1979) have proved that the sum of two ARMA time series will be also an ARMA time series. For demand planning practice, AR(1), with or without model identification, is usually used as the demand time series model. Theorem 2 is, thus, derived to estimate the forecast standard error when the sum of two AR(1) time series is estimated as an AR(1) time series.

Theorem 2:
D_{1t} and D_{2t} are two stationary AR(1) time series:

$$D_{1t} = u_{D1} + {}^{\ddot{}}{}_{1} D_{1t-1} + £_{1t} \quad and \quad £_{1t} \overset{i.i.d.}{\sim} N(0, \pm^2_{£1}) \quad and$$

$$D_{2t} = u_{D2} + {}^{\ddot{}}{}_{2} D_{2t-1} + £_{2t} \quad and \quad £_{2t} \overset{i.i.d.}{\sim} N(0, \pm^2_{£2}) .$$

Let D_t be the sum of D_{1t} and D_{2t}, i.e. $D_t = D_{1t} + D_{2t}$. Suppose that D_t is thought to be an AR(1) time series, i.e. $D_t = u_D + {}^{\ddot{}}{}_{a} D_{t-1} + £_t$, where $£_t \overset{i.i.d.}{\sim} N(0, \pm^2_£)$. If ▪$_a$ and $\pm^2_£$ are estimated using maximum likelihood estimators (MLE), then their expected values can be found to be:

$$E({}^{\hat{}}{}_{a}) = \frac{\left[{}^{\ddot{}}{}_{1}\pm_{D1D1} + \pm_{D1D2}(1) + \pm_{D2D1}(1) + {}^{\ddot{}}{}_{2}\pm_{D2D2}\right] + \left[«_{D1} + «_{D2}\right]^2}{\left[\pm_{D1D1} + 2\pm_{D1D2} + \pm_{D2D2}\right] + \left[«_{D1} + «_{D2}\right]^2}$$

and

$$E(\hat{\pm}^2_£) = \left[\pm_{D1D1} + 2\pm_{D1D2} + \pm_{D2D2} + («_{D1} + «_{D2})^2\right] \left[1 - E({}^{\hat{}}{}_{a})^2\right]$$

where
$_{D1}$ is the mean of time series D_{1t};
$_{D2}$ is the mean of time series D_{2t};
$_{D1D2}(1)$ is the covariance of D_{1t} and $D_{2(t-1)}$ for any t;
$_{D2D1}(1)$ is the covariance of D_{2t} and $D_{1(t-1)}$ for any t; and
$_{D2D1}$ is the covariance of D_{2t} and D_{1t} for any t.

Based on Theorem 1, the two demand time series X_{1t} and X_{2t} can be expressed as $X_{1t} = V_{1t} + V_{2t}$ and $X_{2t} = W_{1t} + W_{2t}$, respectively. Under Approach 3, the two time series are estimated as two separated AR(1) time series as follows:

$$X_{1t}=\hat{\phi}^{x_1}X_{1t-1}+a_t^{x_1} \quad \text{and} \quad X_{2t}=\hat{\phi}^{x_2}X_{2t-1}+a_t^{x_2}.$$

From Theorem 2, we can obtain the one-step-forecast mean squared errors as:

$$E(\hat{\sigma}^2_{a_t^{x_1}})=\left[\sigma^2_{V1}+2\sigma_{V1V2}+\sigma^2_{V2}+(\mu_{V1}+\mu_{V2})^2\right]\left[1-E(\hat{\phi}^{x_1})^2\right]$$

$$E(\hat{\sigma}^2_{a_t^{x_2}})=\left[\sigma^2_{W1}+2\sigma_{W1W2}+\sigma^2_{W2}+(\mu_{W1}+\mu_{W2})^2\right]\left[1-E(\hat{\phi}^{x_2})^2\right].$$

where

$$E(\hat{\phi}^{x_1})=\frac{\left[\phi_1\sigma^2_{V1}+\sigma_{V1V2}(1)+\sigma_{V2V1}(1)+\phi_2\sigma^2_{V2}\right]+\left[\mu_{V1}+\mu_{V2}\right]^2}{\left[\sigma^2_{V1}+2\sigma_{V1V2}+\sigma^2_{V2}\right]+\left[\mu_{V1}+\mu_{V2}\right]^2}$$

$$E(\hat{\phi}^{x_2})=\frac{\left[\phi_1\sigma^2_{W1}+\sigma_{W1W2}(1)+\sigma_{W2W1}(1)+\phi_2\sigma^2_{W2}\right]+\left[\mu_{W1}+\mu_{W2}\right]^2}{\left[\sigma^2_{W1}+2\sigma_{W1W2}+\sigma^2_{W2}\right]+\left[\mu_{W1}+\mu_{W2}\right]^2}$$

$$\sigma_{V1V2}=\frac{-\left[(\frac{e_{11}e_{22}e_{21}e_{12}}{C^2})\sigma_{11}+(\frac{e_{11}e_{21}}{C})^2\sigma_{22}\right]}{1-\phi_1\phi_2};$$

$$\sigma_{W1W2}=\frac{-\left[(\frac{e_{12}e_{22}}{C})^2\sigma_{11}+(\frac{e_{11}e_{22}e_{21}e_{12}}{C^2})\sigma_{22}\right]}{1-\phi_1\phi_2};$$

$\sigma_{V1V2}(1)=\phi_1\sigma_{V1V2}$; $\sigma_{V2V1}(1)=\phi_2\sigma_{V1V2}$;

$\sigma_{W1W2}(1)=\phi_1\sigma_{W1W2}$; $\sigma_{W2W1}(1)=\phi_2\sigma_{W1W2}$;

$$\sigma^2_{V1}=\frac{\sigma^2_{a_1^v}}{1-\phi_1^2};\quad \sigma^2_{a_1^v}=\left(\frac{e_{11}e_{22}}{C}\right)^2\sigma_{11}+\left(\frac{e_{11}e_{21}}{C}\right)^2\sigma_{22};$$

$$\sigma^2_{V2}=\frac{\sigma^2_{a_2^v}}{1-\phi_2^2};\quad \sigma^2_{a_2^v}=\left(\frac{e_{21}e_{12}}{C}\right)^2\sigma_{11}+\left(\frac{e_{21}e_{11}}{C}\right)^2\sigma_{22};$$

$$\sigma^2_{W1}=\frac{\sigma^2_{a_1^w}}{1-\phi_1^2};\quad \sigma^2_{a_1^w}=\left(\frac{e_{12}e_{22}}{C}\right)^2\sigma_{11}+\left(\frac{e_{12}e_{21}}{C}\right)^2\sigma_{22};$$

$$\sigma^2_{W2}=\frac{\sigma^2_{a_2^w}}{1-\phi_2^2};\quad \sigma^2_{a_2^w}=\left(\frac{e_{22}e_{12}}{C}\right)^2\sigma_{11}+\left(\frac{e_{22}e_{11}}{C}\right)^2\sigma_{22};$$

and $C=e_{11}e_{22}-e_{12}e_{21}$.

Therefore, under Approach 3 the safety stock (or production capacity) will be prepared for each demand separately. The total safety stock will be then based on a multiple of the sum of forecast standard errors, $\sqrt{E(\hat{\sigma}^2_{a_t^{x_1}})}+\sqrt{E(\hat{\sigma}^2_{a_t^{x_2}})}$.

To derive the forecast standard error of Approach 4, we first derive the form of the aggregated time series, which is the sum of two time series obeying a VAR(1) time series model. This can be easily done by the following Corollary.

Corollary 1:

X_{1t} and X_{2t} follow the AR(1) model in (2). If Y_t is the sum of X_{1t} and X_{2t}, then Y_t can be expressed as $U_{1t}+U_{2t}$ where U_{1t} and U_{2t} are two AR(1) time series:

$U_{1t}=\phi_1U_{1t-1}+a^u_{1t}$ *and*

$$a^u_{1t}=(\frac{e_{12}+e_{11}}{e_{11}e_{22}-e_{12}e_{21}})(e_{22}a_{1t}-e_{21}a_{2t})$$

$U_{2t}=\phi_2U_{2t-1}+a^u_{2t}$ *and*

$$a^u_{2t}=(\frac{e_{22}+e_{21}}{e_{11}e_{22}-e_{12}e_{21}})(-e_{12}a_{1t}+e_{11}a_{2t}).$$

Corollary 1 shows that the aggregated time series is again a sum of two AR(1) time series. To estimate the one-step-forecast standard error of Approach 4, the results of Theorem 2 can be again applied. Suppose the following AR(1) time series model is used to estimate the aggregated time series.

$$Y_t=\hat{\phi}^yY_{t-1}+a_t^y$$

Then, Theorem 2 tells us that the one-step-forecast mean squared error will be:

$$E(\hat{\sigma}^2_{a_t^y})=\left[\sigma^2_{U1}+2\sigma_{U1U2}+\sigma^2_{U2}+(\mu_{U1}+\mu_{U2})^2\right]\left[1-E(\hat{\phi}^y)^2\right]$$

where

$$E(\hat{\phi}^y)=\frac{\left[\phi_1\sigma^2_{U1}+\sigma_{U1U2}(1)+\sigma_{U2U1}(1)+\phi_2\sigma^2_{U2}\right]+\left[\mu_{U1}+\mu_{U2}\right]^2}{\left[\sigma^2_{U1}+2\sigma_{U1U2}+\sigma^2_{U2}\right]+\left[\mu_{U1}+\mu_{U2}\right]^2};$$

$$\sigma_{U1U2}=[(\frac{e_{12}^2e_{22}^2+e_{12}^2e_{21}e_{22}+e_{22}^2e_{11}e_{12}+e_{11}e_{22}e_{21}e_{12}}{C^2})\sigma_{11}+(\frac{e_{12}e_{21}e_{22}e_{11}+e_{21}^2e_{11}e_{12}+e_{11}^2e_{21}e_{22}+e_{11}^2e_{21}^2}{C})^2\sigma_{22}]/(\phi_1\phi_2-1)$$

$\sigma_{U1U2}(1)=\phi_1\sigma_{U1U2}$; $\sigma_{U2U}(1)=\phi_2\sigma_{U1U2}$

$$\sigma^2_{U1}=\frac{\sigma^2_{a_1^u}}{1-\phi_1^2};$$

$$\sigma^2_{a_1^u}=\left(\frac{(e_{12}+e_{11})e_{22}}{C}\right)^2\sigma_{11}+\left(\frac{(e_{12}+e_{11})e_{21}}{C}\right)^2\sigma_{22};$$

$$\sigma^2_{U2}=\frac{\sigma^2_{a_2^u}}{1-\phi_2^2};$$

$$\sigma^2_{a_2^u}=\left(\frac{(e_{22}+e_{21})e_{12}}{C}\right)^2\sigma_{11}+\left(\frac{(e_{22}+e_{21})e_{11}}{C}\right)^2\sigma_{22};$$

and $C=e_{11}e_{22}-e_{12}e_{21}$. Thus, under Approach 4 the safety stock (or production capacity) will be prepared based on a multiple of the aggregated forecast standard error,

$\sqrt{E(\hat{\pm}^{2}_{a_t^y})}$.

For Approach 5, the one-step-forecast standard error will be simply the standard deviations of white noises: $\sqrt{\pm_{11}}$ and $\sqrt{\pm_{22}}$, since the correct VAR(1) model will be used for the statistical forecast and only the white noises are unpredictable and will be let out by the forecast. The safety stock (or production capacity) will be prepared for each demand separately. The total safety stock preparation will be then based on the sum of the two forecast standard errors, $\sqrt{\pm_{11}} + \sqrt{\pm_{22}}$

5. Evaluation Results and Summaries

With the understanding of the aggregated time series in the previous section, five demand planning approaches can be now analytically evaluated and compared. Overall, we have the following observations based on evaluation and comparison results.

a. With aggregation and statistical forecasting capabilities, Approach 4 appears to be the best approach regardless of the scenarios. Approach 4's effetiveness is also the most stable and less affected by changes of demand correlation and ratio of variation sizes.

b. The simple aggregation approach, Approach 2, performs quite as good as Approach 4 and outperforms Approach 5, the most sophisticated statistical forecasting approach, when the demand correlation is weak or negative. Its performance, however, worsens quickly as the demand correlation becomes positive and large and the two variation sizes become significantly different.

c. Approach 3, even with its statistical forecasting capability, appears to be the worst approach. It outperforms Approach 2 only in Scenario 1 when two demands are more positively correlated and the variations sizes are different.

Now, we summarize our observations and provide the following principles and guidelines for practitioners to adopt appropriate demand planning approaches under different situations.

(1) Demand correlation is negative (< 0)

a. If aggregating demands only incurs a limited extra cost, Approach 2 appears to be the best choice since it requires only simple aggregation and does not need to build statistical model for forecasting.

b. If demand aggregation will incur a substantial extra cost, Approach 5 should be adopted. However, Approach 5 requires a correct multivariate statistical model for accurate forecasts. When the demand correlation is insignificant but individual demands have significant autocorrelations, Approach 3 is a good choice for making more than 10% cost reduction.

(2) Demand correlation is positive (> 0)

a. If the correlation is low, 0< <0.2, and the extra aggregation cost is minimum, Approach 2 is still a good choice given that no statistical model is required for this approach.

b. If the correlation is high, >0.2, and the extra aggregation cost is limited, Approach 4 is more preferable. It should be noted that building a univariate time series model for an aggregated demand in Approach 4 is much more reliable and simpler than building the multivariate time series model in Approach 5.

c. If the extra aggregation cost is substantially large, Approach 5 has to be adopted. Again, Approach 3 can be used instead when low demand correlation and high autocorrelation are observed.

References

1. Granger, C.W.J., and M. J. Morris, "Time Series Modelling and Interpretation," *Journal of the Royal Statistical Society – Series A*, 139, 246-257, 1976.
2. Lee, H. L. , V. Padmanabhan , and S. Whang , "Information distortion in a supply chain : bullwhip effect," *Management Science* , 43 , 4 , 546-558 , 1997
3. Simchi-Levi, David, Philip Kaminsky, and E. Simchi-Levi, *Designing and Managing the Supply Chain*, McGraw Hill, 2000.