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Design of EWMA and CUSUM control charts subject to random shift sizes and quality impacts

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Statistical process control charts are important tools for detecting process shifts. To ensure accurate, responsive fault detection, control chart design is critical. In the literature, control charts are typically designed by minimizing the control chart's responding time, i.e., average run length (ARL), to an anticipated shift size under a tolerable false alarm rate. However, process shifts, originating from various variation sources, often come with different sizes and result in different degrees of quality impacts. In this paper, we propose a new performance measure for EWMA and CUSUM control chart design to take into consideration the variable shift sizes and corresponding quality impacts. Unlike economic designs of control charts that suffer from a complex cost structure and intensive numerical computation, the proposed design methodology does not involve any cost estimation and the design procedure is as simple as looking up tables. Given the Gaussian random shifts and quadratic quality loss function, we show that the proposed design has a significant reduction in the quality impact as compared to conventional ARL-based designs. Guidelines and useful worksheets for practical implementation of the proposed designs are then suggested to practitioners with different knowledge levels of the process excursions.

Keywords: Control chart design, EWMA chart, CUSUM chart, random shift, quadratic quality loss

1. Introduction

Statistical Process Control (SPC) charts are extensively used to detect process excursions and thus prevent the production of defective products. Among the many forms of process excursions, process mean shifts are the primary focus of most control chart design. In particular, the control chart most used in practice, the Shewhart $\bar{X}$ chart, is used to monitor the process trend and to detect process shifts. Despite their lower levels of industrial use, Exponentially Weighted Moving Average (EWMA) and CUmulative SUM (CUSUM) charts have been shown in the literature to be more effective than the Shewhart chart. As computing power increases and becomes easily accessible, EWMA and CUSUM charts are gaining their share of practical uses, especially in hi-tech industries where the tolerance level for process deviations is becoming extremely tight. For example, because of the dramatic advances in semiconductor fabrication technology over the past two decades, the integrated circuit feature size has shrunk from 1μm to below 0.09 μm. With over 300 process steps and rapidly tightening process tolerances, the SPC chart is a critical means to achieve high yields in semiconductor manufacturing. EWMA and CUSUM charts are thus becoming much better received by practitioners because of their superior ability to detect small process shifts. In the literature, researchers (see, for example, Lucas (1982) and Klein (1996)) propose combining EWMA/CUSUM and Shewhart control schemes to detect both large and small shifts. Although not the focus of this paper, such charts should be of increasing interest to practitioners.

Both EWMA and CUSUM charts have to be carefully designed for effective use. Each chart has two parameters to be properly set for certain types of process shifts. The literature on EWMA and CUSUM chart design is extensive (Robinson and Ho, 1978; Woodall, 1986; Crowder, 1987a, 1987b; Lucas and Saccucci, 1990; Srivastava and Wu, 1997; Lucas and Crosier, 2000) and usually involves the evaluation of the control chart performance based on the Average Run Length (ARL). When a process is in an in-control state, the ARL to give a false alarm is denoted as ARL_0. ARL_1, on the other hand, is the ARL for a control chart to signal an out-of-control process. ARL_0 characterizes a control chart's reliability whereas ARL_1 measures the control chart's sensitivity to process excursions. ARL-based control chart design has three steps:

1. determine a tolerable false alarm rate, i.e., a predetermined ARL_0 value;
2. select the most likely process-shift size; and

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3. select a control chart design with the greatest detection power; i.e., select the plan with the smallest ARL_1 value.

Most researchers have focused their attention on how to find the smallest ARL_1 given a fixed ARL_0 and an anticipated shift size.

ARL-based control chart designs are optimized for specific process shifts. Given a shift size, a control chart can be designed to maximize its detection power. This shift-specific design, however, has limited applicability as processes are typically subject to various deviation sizes. Sparks (2000) and Capizzi and Masarotto (2003) addressed the varying-size problem by predicting the future shift size on which the control chart design is dynamically adjusted. For processes where successive shifts follow a clear estimable pattern, these design methodologies are useful. But for most processes where shifts occur arbitrarily with random sizes, should a different control chart design be proposed? This is the first question we attempt to answer in this paper by proposing a random-shift control chart design methodology.

Since different shift sizes incur different quality losses, the design methodology should also take into consideration the impact of the shift on the quality. Economic designs of control charts have considered various quality costs under a predefined stochastic model of shift process (see Lorenzen and Vance (1986), Ho and Case (1994) and Keats *et al.* (1997)). Multiple shift sizes have also been considered in Knappenberger and Grandge (1969) and Duncan (1971) while quadratic quality losses are discussed in Elsayed and Chen (1994). Knowing the complex economic model's limitation, Montgomery *et al.* (1995) propose a simplified ARL-based model that does not consider the stochastic process of shift occurrence. While the body of "economic design" literature is huge, the design's practicality and applicability are always in question. The design method is controversial because of its limited practical use. Another concern about economic design is the tradeoff between quality costs and sampling costs. Many practitioners find it difficult to justify reducing the sampling cost at the cost of the quality, which can be as large as the cost of regaining a lost customer. Difficulties in estimating the dynamic costs and modeling the stochastic out-of-control process are also cited as major obstacles to implementation. While Keats *et al.* (1997) have proposed steps to overcome the difficulties, actual implementation is still rare.

In this paper, we propose a more plausible model considering random shift sizes and ensuing quality impacts without getting into the complexity of cost estimation, stochastic process modeling and numerical computation. The proposed design procedure is a simple extension of the conventional ARL-based designs. The control chart performances under random shifts are evaluated in terms of resulting quality impacts and are compared to the conventional ARL-based designs. This paper is organized into seven sections. Following the Introduction in this section, we first describe EWMA and CUSUM chart designs and the random-shift model. In the third section, the quality impact model of a control chart under random shifts is established. The EWMA and CUSUM charts are then designed to minimize the quality impact in Section 4 followed by evaluation of the proposed control chart designs against conventional designs. In Section 6, we first provide practitioners with worksheets and useful guidelines to implement the proposed control chart design and then compare the EWMA and CUSUM chart performances to assist users in selecting the least-quality-impact chart design. In the final section, implementation issues with the proposed control chart designs are discussed.

2. EWMA and CUSUM charts and random shifts

Unlike the Shewhart $\bar{X}$ chart where only one design is possible given a fixed ARL_0 value, both EWMA and CUSUM charts can have different designs under the same false alarm level. Here, we briefly introduce the two control chart schemes and their design parameters. Let X_i be the ith quality observation to be statistically monitored by a control chart. The observations X_i, $i = 1, 2, 3, \ldots$, are said to be independent and identically distributed (iid) and follow a normal distribution with mean μ_x and standard deviation σ_x, that is

$$X_i \overset{\text{iid}}{\sim} N(\mu_x, \sigma_x^2)\ i = 1, 2, 3, \ldots.$$

The EWMA scheme uses the rolling exponentially weighted averages (Z_i) as the test statistic:

$$Z_i = \lambda X_i + (1 - \lambda) Z_{i-1}, \quad Z_0 = \mu_x, \quad 0 < \lambda \le 1, i = 1, 2, \ldots. \tag{1}$$

The control limits are set at $T \pm L\sigma_z$ where T is the desired process target and σ_z is the standard deviation of the test statistic Z_i. For an in-control process, the process mean (μ_x), and thus the test statistic mean (μ_z), is equal to the target (T). When the control statistic Z_i is observed to exceed any of the control limits, the process is said to be out of statistical control; i.e., the process mean is believed to be no longer in accord with the target T. By adjusting the weight decreasing rate λ (Roberts, 1959) and the control window width (L), the EWMA chart can be designed to best detect a certain shift size under a constant false alarm rate.

To be more sensitive to smaller shifts, the two-sided tabular CUSUM test statistics (Page, 1961) is designed so that small deviations can mount up to a larger identifiable deviation:

$$\begin{aligned} C_i^+ &= \max\,[0, X_i - (T + k\sigma_x) + C_{i-1}^+] \\ C_i^- &= \max\,[0, X_i - (T + k\sigma_x) + C_{i-1}^-], \\ &\qquad C_0^+ = C_0^- = 0, i = 1, 2, 3, \ldots, \end{aligned} \tag{2}$$

where k controls the accumulation span. If either C_i^+ or C_i^- exceeds $h\sigma_x$, the process is considered to be out of control. CUSUM chart design is to specify the values of the parameters k and h for the desirable memory properties and false alarm rates of the control scheme.

In this research, the EWMA and CUSUM charts are designed to have $ARL_0 = 500$, or equivalently a false alarm rate of 0.002. This in-control ARL is chosen for study because it is close to, but somewhat larger than, that of a standard Shewhart $\bar{X}$ chart ($ARL_0 = 370$). Due to better EWMA/CUSUM control sensitivity, a larger ARL_0 value is often set without giving away too much detecting power. For example, the most often seen CUSUM design is $k = 0.5$ and $h = 5$ with $ARL_0 = 465$.

When a process shift occurs, the process mean is deviated from the target T by a random amount S, i.e., $\mu_x = T + S$. The random shift size is assumed to follow a probability density function $g(\cdot)$ with mean μ_s and standard deviation σ_s. Examples in this paper are given assuming a normally distributed shift size and, without loss of generality, a target at zero, i.e., $T = 0$. The process shift, once taking place, remains constant till the control chart alarm goes off. In the next section, we will introduce the control chart performance measures by considering the quality impacts by the process deviation during the lag to detection.

3. Quality impact induced by control charts

Different sizes of process deviations will have different impacts on the product quality. The impacts of process excursions on the product quality are referred to as *quality impacts*. Usually, the larger the shift size, the greater the quality impact. A quality loss function describes the relationship between the shift size and the quality impact. Let $\mathrm{QL}(X_i)$ denote the quality impact incurred by the ith quality observation X_i. The most celebrated quality loss function is a quadratic function (Taguchi):

$$\mathrm{QL}(X_i) = w(X_i - T)^2,$$

where w is the quality loss per unit squared deviation of the quality measure from the target. When the shift size in the process mean is s, the quality loss is denoted as QL_s, i.e., $\mathrm{QL}_s = \mathrm{QL}(X_i \mid \mu_x = T + s, \sigma_x)$. The quality impact of one observation with a mean shift s is then expressed by the expected quality loss:

$$E(\mathrm{QL}_s) = \int_{-\infty}^{\infty} \mathrm{QL}(x) f(x \mid \mu_x = T + s, \sigma_x)\mathrm{d}x, \quad (3)$$

where $f(\cdot)$ is the probability density function (pdf) of quality observations with mean μ_x and standard deviation σ_x. Taking the quadratic loss function as an example, the expected quality impact becomes:

$$E(\mathrm{QL}_s) = E[w(x_i + T)^2 \mid \mu_x = T + s, \sigma_x] = w\left(s^2 + \sigma_x^2\right). \quad (4)$$

To evaluate a control chart's performance, instead of the average out-of-control run length, ARL_1, we can further calculate the quality loss incurred during the out-of-control period. Let $\mathrm{RL}_{1|s}$ denote the run length for the control chart to detect the process shift given σ_x and the shift size s. The quality loss during the out-of-control period is then:

$$\mathrm{QL}_{1|s} = \sum_{i=1}^{\mathrm{RL}_{1|s}} \mathrm{QL}(X_i \mid \mu_x = T + s, \sigma_x),$$

where $\mathrm{QL}_{1|s}$ is a random sum of random variables QL_s. Based on Wald's equation, $\mathrm{RL}_{1|s}$ is the stopping time of the random sum and $E(\mathrm{QL}_{1|s})$ can be thus calculated as (see Appendix A):

$$\begin{aligned} E(\mathrm{QL}_{1|s}) &= E(\mathrm{RL}_{1|s})E[\mathrm{QL}(X_i \mid \mu_x = T + s, \sigma_x)] \\ &= \mathrm{ARL}_{1|s} \times E(\mathrm{QL}_s). \end{aligned} \quad (5)$$

When the shift size is known to follow a distribution with pdf $g(\cdot)$, the final expected quality impact without conditioning on the shift size can then be calculated by the law of total expectation:

$$\begin{aligned} E(\mathrm{QL}_1) &= \int_{-\infty}^{\infty} E(\mathrm{QL}_{1|s}) g(s)\mathrm{d}s \\ &= \int_{-\infty}^{\infty} \mathrm{ARL}_{1|s} E(\mathrm{QL}_s) g(s)\mathrm{d}s. \end{aligned} \quad (6)$$

Calculation of $E(\mathrm{QL}_1)$ involves the estimation of $\mathrm{ARL}_{1|s}$. Two methodologies can be found in the literature. One is Monte Carlo simulation and the other is numerical approximation. Markov chain (Lucas and Saccucci, 1990) and integral equation (Lorenzen and Vance, 1986; Crowder, 1987a, 1987b) solutions are two popular approaches to numerical approximation. In this research, the integral equation approach is adopted to estimate $\mathrm{ARL}_{1|s}$ since the size of the mean shift is also a continuous random variable.

Page (1954) first formulated the integral equation for CUSUM ARL evaluation by first-step analysis. Later, Goel and Wu (1971) and Lucas (1976) used the same integral equation but solved it with different numerical approaches. Given the distribution of the quality observations, Lorenzen and Vance (1986) evaluated the upper-sided ARL, ARL^+, and lower-sided ARL, ARL^-, using the following integral equations, respectively:

$$\mathrm{ARL}^+(u) = 1 + \mathrm{ARL}^+(0)F(K - u) + \int_0^{h\sigma_x} \mathrm{ARL}^+(x) f(x + K - u)\mathrm{d}x, \quad (7a)$$

$$\mathrm{ARL}^-(u) = 1 + \mathrm{ARL}^-(0)[1 - F(K' + u)] + \int_0^{h\sigma_x} \mathrm{ARL}^-(x) f(K' + u - x)\mathrm{d}x, \quad (7b)$$

where $K = \mu_x + k\sigma_x$, $K' = \mu_x - k\sigma_x$, u is the value of the initial quality observation and is usually set to be the target T, and $F(\cdot)$ is the cumulative distribution function of the quality observation distribution, respectively. By Kemp's reciprocal rule (Kemp, 1961), the final CUSUM ARL is obtained by $(\mathrm{ARL})^{-1} = (\mathrm{ARL}^+)^{-1} + (\mathrm{ARL}^-)^{-1}$.

Similarly, Crowder (1987a, 1987b) derived the following integral equation to evaluate the EWMA ARL given the distribution of quality observations:

$$\mathrm{ARL}(u) = 1 + \frac{1}{\lambda}\int_{-L\sigma_x}^{L\sigma_x} \mathrm{ARL}(x) f\left(\frac{x-(1-\lambda)u}{\lambda}\right)\mathrm{d}x. \quad (8)$$

The above integral equations are Fredholm integral equations of the second kind which we solved by numerical recipes provided by Press *et al.* (1992). With the $\mathrm{ARL}_{1|s}$ estimated by assuming iid normal distributions of the quality observations, $E(\mathrm{QL}_1)$ in Equation (6) can be further carried out numerically to evaluate the control chart performance.

4. Control chart design under random shifts

With $E(\mathrm{QL}_1)$ as the performance measure, control charts can be now designed to minimize the quality impact while an acceptable ARL_0 is maintained. While Equation (6) can be solved with a general $f(\cdot)$ and $g(\cdot)$, this paper evaluates control chart designs under the following settings:

1. $\mathrm{ARL}_0 = 500$;
2. $X_i \overset{\text{iid}}{\sim} N(\mu_x,\ 1 \mid \mu_x = 0 + S)$;
3. $S \sim N(\mu_s, \sigma_s^2)$; and
4. a quadratic quality loss function in which w is set to one without loss of generality.

In addition, evaluation will be conducted for μ_s over (0, $4\sigma_x$) with a step size of $0.5\sigma_x$ and for σ_s over (0, $4\sigma_x$) with a step size of $0.1\sigma_x$.

4.1. *EWMA control chart design under random shifts*

Figure 1 shows all possible EWMA chart designs, (λ, L), for ARL_0 to be at least 500. This figure was first presented by Crowder (1989) and is recreated here for numerical validation and for optimum EWMA chart design under random shifts.

For each (μ_s, σ_s), we compute $E(\mathrm{QS}_1)$ for all possible EWMA designs with $ARL_0 = 500$ and select the optimum EWMA design with the lowest $E(\mathrm{QS}_1)$. Table 1 shows the optimum λ^* values for various combinations of (μ_s, σ_s). For a specified (μ_s, σ_s), the optimum λ^* value can be first found in Table 1. With λ^*, the corresponding L^* value can then be obtained from Fig. 1 and thus the optimum EWMA design (λ^*, L^*) is determined.

Example 1: Suppose the process shift is known to center around $T + 2\sigma_x$ and deviate over a small range with a standard deviation $\sigma_s = 0.5\sigma_x$. To design a EWMA chart with the smallest quality impact, we first look up Table 1 with $\mu_s = 2$ and $\sigma_s = 0.5$ to find $\lambda* = 0.31$. From Fig. 1, a corresponding $L^* = 3.04$ is found such that $ARL_0 =$ 500. ◀

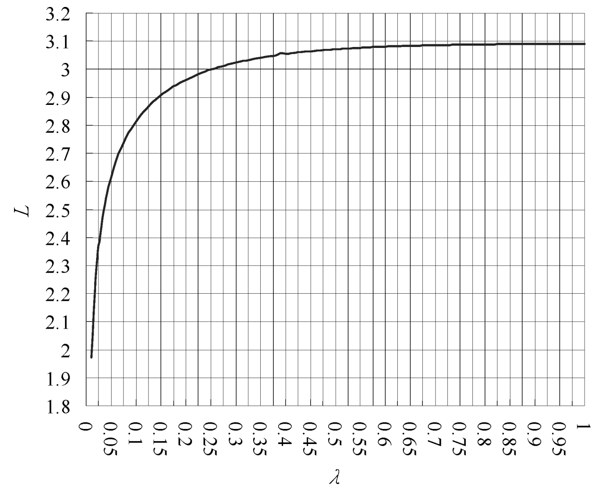


Fig. 1. EWMA chart design with $ARL_0 = 500$.

The first row of Table 1 consists of the conventional EWMA optimal designs for fixed shift sizes ($\sigma_s = 0$). The optimal values of λ here can be compared and found to be identical to those reported by Lucas and Saccucci (1990). The first column of Table 1 manifests the designs suggested by this research for uncertain shifts occurring about the target ($\mu_s = 0$ and $\sigma_s \neq 0$).

λ^* and $L^*$for $\mu_s = 0$

Figure 2 shows the trend of the optimal EWMA chart designs over various σ_s. This figure can be very useful in practice since most shifts are likely to occur on both sides of the target. It is observed that both λ^* and L^* increase as the variability of the shift size increases. This is because the greater the shift spread, the higher the occurrence chance of large shifts. In particular, we look at the optimal EWMA designs for $\sigma_s = \sigma_x$, $2\sigma_x$ and $3\sigma_x$ (highlighted with boldface in Table 1). For $\sigma_s = \sigma_x$, most shifts are small and their distribution is exactly the same as the in-control quality observations' distribution; i.e., about 90% of the shifts occur within the range from $-1.6\sigma_x$ to $1.6\sigma_x$. λ^* of the optimal EWMA chart is 0.02 in this case. When $\sigma_s = 2\sigma_x$, i.e., over 30% of shifts are greater than $2\sigma_x$, λ^* increases to 0.06. Only when $\sigma_s = 3\sigma_x$, i.e., over 30% of shift sizes are greater than $3\sigma_x$, does λ^* increase to 0.12. Figure 3 illustrates the shift distributions with $\sigma_s = 2\sigma_x$ and $3\sigma_x$ and is intended to give readers a feel about how the uncertain shifts spread as compared to the in-control quality distribution. It is interesting to note that these optimal designs for variable shifts are equivalent to the optimal conventional designs for fixed shift sizes equal to $0.25\sigma_x$,$0.6\sigma_x$, and σ_x, respectively. That is, the values of the optimal design parameters for detecting uncertain shifts are smaller than one would expect when thinking in a deterministic sense.

For cases with $\mu_s \neq 0$, $\mu_s = 1$ is used as an example to show how the optimal design changes as the shift variance changes. Figure 4 shows the values of λ^* and L^* for

Table 1. Optimum λ* for EWMA chart design with $ARL_0 = 500$ under random shifts

	μ_s																
σ_s	*0*	*0.25*	*0.5*	*0.75*	*1*	*1.25*	*1.5*	*1.75*	*2*	*2.25*	*2.5*	*2.75*	*3*	*3.25*	*3.5*	*3.75*	*4*
0		0.02	0.05	0.09	0.13	0.19	0.24	0.3	0.37	0.43	0.52	0.6	0.68	0.74	0.8	0.84	0.89
0.1	0.01	0.01	0.04	0.08	0.13	0.18	0.24	0.3	0.36	0.43	0.52	0.6	0.67	0.74	0.8	0.84	0.88
0.2	0.01	0.01	0.03	0.07	0.12	0.17	0.23	0.29	0.36	0.43	0.51	0.59	0.67	0.73	0.79	0.84	0.88
0.3	0.01	0.01	0.02	0.05	0.1	0.16	0.22	0.28	0.35	0.42	0.51	0.59	0.66	0.72	0.78	0.83	0.87
0.4	0.01	0.01	0.02	0.04	0.07	0.13	0.2	0.27	0.34	0.41	0.49	0.58	0.65	0.71	0.77	0.82	0.85
0.5	0.01	0.02	0.02	0.03	0.06	0.1	0.17	0.24	0.31	0.4	0.48	0.56	0.63	0.69	0.75	0.8	0.84
0.6	0.02	0.02	0.02	0.03	0.05	0.08	0.14	0.21	0.28	0.37	0.46	0.54	0.61	0.68	0.73	0.78	0.82
0.7	0.02	0.02	0.02	0.03	0.04	0.07	0.11	0.17	0.25	0.34	0.43	0.51	0.59	0.65	0.7	0.75	0.79
0.8	0.02	0.02	0.02	0.03	0.04	0.06	0.09	0.14	0.21	0.3	0.4	0.48	0.56	0.62	0.68	0.72	0.77
0.9	0.02	0.02	0.03	0.03	0.04	0.06	0.08	0.12	0.18	0.26	0.36	0.45	0.52	0.59	0.65	0.7	0.74
1	**0.02**	0.03	0.03	0.03	0.04	0.05	0.08	0.11	0.16	0.23	0.32	0.41	0.48	0.55	0.61	0.66	0.71
1.1	0.03	0.03	0.03	0.03	0.04	0.05	0.07	0.1	0.14	0.2	0.29	0.37	0.45	0.52	0.57	0.63	0.67
1.2	0.03	0.03	0.03	0.04	0.04	0.05	0.07	0.09	0.13	0.18	0.26	0.33	0.41	0.47	0.54	0.59	0.63
1.3	0.03	0.03	0.04	0.04	0.05	0.06	0.07	0.09	0.12	0.17	0.23	0.31	0.37	0.44	0.5	0.55	0.6
1.4	0.04	0.04	0.04	0.04	0.05	0.06	0.07	0.09	0.11	0.16	0.21	0.28	0.35	0.4	0.46	0.51	0.56
1.5	0.04	0.04	0.04	0.05	0.05	0.06	0.07	0.09	0.11	0.15	0.2	0.26	0.32	0.37	0.43	0.47	0.52
1.6	0.04	0.04	0.04	0.05	0.05	0.06	0.07	0.09	0.11	0.15	0.19	0.25	0.3	0.35	0.4	0.44	0.49
1.7	0.05	0.05	0.05	0.05	0.06	0.06	0.07	0.09	0.11	0.14	0.19	0.23	0.28	0.33	0.37	0.41	0.46
1.8	0.05	0.05	0.05	0.05	0.06	0.07	0.08	0.09	0.11	0.14	0.18	0.22	0.26	0.3	0.35	0.38	0.43
1.9	0.05	0.05	0.06	0.06	0.06	0.07	0.08	0.09	0.12	0.14	0.18	0.21	0.25	0.29	0.33	0.36	0.4
2	**0.06**	0.06	0.06	0.06	0.07	0.07	0.08	0.1	0.12	0.15	0.18	0.21	0.24	0.27	0.31	0.34	0.37
2.1	0.06	0.06	0.06	0.07	0.07	0.08	0.09	0.1	0.12	0.15	0.17	0.21	0.23	0.26	0.3	0.33	0.35
2.2	0.06	0.07	0.07	0.07	0.08	0.08	0.09	0.11	0.13	0.15	0.17	0.2	0.23	0.25	0.28	0.31	0.34
2.3	0.07	0.07	0.07	0.08	0.08	0.09	0.1	0.11	0.13	0.15	0.17	0.2	0.22	0.25	0.27	0.3	0.32
2.4	0.07	0.08	0.08	0.08	0.09	0.09	0.11	0.12	0.14	0.16	0.17	0.2	0.22	0.24	0.26	0.29	0.31
2.5	0.08	0.08	0.08	0.09	0.09	0.1	0.11	0.13	0.14	0.16	0.17	0.19	0.21	0.23	0.25	0.27	0.3
2.6	0.09	0.09	0.09	0.09	0.1	0.11	0.12	0.13	0.15	0.16	0.18	0.19	0.21	0.23	0.25	0.27	0.29
2.7	0.09	0.09	0.1	0.1	0.11	0.12	0.12	0.13	0.15	0.16	0.18	0.19	0.21	0.23	0.24	0.26	0.28
2.8	0.1	0.1	0.1	0.11	0.11	0.12	0.13	0.14	0.15	0.17	0.18	0.19	0.21	0.22	0.24	0.25	0.27
2.9	0.11	0.11	0.11	0.12	0.12	0.13	0.13	0.15	0.16	0.17	0.18	0.19	0.21	0.22	0.23	0.25	0.26
3	**0.12**	0.12	0.12	0.12	0.13	0.13	0.14	0.15	0.16	0.17	0.18	0.19	0.21	0.22	0.23	0.24	0.26
3.1	0.12	0.12	0.12	0.13	0.13	0.14	0.15	0.15	0.16	0.17	0.18	0.19	0.21	0.21	0.23	0.24	0.25
3.2	0.13	0.13	0.13	0.13	0.14	0.14	0.15	0.16	0.17	0.17	0.18	0.19	0.2	0.21	0.23	0.24	0.25
3.3	0.13	0.13	0.14	0.14	0.14	0.15	0.15	0.16	0.17	0.18	0.19	0.19	0.2	0.21	0.22	0.23	0.25
3.4	0.14	0.14	0.14	0.15	0.15	0.15	0.16	0.16	0.17	0.18	0.19	0.19	0.2	0.21	0.22	0.23	0.24
3.5	0.15	0.15	0.15	0.15	0.15	0.16	0.16	0.17	0.17	0.18	0.19	0.2	0.2	0.21	0.22	0.23	0.24
3.6	0.15	0.15	0.15	0.16	0.16	0.16	0.17	0.17	0.18	0.18	0.19	0.2	0.2	0.21	0.22	0.23	0.24
3.7	0.16	0.16	0.16	0.16	0.16	0.17	0.17	0.17	0.18	0.18	0.19	0.2	0.2	0.21	0.22	0.23	0.23
3.8	0.16	0.16	0.16	0.16	0.17	0.17	0.17	0.18	0.18	0.19	0.19	0.2	0.2	0.21	0.22	0.22	0.23
3.9	0.16	0.17	0.17	0.17	0.17	0.17	0.17	0.18	0.18	0.19	0.19	0.2	0.21	0.21	0.22	0.22	0.23
4	0.17	0.17	0.17	0.17	0.17	0.17	0.18	0.18	0.19	0.19	0.19	0.2	0.21	0.21	0.22	0.22	0.23

$\mu_s = 1$ and various σ_s. When $\sigma_s = 0$, i.e., the shift size is known definitely to be one σ_x, the EWMA chart design is equivalent to the conventional design with $\lambda^* = 0.13$ and $L^* = 2.88$. When the shift size becomes uncertain, both λ^* and L^* first decrease since a half of the uncertain shifts occur around the target as shown in the shaded area of Fig. 5 with $\sigma_s = \sigma_x$. However, when the uncertainty continues to increase, the optimal values of λ^* and L^* pick up again due to the widespread shifts over both sides of the target. It is interesting to note that the values of λ^* and L^* are not greater than those in the conventional design until σ_s reaches 3 σ_x. Again, the optimal values of EWMA chart parameters are smaller than those suggested in conventional designs.

4.2. *CUSUM chart design under random shifts*

Similar to Fig. 1 for EWMA charts; Fig. 6 shows all possible CUSUM chart designs, (k, h), for ARL_0 equal to 500. With this figure, if an optimal value of k^* is found for a random shift, the optimal value of h^* can be found accordingly.

Again, for each (μ_s, σ_s) we compute E (QS_1) for all possible CUSUM designs with $ARL_0 = 500$ and select the

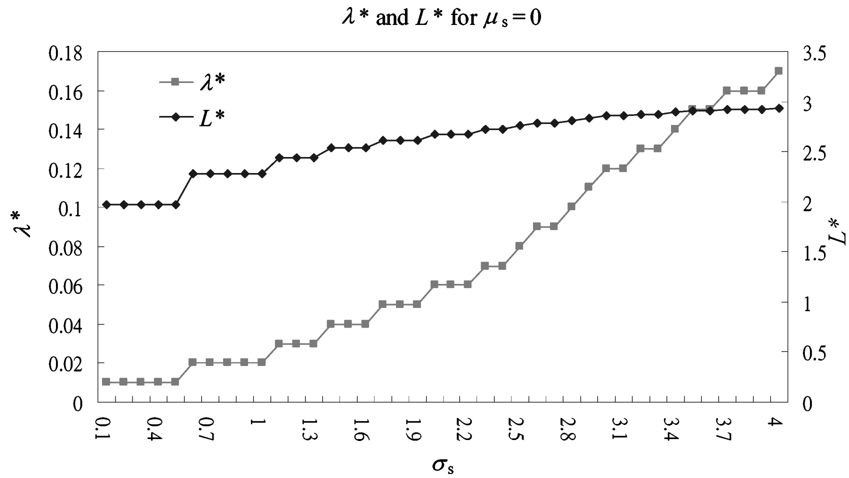


Fig. 2. Optimal EWMA chart design: λ^* and L^* for $\mu_s = 0$.

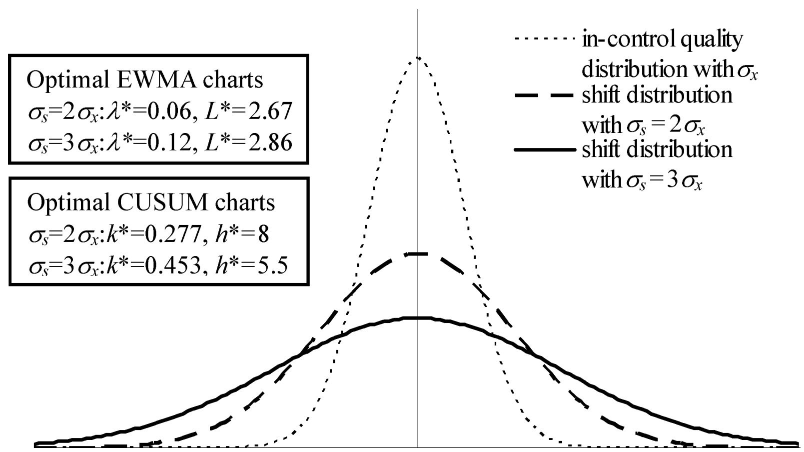


Fig. 3. Distributions and optimal designs for $\sigma_s = 2\sigma_x$ (dashed lines) and $\sigma_s = 3\sigma_x$ (solid line).

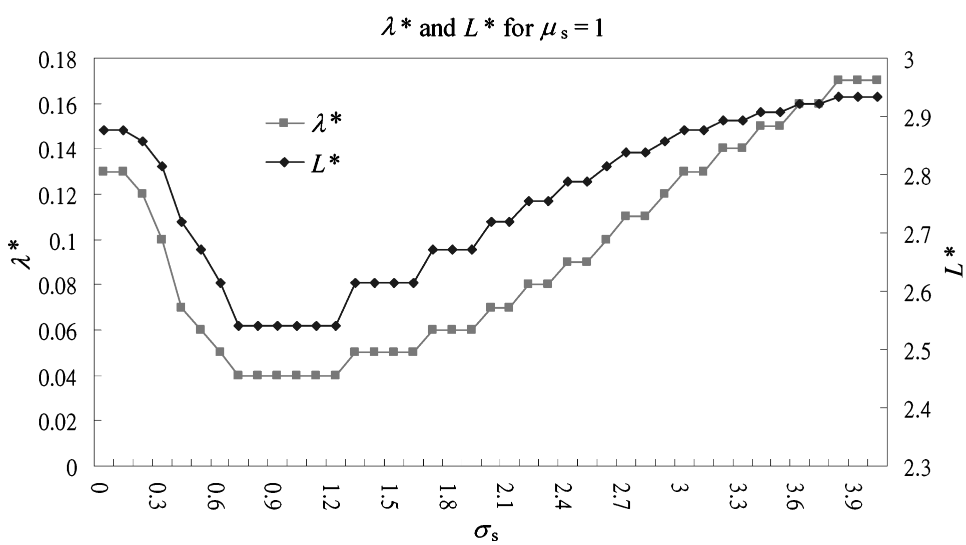


Fig. 4. Optimal EWMA chart design: λ^* and L^* for $\mu_s = 1$.

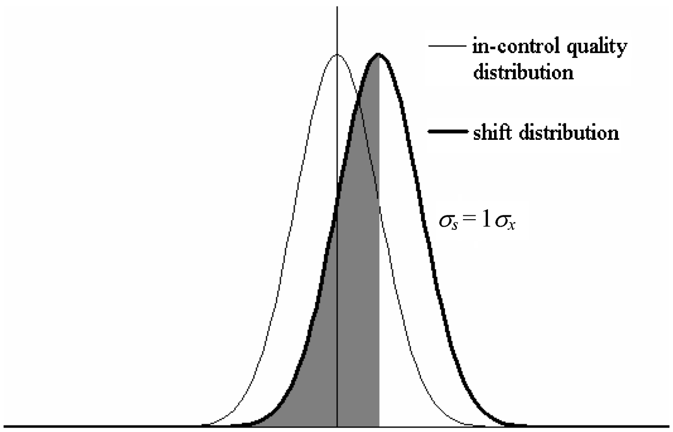


Fig. 5. Half of uncertain shifts occurring around target.

optimum CUSUM design with the lowest $E(\mathrm{QS}_1)$. Table 2 shows the optimum k^* values for various combinations of (μ_s, σ_s). For a specified (μ_s, σ_s), the optimum k^* value can be first found in Table 2. With the value of k^* known, the corresponding h^* value can then be obtained from Fig. 6 and thus the optimum CUSUM design (k^*, h^*) is determined.

Example 1 (continued): With $\mu_s = T + 2\sigma_x$ and $\sigma_s = 0.5\sigma_x$, the least-quality-impact CUSUM design can be found by first looking up Table 2 for the optimum value of $k^* = 0.89$. From Fig. 6, the corresponding value of h^* is then found to be three. ◄

Similar to Table 1, the first row of Table 2 consists of the conventional CUSUM optimal designs for fixed shift sizes ($\sigma_s = 0$). The optimal values here are comparable to the values given by Lucas (1976). The first column of Table 2 shows the designs exclusively suggested by this research for uncertain shifts occurring around the target ($\mu_s = 0$ and $\sigma_s \neq 0$). Figure 7 shows the trend of the optimal CUSUM

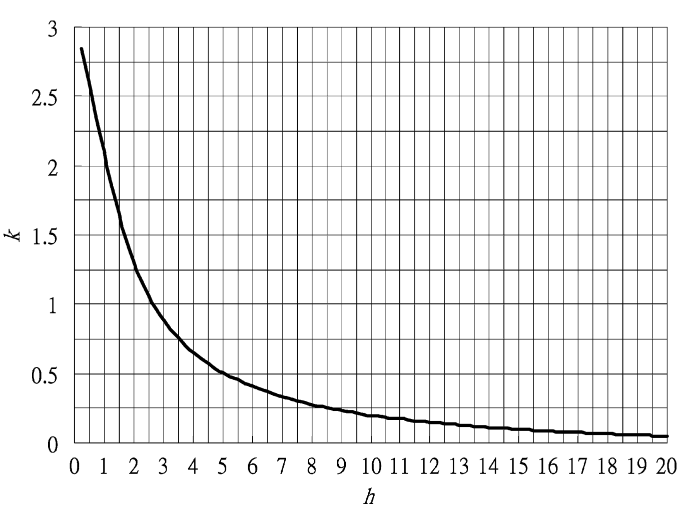


Fig. 6. CUSUM chart design with $ARL_0 = 500$.

parameters over various σ_s. It can be observed that as the shift uncertainty increases, k^* increases and h^* decreases. We also look at the optimal designs for $\sigma_s = \sigma_x$, $2\sigma_x$ and $3\sigma_x$ (highlighted with boldface in Table 2). These optimal designs are the same as the optimal designs for fixed shift sizes equal to $0.37\,\sigma_x$, $0.56\sigma_x$ and $0.9\sigma_x$, respectively. Again, the values of the optimal design parameters for variable shifts are smaller than one would expect when thinking in a deterministic sense. The optimal CUSUM parameters for $\sigma_s = 2\sigma_x$ and $3\,\sigma_x$ are also shown in Fig. 3.

For $\mu_s = 1$, the optimal CUSUM design parameters are shown in Fig. 8. Similarly, when $\sigma_s = 0$, the CUSUM chart design is equivalent to the conventional design where $k^* = 0.51$ and $h^* = 5$. When the shift size becomes uncertain, k^* first decreases for the same reasons as shown in Fig. 5. The value of k^* continues to decrease until the uncertainty widely spreads over both sides of the target. Accordingly, the value of h^* first increases and then decreases as the uncertainly continues to widen. The values of k^* and h^* do not reach the levels of those in the conventional design until σ_s reaches $3.2\sigma_x$. As in the EWMA cases, the optimal values of CUSUM chart parameters are generally lower than suggested in the literature.

5. Quality impact reduced by designs considering random shifts

When the shifts are indeed random in size, we can evaluate the quality impact reduction by the proposed design over the conventional designs. The loss saving is calculated as follows:

$$\textit{Loss saving (\%)} = \frac{E'(\mathrm{QL}_1) - E^*(\mathrm{QL}_1)}{E'(\mathrm{QL}_1)} \times 100\%, \qquad (9)$$

where $E'(\mathrm{QL}_1)$ is the quality impact induced by conventional optimal chart designs assuming $\sigma_s = 0$ and $E^*(\mathrm{QL}_1)$ is the quality impact caused by the chart designs optimized for a known σ_s. In Fig. 9(a) and Fig. 9(b), we calculate and show the loss savings by the proposed EWMA and CUSUM designs over the conventional designs, respectively, for $\mu_s = 1$ and various σ_s.

For EWMA control charts, the saving can be up to 11.3% and the maximum saving occurs at $\sigma_s = 0.9$. The saving for CUSUM charts can be up to 13% for $\sigma_s = 0.8$. For both control charts, the saving becomes quite insignificant ($\leq 1\%$) when σ_s is larger than $2.6\sigma_x$. That is, the proposed designs are not particularly preferred for shifts centering around $\mu_s = 1$ and uncertainly spreading over a very wide range.

6. Chart implementation and best designs for random shifts

Without prior knowledge on how the shift varies, implementing the proposed chart designs would be difficult. It is

Table 2. Optimum k^* for CUSUM chart design with $ARL_0 = 500$ under random shifts

	μ_s																
σ_s	*0*	*0.25*	*0.5*	*0.75*	*1*	*1.25*	*1.5*	*1.75*	*2*	*2.25*	*2.5*	*2.75*	*3*	*3.25*	*3.5*	*3.75*	*4*
0		0.12	0.25	0.37	0.51	0.61	0.76	0.89	0.97	1.18	1.31	1.31	1.47	1.66	1.66	1.87	2.1
0.1	0.06	0.11	0.22	0.37	0.48	0.61	0.76	0.89	0.97	1.18	1.31	1.31	1.47	1.66	1.66	1.87	2.1
0.2	0.08	0.1	0.18	0.32	0.45	0.61	0.7	0.89	0.97	1.06	1.18	1.31	1.47	1.66	1.66	1.87	1.87
0.3	0.09	0.11	0.15	0.25	0.41	0.54	0.7	0.82	0.97	1.06	1.18	1.31	1.47	1.66	1.66	1.87	1.87
0.4	0.1	0.12	0.15	0.22	0.33	0.48	0.66	0.82	0.97	1.06	1.18	1.31	1.47	1.47	1.66	1.87	1.87
0.5	0.12	0.12	0.15	0.2	0.28	0.41	0.58	0.7	0.89	1.06	1.18	1.31	1.47	1.47	1.66	1.66	1.87
0.6	0.12	0.13	0.15	0.18	0.25	0.37	0.51	0.66	0.82	0.97	1.18	1.31	1.31	1.47	1.66	1.66	1.87
0.7	0.13	0.14	0.15	0.18	0.24	0.32	0.43	0.58	0.76	0.89	1.06	1.18	1.31	1.47	1.47	1.66	1.66
0.8	0.15	0.15	0.16	0.18	0.23	0.3	0.41	0.51	0.66	0.82	0.97	1.18	1.31	1.31	1.47	1.66	1.66
0.9	0.15	0.16	0.18	0.2	0.22	0.28	0.37	0.48	0.61	0.76	0.97	1.06	1.18	1.31	1.47	1.47	1.66
1	**0.17**	0.17	0.18	0.2	0.23	0.28	0.35	0.43	0.54	0.7	0.89	0.97	1.18	1.18	1.31	1.47	1.47
1.1	0.18	0.18	0.18	0.21	0.24	0.28	0.33	0.41	0.51	0.66	0.82	0.97	1.06	1.18	1.31	1.31	1.47
1.2	0.18	0.2	0.2	0.22	0.24	0.28	0.33	0.41	0.48	0.61	0.76	0.89	0.97	1.06	1.18	1.31	1.31
1.3	0.2	0.2	0.22	0.22	0.25	0.28	0.33	0.39	0.45	0.58	0.7	0.82	0.97	1.06	1.18	1.18	1.31
1.4	0.22	0.22	0.22	0.24	0.25	0.29	0.33	0.39	0.45	0.54	0.66	0.82	0.89	0.97	1.06	1.18	1.18
1.5	0.22	0.22	0.23	0.25	0.28	0.3	0.33	0.39	0.43	0.54	0.66	0.76	0.82	0.97	1.06	1.06	1.18
1.6	0.24	0.24	0.24	0.25	0.28	0.3	0.33	0.39	0.43	0.54	0.61	0.7	0.82	0.89	0.97	1.06	1.06
1.7	0.25	0.25	0.25	0.28	0.28	0.3	0.35	0.39	0.43	0.51	0.61	0.7	0.76	0.89	0.89	0.97	1.06
1.8	0.25	0.26	0.28	0.28	0.3	0.32	0.35	0.39	0.45	0.51	0.61	0.66	0.76	0.82	0.89	0.97	0.97
1.9	0.28	0.28	0.28	0.3	0.3	0.33	0.37	0.41	0.45	0.51	0.58	0.66	0.7	0.82	0.82	0.89	0.97
2	**0.28**	0.29	0.3	0.3	0.32	0.35	0.37	0.41	0.45	0.54	0.58	0.66	0.7	0.76	0.82	0.89	0.97
2.1	0.3	0.3	0.3	0.32	0.33	0.35	0.39	0.43	0.48	0.54	0.58	0.66	0.7	0.76	0.82	0.82	0.89
2.2	0.32	0.32	0.32	0.33	0.35	0.37	0.41	0.43	0.48	0.54	0.58	0.61	0.7	0.7	0.76	0.82	0.89
2.3	0.33	0.33	0.33	0.35	0.37	0.39	0.41	0.45	0.48	0.54	0.58	0.61	0.66	0.7	0.76	0.82	0.82
2.4	0.35	0.35	0.35	0.37	0.39	0.41	0.43	0.45	0.51	0.54	0.58	0.61	0.66	0.7	0.76	0.76	0.82
2.5	0.37	0.37	0.37	0.39	0.41	0.41	0.45	0.48	0.51	0.54	0.58	0.61	0.66	0.7	0.7	0.76	0.82
2.6	0.39	0.39	0.39	0.41	0.41	0.43	0.45	0.48	0.51	0.54	0.58	0.61	0.66	0.7	0.7	0.76	0.76
2.7	0.41	0.41	0.41	0.41	0.43	0.45	0.48	0.48	0.54	0.54	0.58	0.61	0.66	0.66	0.7	0.76	0.76
2.8	0.41	0.41	0.43	0.43	0.45	0.48	0.48	0.51	0.54	0.58	0.58	0.61	0.66	0.66	0.7	0.7	0.76
2.9	0.43	0.43	0.43	0.45	0.45	0.48	0.48	0.51	0.54	0.58	0.58	0.61	0.66	0.66	0.7	0.7	0.76
3	**0.45**	0.45	0.45	0.48	0.48	0.48	0.51	0.54	0.54	0.58	0.58	0.61	0.61	0.66	0.7	0.7	0.7
3.1	0.48	0.48	0.48	0.48	0.48	0.51	0.51	0.54	0.54	0.58	0.58	0.61	0.61	0.66	0.66	0.7	0.7
3.2	0.48	0.48	0.48	0.48	0.51	0.51	0.54	0.54	0.58	0.58	0.58	0.61	0.61	0.66	0.66	0.7	0.7
3.3	0.48	0.48	0.51	0.51	0.51	0.54	0.54	0.54	0.58	0.58	0.61	0.61	0.61	0.66	0.66	0.7	0.7
3.4	0.51	0.51	0.51	0.51	0.54	0.54	0.54	0.58	0.58	0.58	0.61	0.61	0.61	0.66	0.66	0.7	0.7
3.5	0.51	0.51	0.54	0.54	0.54	0.54	0.54	0.58	0.58	0.58	0.61	0.61	0.61	0.66	0.66	0.66	0.7
3.6	0.54	0.54	0.54	0.54	0.54	0.54	0.58	0.58	0.58	0.58	0.61	0.61	0.61	0.66	0.66	0.66	0.7
3.7	0.54	0.54	0.54	0.54	0.54	0.58	0.58	0.58	0.58	0.58	0.61	0.61	0.61	0.66	0.66	0.66	0.7
3.8	0.54	0.54	0.54	0.54	0.58	0.58	0.58	0.58	0.58	0.61	0.61	0.61	0.61	0.66	0.66	0.66	0.7
3.9	0.58	0.58	0.58	0.58	0.58	0.58	0.58	0.58	0.58	0.61	0.61	0.61	0.61	0.66	0.66	0.66	0.66
4	0.58	0.58	0.58	0.58	0.58	0.58	0.58	0.58	0.61	0.61	0.61	0.61	0.61	0.66	0.66	0.66	0.66

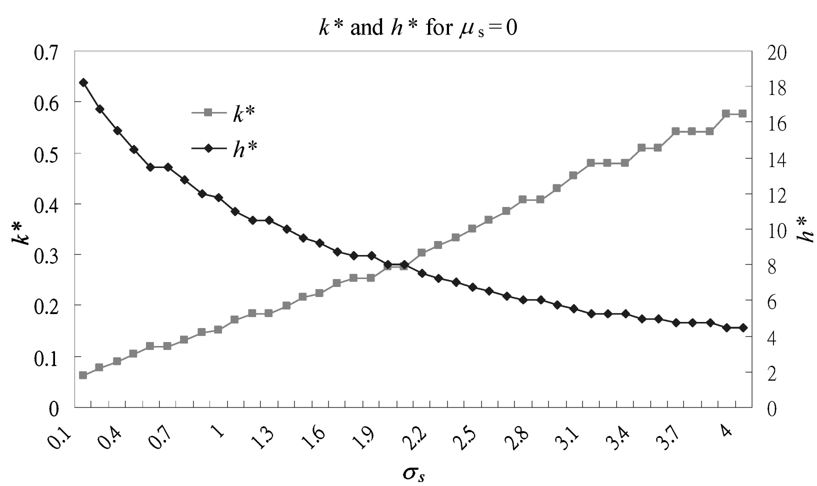


Fig. 7. Optimal CUSUM chart design: k^* and h^* for $\mu_s = 0$.

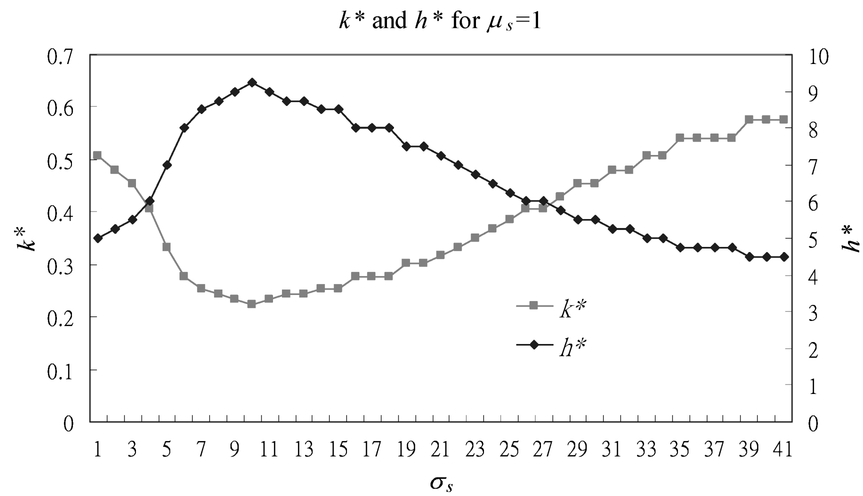
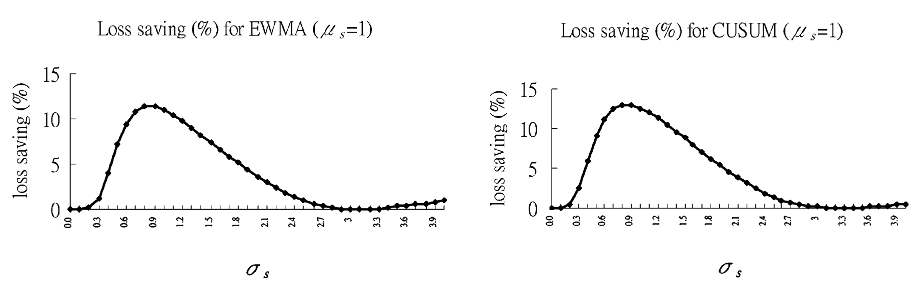


Fig. 8. Optimal CUSUM chart design: k^* and h^* for $\mu_s = 1$.

suggested that users should start with designs for $\mu_s = 0$. By choosing the control chart designs with $\mu_s = 0$, it implies that the variable shift size is thought to center around the target process mean and the larger the shift size the smaller the occurrence likelihood. This should be conceivable when there is no better idea about the shift location. As for what σ_s should be set for choosing a design, we provide worksheets (Appendix B) to help users choose and implement a suitable control chart. The worksheets contain the optimum control chart designs for $\mu_s = 0$ and various values of σ_s and their corresponding quality impacts incurred by shifts with a fixed size in the range from $0.25\sigma_x$ to $4.00\ \sigma_x$. For each shift size, the quality impact is calculated as if the shift is fixed at the location, i.e., $\sigma_s = 0$. Also shown in the table is the weight assigned to each shift size. For the default, all the weights are set equal (in fact equal to one). Practitioners can place emphases on certain shift sizes by increasing their weights. Then, we calculate the weighted average quality impacts by the proposed control chart designs and choose the design with the least average quality impact.

It is interesting to observe that both EWMA and CUSUM designs for $\sigma_s = 3\sigma_x$ (highlighted with boldface in Tables A1 and A2, respectively) have the smallest average quality impact when the shift sizes are deemed equally important, i.e., with equal weights, over the range of $[0.25\sigma_x, 4.00\sigma_x]$. Consequently, the chart designs for $\mu_s = 0$ and $\sigma_s = 3\sigma_x$, which have been discussed in detail in Section 4, are firstly recommended to naïve practitioners. If either an EWMA or a CUSUM chart has to be picked, the EWMA chart with $\lambda^* = 0.12$ and $L^* = 2.86$ appears to have the smallest average quality impact and should be most preferable. As more knowledge on the shift size is learned through proper estimates of shift sizes (see, for example, Chen and Elsayed (2000, 2002)), users can change their emphases on certain shift sizes by adjusting the weights on the worksheets accordingly to choose a more suitable chart design.

Example 2 When a quality engineer believes that the shift size varies over a range centering on the target mean but is particularly concerned about shift sizes not larger than $2\sigma_x$, the engineer can set the weights for shift sizes above $2\sigma_x$ to be zero on the worksheets. After using the worksheets to calculate the weighted average of the quality impacts for each design, it is found that both EWMA and CUSUM chart designs for $\sigma_s = 2\sigma_x$ are the design having the smallest average quality impact within its own class and the EWMA design with $\lambda^* = 0.06$ $L^* = 2.67$ has the smallest average



(a) (b)

Fig. 9. Loss savings for $\mu_s = 1$: (a) EWMA charts; and (b) CUSUM charts.

Table 3. Least quality impacts under $ARL_0 = 500$ and random shifts

	μ_s																
σs	0	0.25	0.5	0.75	1	1.25	1.5	1.75	2	2.25	2.5	2.75	3	3.25	3.5	3.75	4
0	0	841.24	359.57	248.56	204.12	185.84	176.99	172.30	170.76	171.55	173.34	176.07	179.19	183.41	188.82	195.56	204.55
0.1	1189.73	953.15	388.97	254.37	206.99	186.95	177.98	172.80	171.19	171.83	173.63	176.22	179.37	183.56	188.99	195.73	204.81
0.2	1136.53	905.78	495.73	285.84	217.13	191.55	180.96	174.77	172.45	172.77	174.40	176.76	179.87	184.10	189.48	196.34	205.47
0.3	979.29	848.68	576.33	356.65	246.04	202.66	186.31	178.33	174.88	174.41	175.60	177.72	180.76	185.06	190.33	197.38	206.52
0.4	859.39	785.55	607.07	426.17	298.01	228.79	198.18	185.59	179.29	177.17	177.53	179.23	182.14	186.46	191.60	198.88	208.00
0.5	772.18	727.70	607.83	470.63	352.40	269.63	221.85	198.87	187.16	182.08	180.69	181.58	184.21	188.02	193.36	200.70	209.94
0.6	705.17	674.19	594.57	493.46	394.57	313.12	255.50	220.45	201.93	190.75	186.29	185.42	187.00	190.26	195.75	202.86	212.38
0.7	651.13	630.74	576.49	501.90	422.57	350.24	291.43	249.02	222.53	205.52	195.23	191.14	190.89	193.61	198.62	205.66	215.14
0.8	609.65	595.76	557.97	501.95	439.51	378.28	323.83	279.63	247.54	226.42	209.70	200.65	197.50	198.48	202.33	209.34	218.37
0.9	577.37	567.68	540.23	497.66	448.78	398.40	350.27	308.10	274.21	248.96	228.89	214.31	207.02	205.19	207.88	213.59	222.49
1	552.05	544.88	523.09	491.46	453.01	411.93	370.83	332.60	299.65	272.76	251.29	232.23	220.41	215.34	215.14	219.54	227.44
1.1	530.00	524.34	508.35	484.76	454.15	420.79	386.16	352.69	322.32	295.82	272.78	252.42	236.57	227.76	225.12	227.40	233.78
1.2	511.89	507.74	495.94	477.40	453.54	426.44	397.51	368.82	341.72	316.87	293.81	273.48	255.29	243.26	237.39	237.23	242.00
1.3	497.44	494.39	485.54	470.37	451.84	429.85	405.86	381.55	357.94	335.39	313.37	292.75	274.68	260.12	251.91	249.30	251.70
1.4	485.81	483.19	475.67	464.22	449.25	431.58	411.97	391.67	371.42	351.33	330.90	310.90	293.25	278.01	267.55	262.81	263.44
1.5	475.18	473.23	467.62	459.03	446.69	432.42	416.49	399.69	382.48	364.84	346.27	327.45	310.18	295.95	284.25	277.58	276.03
1.6	466.76	465.32	461.17	453.79	444.40	432.81	419.92	406.09	391.61	376.28	359.56	342.25	325.83	311.81	300.50	292.73	289.83
1.7	460.10	458.84	455.19	449.54	442.14	433.04	422.65	411.25	399.13	385.81	370.95	355.25	340.05	326.63	315.98	307.91	303.51
1.8	453.96	453.04	450.37	446.26	440.10	432.99	424.72	415.48	405.33	393.77	380.62	366.58	352.71	340.18	329.87	322.47	317.44
1.9	449.29	448.63	446.72	443.12	438.63	432.94	426.41	419.01	410.44	400.40	388.82	376.34	363.87	352.36	342.61	335.24	330.36
2	445.56	444.99	443.35	440.83	437.41	433.12	427.95	421.82	414.54	405.86	395.74	384.71	373.62	363.19	354.11	346.94	342.05
2.1	442.45	442.08	440.99	439.17	436.46	433.18	429.09	424.14	417.93	410.35	401.53	391.90	382.06	372.66	364.30	357.46	352.47
2.2	440.39	440.12	439.19	437.77	435.96	433.39	430.15	425.96	420.62	414.04	406.37	397.93	389.26	380.86	373.22	366.75	361.75
2.3	438.58	438.39	437.85	436.98	435.46	433.51	430.89	427.43	422.76	417.05	410.37	403.00	395.38	387.86	380.89	374.82	369.89
2.4	437.54	437.45	436.97	436.24	435.22	433.73	431.53	428.42	424.48	419.49	413.65	407.23	400.49	393.78	387.44	381.75	376.89
2.5	436.64	436.56	436.33	435.82	435.00	433.72	431.86	429.23	425.72	421.35	416.31	410.67	404.69	398.72	392.92	387.57	382.81
2.6	436.13	436.05	435.81	435.47	434.76	433.69	432.01	429.68	426.68	422.80	418.39	413.42	408.10	402.73	397.40	392.36	387.74
2.7	435.67	435.64	435.49	435.10	434.55	433.58	432.07	430.03	427.25	423.88	419.97	415.57	410.81	405.96	401.04	396.25	391.74
2.8	435.26	435.21	435.10	434.77	434.22	433.24	431.89	430.06	427.61	424.64	421.13	417.19	412.90	408.46	403.88	399.33	394.88
2.9	434.87	434.82	434.65	434.37	433.74	432.90	431.70	429.99	427.76	425.04	421.90	418.34	414.44	410.31	406.03	401.65	397.31
3	434.43	434.36	434.15	433.81	433.29	432.45	431.29	429.68	427.65	425.19	422.34	419.08	415.48	411.62	407.54	403.35	399.04
3.1	433.84	433.79	433.64	433.25	432.71	431.92	430.81	429.31	427.40	425.12	422.48	419.45	416.09	412.44	408.53	404.44	400.20
3.2	433.23	433.17	432.97	432.65	432.09	431.31	430.17	428.75	427.00	424.86	422.37	419.50	416.31	412.80	409.07	405.06	400.84
3.3	432.61	432.56	432.33	431.93	431.40	430.54	429.51	428.09	426.39	424.39	422.02	419.27	416.20	412.80	409.17	405.23	401.06
3.4	431.84	431.77	431.56	431.17	430.54	429.76	428.66	427.35	425.66	423.72	421.42	418.79	415.80	412.48	408.92	405.02	400.85
3.5	431.02	430.94	430.69	430.27	429.70	428.85	427.78	426.44	424.83	422.89	420.64	418.08	415.15	411.89	408.37	404.50	400.34
3.6	430.10	430.02	429.78	429.37	428.71	427.87	426.81	425.45	423.87	421.94	419.70	417.17	414.29	411.05	407.56	403.71	399.55
3.7	429.14	429.04	428.77	428.31	427.68	426.85	425.71	424.39	422.76	420.87	418.61	416.09	413.23	410.01	406.53	402.69	398.51
3.8	428.05	427.96	427.69	427.25	426.58	425.67	424.56	423.22	421.56	419.64	417.40	414.86	412.01	408.78	405.31	401.47	397.26
3.9	426.96	426.87	426.56	426.06	425.36	424.45	423.35	421.92	420.27	418.30	416.06	413.50	410.64	407.41	403.93	400.07	395.85
4	425.70	425.60	425.29	424.79	424.09	423.19	421.98	420.55	418.87	416.87	414.63	412.02	409.14	405.90	402.41	398.52	394.29

Note: Shaded area: least quality impacts by CUSUM; Clear area: least quality impacts by EWMA.

impact among all designs. If the quality engineer is instead concerned about shifts with sizes not smaller than $2\sigma_x$, then the weights for shift sizes below $2\sigma_x$ are set to zero to find that the CUSUM chart design, $k^* = 0.57$, $h^* = 4.5$, for $\sigma_s = 4\sigma_x$ has the smallest average quality impact among all designs and should be chosen. ◄

Only when sufficient knowledge of the shift location (μ_s) and its spread (σ_s) has been accumulated and ascertained, may the users advance to design the control chart for the anticipated μ_s and σ_s using procedures provided in Section 4. With both the optimum EWMA and CUSUM chart designs available, we are naturally led to ask which type of control chart, EWMA or CUSUM, is best for a given random-shift situation. To naïve practitioners without preference for the type of control charts, the EWMA chart design would be recommended by this research because of the EWMA chart's less average quality impacts shown in Tables A1 and A2.

In the literature, the CUSUM procedure for detecting a fixed shift has been shown by Lorden (1971), and further by Moustakides (1986), to minimize the essential supremum of conditional average delay time. It is shown that this minimum value is actually equal to the ARL_1 for the case of CUSUM charts. However, as pointed out by Srivastava and Wu (1993), this equality does not hold in the case of EWMA charts. Comparison of the ARL_1 performance between CUSUM and EWMA charts is thus not conclusive until more thorough comparisons done by Lucas and Saccuci (1990) (L-S) and by Srivastava and Wu (1997). In particular, the L-S comparison has concluded that the EWMA charts perform relatively better in the case of smaller fixed shifts.

To select a superior type of control charts given a random shift (μ_s, σ_s), instead of a fixed shift, we compare the quality impacts, rather than ARL_1, made by respective optimal EWMA and CUSUM designs and select the design with a lower quality impact in Table 3. The shaded cells in Table 3 show the quality impacts by the CUSUM chart designs while the rest of the table shows the quality impacts by the EWMA chart designs. Examining the shaded area in the table, we can conclude that only when the uncertain shift is relatively large ($\mu_s \geq 1.5\sigma_x$) with a small spread ($\sigma_s \leq 1.9\sigma_x$), would the CUSUM chart designs be preferred over the EWMA charts. This result is quite comparable to the L-S comparison where CUSUM charts are shown to perform better than EWMA charts in the case of larger shifts. In addition, our study further shows that the uncertain large shift should also have a small spread for the CUSUM charts to perform better in terms of impacts on the quality.

Table 3 is very useful for practitioners to pick the best chart design. Given a random-shift situation (μ_s, σ_s), the practitioner can first look up Table 3 to see whether or not the (μ_s, σ_s) cell is shaded. If the cell is shaded, the CUSUM chart design should be used and Table 2 and Fig. 6 are used to find the best CUSUM design. Otherwise, Table 1 and Fig. 1 should be used to find the optimal EWMA design.

Example 3 After the process excursion is better understood and μ_s and σ_s are estimated to be three and 1.5, respectively, the engineer can look up Table 3 and observe that it falls in the unshaded area. That is, a EWMA chart is preferred over the CUSUM chart. The engineer can then turn to Table 1 to find $\lambda^* = 0.32$. $L^* = 3.02$ is then obtained from Fig. 1. ◄

7. Conclusions

This paper is the first in the literature to design EWMA and CUSUM charts for uncertain shift sizes with different levels of quality impacts. It is found that when shifts are uncertain in size the optimal designs for both EWMA and CUSUM charts should be more conservative, i.e., the optimal designs for random shifts are comparable to conventional designs for smaller deterministic shifts. For naïve practitioners, the EWMA chart design for $\mu_s = 0$ and $\sigma_s = 3$, i.e., $\lambda^* = 0.12$ and $L^* = 2.86$, is recommended as a very good control chart to start with. We also find that the CUSUM chart performs better only when shifts are more certain and large. While more advanced control charts, such as combined and adaptive control charts, are not studied here and only cases of normally distributed shifts with $ARL_0 = 500$ are discussed in this research, the proposed chart design procedure can be easily extended by following the same procedure presented in Sections 3 and 4 of this paper.

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Appendices

Appendix A

Definition of stopping time: An integer-valued random variable N is said to be a *stopping time* for the sequence $X_1, X_2, \ldots$ if the event $\{N = n\}$ is independent of $X_{n+1}, X_{n+2}, \ldots$ for all $n = 1, 2, \ldots$

Wald's Equation: If $X_1, X_2, \ldots$ are iid random variables having finite expectations, and if N is a stopping time for $X_1, X_2, \ldots$ such that $E[N] < \infty$, then:

$$E\left[\sum_{i=1}^{n} X_i\right] = E[N]E[X],$$
$$E(\mathrm{QL}_{1|s}) = \mathrm{ARL}_{1|s}E[\mathrm{QL}_s].$$

Proof. Let random variables $\{X_i\}_{i=1}^{\infty}$ be a sequence of quality observations where: $\{X_i\}_{i=1}^{\infty} \overset{\text{iid}}{\sim} N(T + s, \sigma_x^2)$.

Given the known shift size s, let the event $\{\mathrm{RL}_{1|s} = n\}$ correspond to an out-of-control event detected by a control chart after having observed $X_1, X_2, \ldots, X_n$. Since the out-of-control event is independent of the observations yet to come, namely, $X_{n+1}, X_{n+2}, \ldots$, $\mathrm{RL}_{1|s}$ is a stopping time. The quality loss given a shift size s is then calculated as

$$\mathrm{QL}_{1|s} = \sum_{i=1}^{\mathrm{RL}_{1|s}} \mathrm{QL}(X_i \mid \mu_x = T + s).$$

Since $\mathrm{RL}_{1|s}$ is a stopping time, by Wald's equation:

$$E(\mathrm{QL}_{1|s}) = E(\mathrm{RL}_{1|s})E(QL(X_i \mid \mu_x = T + s))$$
$$= \mathrm{ARL}_{1|s}E(\mathrm{QL}_s).$$

■

Appendix B

Table A1. Worksheet for selection of a starting EWMA control chart design

EWMA designs			*Weights*																*Weight sum 8*
			1	*1*	*1*	*1*	*1*	*1*	*1*	*1*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*avg. quality*
σs	λ	L	$\mu = 0.25$	$\mu = 0.50$	$\mu = 0.75$	$\mu = 1.00$	$\mu = 1.25$	$\mu = 1.50$	$\mu = 1.75$	$\mu = 2.00$	$\mu = 2.25$	$\mu = 2.50$	$\mu = 2.75$	$\mu = 3.00$	$\mu = 3.25$	$\mu = 3.50$	$\mu = 3.75$	$\mu = 4.00$	*impact*
0.1–0.5	0.01	1.973	842.4917	420.3798	337.6636	317.2988	322.6838	339.5717	363.671	392.0474	423.692	457.6055	493.4807	530.8546	569.5651	609.3433	650.556	693.4773	416.975 9709
0.6–1	0.02	2.278	841.2441	382.2891	297.2274	274.7251	276.8683	289.7139	309.1692	332.5155	358.8301	387.2091	417.3883	448.9988	481.974	515.835	550.031	584.5741	375.469 0746
1.1–1.3	0.03	2.4371	870.2137	366.5356	276.7065	251.9636	251.8724	262.2227	278.9318	299.3541	322.6032	347.8247	374.7552	402.943	432.1823	462.3785	494.2644	528.5169	357.225 0545
1.4–1.6	0.04	2.5405	910.9161	360.39	264.6255	237.5598	235.6508	244.1577	258.923	277.3131	298.4483	321.4815	346.1344	372.0356	399.3075	428.1388	458.7843	490.3104	348.692 0129
1.7–1.9	0.05	2.6151	957.0835	359.5726	257.1361	227.6671	224.1546	231.1657	244.4184	261.2541	280.7865	302.1813	325.2143	349.6294	375.5876	402.8745	430.8241	458.0903	345.306 5026
2–2.2	0.06	2.6723	1005.879	362.1513	252.5143	220.5618	215.5672	221.2926	233.2966	248.8737	267.1302	287.2516	309.0568	332.2777	356.848	382.1558	407.2737	431.4779	345.017 0991
2.3–2.4	0.07	2.718	1055.902	367.1144	249.8622	215.344	208.9384	213.5144	224.4457	238.9644	256.1669	275.2458	296.0109	318.0993	341.2119	364.6302	387.72	410.5772	346.760 6675
2.5–2.7	0.08	2.7556	1106.379	373.8646	248.6559	211.4874	203.7116	207.2313	217.2135	230.8151	247.1147	265.2927	285.1122	306.1122	327.8821	349.8358	371.8394	394.5919	349.919 7765
2.6–2.7	0.09	2.7872	1156.803	382.0119	248.5611	208.6584	199.5339	202.0622	211.1848	223.9714	239.4729	256.8427	275.7881	295.79	316.4535	337.435	359.0765	382.3756	354.098 3306
2.8	0.1	2.8143	1207.087	391.3302	249.3725	206.6463	196.1794	197.7612	206.0904	218.1373	232.9171	249.5493	267.6964	286.8294	306.6545	327.0813	348.8009	372.8535	359.075 54
2.9	0.11	2.8378	1256.909	401.6096	250.9254	205.2897	193.4778	194.1449	201.7304	213.0948	227.2119	243.1675	260.5991	279.0048	298.2324	318.4008	340.4015	365.1413	364.647 7247
3–3.1	**0.12**	**2.8583**	**1305.932**	**412.6713**	**253.0905**	**204.4666**	**191.3024**	**191.0773**	**197.9569**	**208.6836**	**222.1868**	**237.5225**	**254.3223**	**272.1352**	**290.9549**	**311.0451**	**333.3639**	**358.5813**	**370.647 6154**
3.2–3.3	0.13	2.8765	1354.461	424.4821	255.8167	204.1152	189.582	188.4779	194.6807	204.8058	217.7374	232.5068	248.7552	266.0913	284.6381	304.7358	327.3059	352.734	377.052 7076
3.4	0.14	2.8928	1402.522	436.971	259.0447	204.1752	188.2512	186.2752	191.8233	201.3765	213.7741	228.0265	243.7952	260.747	279.1054	299.229	321.929	347.2984	383.804 9289
3.5–3.6	0.15	2.9073	1449.525	449.9698	262.6841	204.5739	187.2394	184.3965	189.3077	198.3144	210.2108	223.9905	239.3399	255.9759	274.1906	294.3148	317.0028	342.0715	390.751 3591
3.7–3.9	0.16	2.9204	1495.861	463.4995	266.7278	205.2927	186.5212	182.8105	187.0984	195.5796	207.0043	220.3502	235.3297	251.6977	269.7838	289.857	312.3902	336.9565	397.923 8459
4	0.17	2.9323	1541.554	477.524	271.1467	206.3031	186.0653	181.483	185.1574	193.131	204.1095	217.0549	231.703	247.8332	265.785	285.743	307.992	331.8992	405.29 55504

Table A2. Worksheet for selection of a starting CUSUM control chart design

CUSUM designs			Weights																Weight sum 8
			1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	avg. quality
σ_s	k	h	$\mu = 0.25$	$\mu = 0.50$	$\mu = 0.75$	$\mu = 1.00$	$\mu = 1.25$	$\mu = 1.50$	$\mu = 1.75$	$\mu = 2.00$	$\mu = 2.25$	$\mu = 2.50$	$\mu = 2.75$	$\mu = 3.00$	$\mu = 3.25$	$\mu = 3.50$	$\mu = 3.75$	$\mu = 4.00$	impact
0.2	0.0762	16.75	966.3902	494.0494	400.3978	377.6591	384.5266	404.6739	433.1368	466.5114	503.6143	543.2868	585.1583	628.6964	673.7685	720.1418	767.8462	816.2638	490.918 1456
0.3	0.0904	15.5	950.646	471.7524	378.9359	355.8378	361.4247	379.7751	406.0867	437.0922	471.6629	508.6941	547.8356	588.5869	630.8607	674.3544	718.7678	763.876	467.693 8399
0.4	0.1037	14.5	942.8321	454.7228	362.1036	338.5784	343.0858	359.9693	384.5421	413.6404	446.1771	481.0889	518.0455	556.5732	596.5429	637.5689	679.651	723.264	449.934 311
0.5–0.6	0.119	13.5	939.7678	438.4808	345.5762	321.4867	324.862	340.2509	363.0689	390.2499	420.7441	453.5317	488.298	524.5688	562.1933	600.9704	641.2752	683.0734	432.967 8958
0.7	0.1322	12.75	942.0752	427.1005	333.4989	308.8497	311.3223	325.5624	347.0479	372.7791	401.7345	432.9224	466.0331	500.6019	536.5573	573.8296	612.5321	651.8218	421.029 4964
0.8	0.1472	12	949.3905	416.61	321.7685	296.4054	297.9181	310.9787	331.1142	355.3844	382.7916	412.3709	443.8222	476.7232	511.0752	546.6884	583.1412	619.4875	409.946 2175
0.9	0.1526	11.75	952.7945	413.2884	317.9194	292.2881	293.4694	306.1311	325.8134	349.5943	376.4837	405.5256	436.4264	468.7801	502.5843	537.5609	573.1728	608.6808	406.412 3158
1	0.1705	11	968.234	404.2957	306.7492	280.1462	280.2707	291.7034	310.0069	332.307	357.6348	385.0608	414.3164	445.016	477.0467	509.9096	543.2057	577.2238	396.714 1352
1.1–1.2	0.1838	10.5	982.0535	399.002	299.5477	272.1747	271.5508	282.1418	299.5133	320.818	345.0996	371.4473	399.6019	429.1547	459.8987	491.3701	523.6233	557.4841	390.850 2265
1.3	0.1986	10	1000.602	394.6473	292.6964	264.3919	262.9598	272.6787	289.1011	309.3991	332.6271	357.8906	384.9238	413.2835	442.7417	473.0457	504.7247	538.6243	385.809 5274
1.4	0.215	9.5	1023.835	391.2947	286.2006	256.7904	254.4876	263.3032	278.7588	298.0386	320.2049	344.3723	370.2598	397.4131	425.6977	455.1312	486.4376	519.9149	381.588 6136
1.5	0.2238	9.25	1036.895	389.976	283.072	253.0449	250.2849	258.6381	273.6043	292.371	314.003	337.6172	362.9267	389.4904	417.2534	446.3325	477.3852	510.3005	379.735 7232
1.6	0.2431	8.75	1068.645	388.5864	277.2653	245.7874	242.0351	249.4241	263.3899	281.1159	301.6669	324.1634	348.3218	373.764	400.5947	428.9604	459.0801	490.043	377.031 1769
1.7–1.8	0.2536	8.5	1086.982	388.5062	274.575	242.2625	237.9759	244.8643	258.3194	275.5179	295.5229	317.4577	341.0507	365.9647	392.3522	420.2765	449.647	479.2803	376.125 4438
1.9–2	0.2766	8	1129.62	389.9015	269.7371	235.4761	230.024	235.8638	248.2713	264.3973	283.2996	304.1121	326.6027	350.4957	375.9174	402.6218	429.9482	456.6248	375.411 3973
2.1	0.3027	7.5	1181.535	393.8786	265.7987	229.1198	222.3406	227.0535	238.3707	253.3974	271.1839	290.8798	312.2853	335.1146	359.3103	384.3021	409.2134	433.2965	376.436 7438
2.2	0.3171	7.25	1211.485	397.0658	264.2487	226.1388	218.6201	222.7334	233.4861	247.9514	265.1747	284.3111	305.163	327.4073	350.8676	374.88	398.6506	421.8401	377.716 1687
2.3	0.3325	7	1244.247	401.1821	263.0245	223.3084	214.9906	218.476	228.6491	242.5442	259.1995	277.7708	298.0497	319.6609	342.319	365.3404	388.1332	410.7899	379.552 7161
2.4	0.349	6.75	1280.047	406.3604	262.1758	220.6501	211.4637	214.2885	223.8652	237.1799	253.2601	271.256	290.9353	311.8651	333.6837	355.7677	377.8173	400.3201	382.003 8264
2.5	0.3667	6.5	1318.933	412.7365	261.7554	218.1856	208.051	210.1782	219.139	231.8612	247.3563	264.7618	283.8111	304.0216	325.004	346.2677	367.8529	390.5515	385.104 959
2.6	0.3858	6.25	1361.843	420.5993	261.875	215.968	204.785	206.1676	214.4881	226.6018	241.4977	258.2932	276.6841	296.1567	316.3519	336.9642	358.37	381.5365	389.041 0415
2.7–2.8	0.4064	6	1408.557	430.097	262.5967	214.0216	201.6761	202.2616	209.9141	221.4008	235.6808	251.8456	269.5585	288.3012	307.8011	327.9595	349.4364	373.2229	393.815 5954
2.9	0.4288	5.75	1460.61	441.7068	264.1074	212.4348	198.7782	198.4973	205.4455	216.2808	229.9242	245.4387	262.467	280.5165	299.4436	319.3434	341.0638	365.4683	399.732 578
3	**0.4531**	**5.5**	**1517.337**	**455.587**	**266.4881**	**211.2381**	**196.1024**	**194.8781**	**201.0812**	**211.2384**	**224.2246**	**239.0753**	**255.4295**	**272.8478**	**291.336**	**311.1361**	**333.1602**	**358.0124**	**406.743 7465**
3.1–3.3	0.4795	5.25	1578.697	472.0485	269.8873	210.4977	193.6826	191.4226	196.8333	206.2818	218.5913	232.7723	248.4797	265.3432	283.5116	303.3014	325.554	350.5204	414.918 8292
3.4–3.5	0.5085	5	1647.404	491.9845	274.6719	210.3871	191.6213	188.2003	192.7547	201.4558	213.0676	226.5787	241.6753	258.0574	275.982	295.7525	318.0172	342.6314	424.809 978
3.6–3.8	0.5402	4.75	1721.377	515.5115	280.967	210.9608	189.9374	185.2151	188.8427	196.7561	207.6529	220.5007	235.0239	250.9775	268.6698	288.3059	310.2483	333.9741	436.195 9778
3.9–4	0.5751	4.5	1802.138	543.5187	289.1993	212.4244	188.7459	182.539	185.1502	192.2264	202.3877	214.5761	228.5491	244.0862	261.4777	280.7626	301.9722	324.2791	449.492 6605

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