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An optimal thermal destratification control system synthesis in aquaculture ponds

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A linear dynamic model describing the behavior of temperature variation and thermal stratification in aquaculture ponds is used in developing optimal feedback controllers for a thermal destratification control system. Linear quadratic regulators (LQR's) with output feedback control of a linear-invariant system are chosen for this study. Both impulse and step disturbances are taken into account and an optimal proportional plus integral (PI) feedback controller is synthesized. To illustrate this procedure, the design is applied to control the thermal stratification in a typical shallow aquaculture pond. A sensitivity analysis is also performed to study the sensitivity of the objective function with respect to the controller gains. Feedback gains associated with temperature at middepth are shown to be far more sensitive compared with that of temperatures at other water depths. This result provides useful information in order to install the control devices at an optimum mixing depth.

Keywords: thermal stratification, linear quadratic regulator, optimal PI controller, aquaculture pond

1. Introduction

The transport of energy and mass by advection and diffusion can be reduced as aquaculture ponds become stratified.[1] Benthic conditions of low temperature and dissolved oxygen can severely impact animal growth and survival.[2,3] Mechanical destratification of water can be used to cycle surface water to bottom layers, aiding the destratification of water temperature and the natural digestion of organic matter.[4,5] Moreover, large volumes of water can be transferred with minimal disturbance of the ecobalance.

Another feasible means is to use submersible pumping technology to rectify thermal stratification problems.[6] Solutions include enhancing water exchange and an ingenious destratification concept, i.e., to breathe new life into stagnated water where the natural oxygen exchange between water layers at different temperatures no longer takes place.

There are various pneumatic and electrical devices that can be employed to carry out the control action for artificial mixing such as destratification, hypolimnetic aeration, and epilimnetic mixing.[7] For example, a submersible mixer can be positioned under the water to mechanically stimulate circulation.[8] This promotes oxygen uptake and the breakdown of thermal stratification, stimulates the biological breakdown of dead vegetation, and gives new life to the aquaculture water.

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A mathematical model to simulate stratification in shallow aquaculture ponds based upon discrete, completely mixed, and horizontal volume elements assumptions has been described by Losordo and Piedrahita (referred to as the L-P Model, see Appendices A and B).[9] They used theoretical and empirical relationships applied to heat balance calculations in lakes, reservoirs, and waste treatment ponds to calculate the energy exchanges between the pond's surface and the atmosphere. Energy transfer between volume elements, however, is considered the result of the turbulent mixing, which are the functions of the temperature gradient and an effective diffusion coefficient. Output from the L-P Model allows for accurate predictions of the time of maximum stratification and completely mixed pond conditions.

Classical control engineering is largely concerned with the design of dynamic feedback controllers to compromise between conflicting requirements for accuracy and for closed-loop stability. From the practical engineering point of view, optimal control theory is valuable mainly as a source of insight into the structure required for feedback controllers. A further insight is that dynamics in feedback controllers serve the purpose of estimation. Familiar controllers such as the lead-lag compensator and the PI controller can be regarded as the cascade of a dynamic linear observer and a certainty-equivalent control law, each of which is approximately matched to a reduced order-linearized model of the controlled process.[10]

The purpose of this paper is to present initial considerations on the use of optimal control theory for designing a feedback control system to efficiently control thermal stratification in shallow aquaculture ponds. The

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structure of the primary system or plant is based on the L-P Model. Control logic developed for breakdown of thermal destratification will incorporate control logic algorithms used for dissolved oxygen and other environmental factors of aquaculture ponds. A sensitivity analysis in an optimal control system is also presented.

2. System control model

The temperature of each volume element in the water column and the sediment volume element heat transfer components (*Figure 1*)[9] are determined by first calculating the next flux of heat for the specific volume element. The heat balance calculations are classified according to the sources and sinks of heat involved at the specific depth in the water column. Therefore, taking into account the heat balance for surface, mid-depth, bottom, and sediment volume elements, a vector-matrix ordinary differential equation that describes the heat energy transport in four water volume elements can be derived from the L-P Model as:

$$\{\dot{H}(t)\} = [K_1]\{\phi_1(t)\}[A] \tag{1}$$

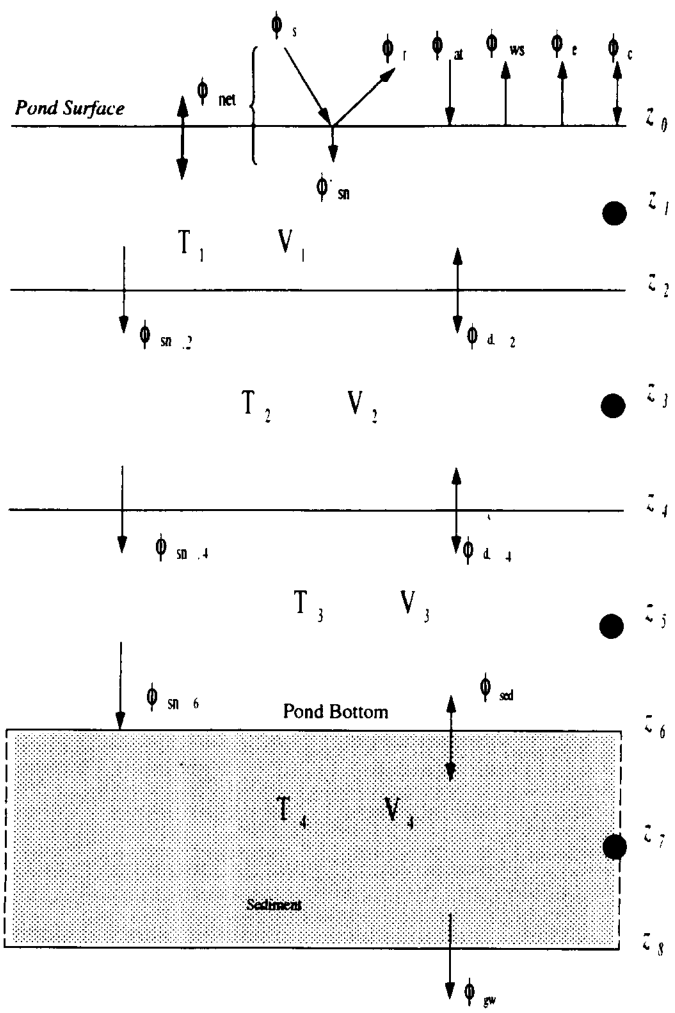

Figure 1. Water column and sediment volume element heat transfer components in a shallow aquaculture pond

where $\{\mathbf{H}\} = \{\mathbf{H}_1, \mathbf{H}_2, \mathbf{H}_3, \mathbf{H}_4\}^{\mathrm{T}}$, heat energy vector (kJ); $\{\phi_1\} = \{\phi_{\mathrm{net}}, \phi_{\mathrm{sn},2}, \phi_{\mathrm{sn},4}, \phi_{\mathrm{sn},6}, \phi_{\mathrm{d},2}, \phi_{\mathrm{d},4}, \phi_{\mathrm{sed}}, \phi_{\mathrm{gw}}\}^T$, heat flux vector (kJ m^{-2} hr^{-1}) (Appendix A); ϕ_{net} = net heat flux passing the air/water interface; $\phi_{\mathrm{sn},i}$ = penetrating short-wave solar irradiance at depth z_i, $i = 2, 4, 6$; $\phi_{\mathrm{d},i}$ = effective diffusion of heat at depth z_i, $i = 2, 4, 6$; ϕ_{sed} = heat transfer between bottom volume element and sediment volume element; ϕ_{gw} = heat loss from the sediment volume element to the ground water table; $[\mathbf{A}] = \mathrm{diag}[\mathbf{A}_1, \mathbf{A}_2, \mathbf{A}_3, \mathbf{A}_4]$, diagonal surface area matrix (m^2); and $[\mathbf{K}_1]$ = a coefficient matrix:

$$[K_1] = \begin{bmatrix} 1 & -1 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & -1 \end{bmatrix}$$

Based on equation (1), a general model that can represent the temperature behavior in any volume element at time t may be written in the form as:

$$\{\dot{T}(t)\} = [K_2]\{\phi_1(t)\} \tag{2}$$

where: $\{\mathbf{T}(t)\} = \{\mathbf{T}_1(t), \mathbf{T}_2(t), \mathbf{T}_2(t), \mathbf{T}_4(t)\}^{\mathrm{T}}$, $[\mathbf{K}_2] = [\mathbf{K}_1][\mathbf{C}]$, and $[\mathbf{C}] = (\rho_w^{-1} C_{\mathrm{pw}}^{-1})\mathrm{diag}[z_1^{-1}, z_2^{-1}, z_3^{-1}, z_4^{-1}]$ (kJ^{-1} m^2 °C), in which ρ_w = water density (kg m^{-3}), and C_{pw} = specific heat of waer (kJ kg^{-1} °C^{-1}).

After some mathematical manipulations, equation (2) can be rewritten as a linear dynamic equation in terms of a state-space representation:

$$\{\dot{T}(t)\} = [A]\{T(t)\} + [B]\{\phi(t)\} + [C]\{W(t)\} \tag{3}$$

where $[\mathbf{A}] = [\mathbf{C}][\mathbf{K}]$, a system state matrix contains the essential dynamic characters of the system being studied; $[\mathbf{B}] = [\mathbf{C}][\mathbf{K}_3]$, a control input matrix; $\{\phi\} = \{\phi_{\mathrm{net}}, \phi_{\mathrm{sn},2}, \phi_{\mathrm{sn},4}, \phi_{\mathrm{sn},6}\}^T$, short-wave solar irradiance-derived heat flux vector; $\{W\} = \{0, 0, 0, w_0\}$, heat conduction-derived heat flux vector. The matrices $[K]$ and $[K_3]$ can be written as follows, respectively:

$$[K] = \begin{bmatrix} -k_1 & k_1 & 0 & 0 \\ k_1 & -(k_1 + k_2) & k_2 & 0 \\ 0 & k_2 & -(k_2 + k_3) & k_3 \\ 0 & 0 & k_3 & -(k_3 + k_4) \end{bmatrix}$$

$$[K_3] = \begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

where $k_1 = \rho_w C_{\mathrm{pw}} E_{z,2}/\Delta z$, $k_2 = \rho_w C_{\mathrm{pw}} E_{z,4}/\Delta z$, $k_3 = k_{\mathrm{sed}}/\Delta z$, $k_4 = k_e/\Delta z$, and $w_0 = (k_e/\Delta z)T_{\mathrm{gw}}(t)$; $E_{z,i}$ = effective diffusion coefficient at depth z_i (m^2 h^{-1}); k_{sed} and k_e = thermal conductivity coefficients for sediment and earth, respectively ($\cong 2.53$ kJ m^{-1} hr^{-1} °C^{-1});[9] T_{gw} = temperature of ground water; and Δz = distance between center of the volume element.

A derivative variable can be defined as, $\Delta y_i = y_i - y_{is}$, in which y_i is the ith element of vector $\{\mathbf{y}\}$, and Δy_i is the difference between y_i and a steady-state value or a set point, y_{is}. By introducing the derivative variables into equation (3), a system control model then can be derived as

$$\{\dot{X}(t)\} = [A]\{X(t)\} + [B]\{U(t)\} + [C]\{V(t)\} \quad (4)$$

in which: $\{\mathbf{X}\} \equiv \{\Delta T\} = \{\mathbf{T}\} - \{\mathbf{T}_S\}$, a system state variable vector; $\{\mathbf{U}\} \equiv \{\Delta\phi\} = \{\phi\} - \{\phi_s\}$, a system controllable input variable vector; and $\{\mathbf{V}\} \equiv \{\Delta W\} = \{\mathbf{W}\} - \{\mathbf{W}_S\}$, a system uncontrollable input variable vector.

Equation (4) can be used to describe the dynamic behavior of temperature variation and thermal stratification in aquaculture ponds. Equation (4) is a continuous form ordinarily employed in modern estimation and control theory. The stability criterion for the solution of a state-space model (e.g., equation [4]) has been discussed by Liao and Feddes.[11]

3. Optimal feedback control synthesis

In order to derive the optimal feedback control strategy for the system, the linear quadratic regularor (LQR) with state feedback will be considered. The LQR has been well developed for the past two decades. It is not possible nor economically feasible, however, to measure all temperatures at all locations within an aquaculture ponds. Because of nonlinearity and uncertainty in even the simplest model, general procedures for designing linear controllers to match linear controlled processes shall be restricted to a suboptimal design and are obtained in the form of a suboptimal control law.[12] Effectiveness of the resulting feedback controller was investigated by simulating its performance when controlling the simplest mathematical model. Therefore the problem of obtaining the suboptimal output feedback control of a time-invariant system is most suitable for the system introduced.

Practically it is not always possible to have all the state variables available for feedback. Rather than reconstructing the state variables via a Kalman filter or some form of a state estimator, it is preferable to generate the control variables by taking linear combinations of the available output variables. The state output vector of interest to the problems is, $\{Z(t)\} = [H]\{X(t)\}$, where: $\{\mathbf{Z}(t)\}$ = state output variable vector, and $[\mathbf{H}]$ = constant matrix of output state vector.

Therefore, when considering again the plant in equation (4), a more general form is as follows:

$$\{\dot{X}\} = [A]\{X\} + [B]\{U\} + [C]\{V\}, \quad (5a)$$

$$\{X(0)\} = \{X_0\}, \{U(0)\} = \{U_0\}, \quad (5b)$$

$$\{Z\} = [H]\{X\} \quad (5c)$$

and is subjected to constant disturbances,

$$\{V_i(t)\} = \{v_i\}, i = 1, 2, \ldots, n \quad (5d)$$

The quadratic cost function to be minimized can be written as[13]:

$$J = 1/2\int_0^{t_f}(\{Z\}^T[Q]\{Z\} + \{\dot{U}\}^T[R]\{\dot{U}\})dt$$

$$= 1/2\int_0^{t_f}(\{X\}^T[M]\{X\} + \{\dot{U}\}^T[R]\{\dot{U}\})dt \quad (6)$$

where: $[\mathbf{M}] = [\mathbf{H}]^T[\mathbf{Q}][\mathbf{H}]$, in which $[\mathbf{Q}]$ is a positive semidefinite weighting matrix, and $[\mathbf{R}]$ is a positive definite control weighting matrix.

If the linear plant in equation (6) is observable, $[\mathbf{M}]$ is a positive semidefinite output weighting matrix when $[\mathbf{Q}]$ is positive semidefinite. To ensure that the controller can derive the system for a specified period of time, the system must be controllable. Observability and controllability of the plant are guaranteed if and only if the following matrices

$$[\Xi] \equiv [[H]^T|[A]^T[H]^T|\cdots|([A]^{n-1})^T[H]^T]$$

and

$$[\Theta] \equiv [[B]|[A][B]|\cdots|[A]^{n-1}[B]]$$

have rank of n (i.e., $|[\Xi][\Xi]^T| \neq 0$, and $|[\Theta][\Theta]^T| \neq 0$), respectively.[14]

The derivative of equation (5) with respect to time, yields:

$$\{\ddot{X}\} = [A]\{\dot{X}\} + [B]\{\dot{U}\}, \quad \{X(0)\} = \{X_0\},$$

$$\{U(0)\} = \{U_0\} \quad (7a)$$

$$\{\dot{X}(0)\} = [A]\{X_0\} + [B]\{U_0\} + [C]\{V\} \quad (7b)$$

$$\{\dot{Z}\} = [H]\{\dot{X}\} \quad (7c)$$

and defining the following new variables:

$$\{\omega\} \equiv \{\dot{X}\}, \quad \{\theta\} \equiv \{\dot{U}\}, \quad \{\xi\} \equiv \{\dot{Z}\}$$

Therefore, equations (5) and (7) can be reduced to the following pair of vector-matrix differential equations:

$$\{\dot{X}\} = \{\omega\}, \{X(0)\} = \{X_0\} \quad (8a)$$

$$\{\dot{\omega}\} = [A]\{\omega\} + [B]\{\theta\}, \quad (8b)$$

$$\{\omega(0)\} = [A]\{X_0\} + [B]\{U_0\} + [C]\{V\}, \quad (8c)$$

$$\{Z\} = [H]\{X\} \quad (8d)$$

$$\{\xi\} = [H]\{\omega\} \quad (8e)$$

Equation (8) can be compactly expressed as:

$$\{\dot{\eta}\} = [A_a]\{\eta\} + [B_a]\{\theta\}, \quad \{\eta(0)\} = \{\eta_i\} \quad (9a)$$

$$\{q\} = [H_a]\{\eta\} \quad (9b)$$

where:

$$\{\eta\} = \{\{X\}|\{\omega\}\}^T$$

$$\{\eta_i\} = \{\{X_0\}|[A]\{X_0\} + [B]\{U_0\} + [C]\{V\}\}^T$$

$$[A_a] = \begin{bmatrix} [O] & [I] \\ [O] & [A] \end{bmatrix}, \quad [B_a] = \begin{bmatrix} [O] \\ [B] \end{bmatrix}$$

$$[H_a] = \mathrm{diag}[[H]|[H]], \quad \{q\} = \{\{Z\}|\{\xi\}\}^T$$

Therefore, equation (6) in terms of the new variables becomes:

$$J = 1/2\int_0^{t_f}(\{\eta\}^T[M_a]\{\eta\} + \{\theta\}^T[R]\{\theta\})\,dt \quad (10)$$

where:

$$[\mathbf{M}_a] = \mathrm{diag}[[\mathbf{M}]|[0]] = \mathrm{diag}\,[[\mathbf{H}]^T[\mathbf{Q}]\,[\mathbf{H}]|[0]].$$

Matrix $[\mathbf{M}_a]$ becomes positive semidefinite since $[\mathbf{M}]$ is positive semidefinite. Thus, the original output LQR in equations (5) and (6) can be restated in equation (10).

An optimal control vector ($\{\hat{\theta}(t)\}$) which minimizes equation (10) subject to a given linear plant in equation (9) can be found. This alternative problem is recognized as a LQR. The solution of the optimal control vector is given by the well-known expression:

$$\{\hat{\theta}(t)\} = -[R]^{-1}[B_a]^T[P]\{\hat{\eta}(t)\} \tag{11}$$

where $[\mathbf{P}] = [\mathbf{P}]^T$ is the positive definite solution of the Riccati equation:

$$\begin{aligned}[\dot{P}] = &-[P][A_a] - [A_a]^T[P] \\ &+ [P][B_a][R]^{-1}[B_a]^T[P] \\ &- [M_a], [P(t_f)] = 0\end{aligned} \tag{12}$$

If the plant in equation (9) is observable and controllable, then as t_f approaches infinity in equation (10), the LQR becomes a suboptimal controller design, and the suboptimal control vector in equation (11) becomes:

$$\begin{aligned}\{\hat{\theta}(t)\} &= -[R]^{-1}[B_a]^T[P^*]\{\hat{\eta}(t)\} \\ &= -[R]^{-1}[B_a]^T[P^*][H_a]^{-1}\{\hat{q}(t)\}\end{aligned} \tag{13}$$

where $[\mathbf{P}^*] = [\mathbf{P}^*]^T$ is the unique, positive definite solution of the following algebraic Riccati equation:

$$\begin{aligned}&-[P^*][A_a] - [A_a]^T[P^*] \\ &+ [P^*][B_a][R]^{-1}[B_a]^T[P^*] - [M_a] \\ &= 0\end{aligned} \tag{14}$$

Integrating equation (13) with respect to time and using the relations of $\{\mathbf{Z}\} = [\mathbf{H}]\{X\}$, gives:

$$\begin{aligned}\{\hat{U}(t)\} = &[[L_{12}] + [L_{22}]][H]\{\hat{X}(t)\} \\ &+ [[L_{11}] + [L_{21}]][H] \\ &\times \int_0^t \{\hat{X}(\tau)\}\, d\tau + \{\tilde{U}_0\}\end{aligned} \tag{15a}$$

where

$$\{\tilde{U}_0\} = \{U_0\} - [[L_{12}] + [L_{22}]][H]\{X_0\} \tag{15b}$$

The elements $[\mathbf{L}_{ij}]$ in equation (15) are appropriately partitioned submatrices $[-[R]^{-1}[B_a]^T[P^*][H_a]^{-1}]$.

This is the desired optimal feedback controller to the original LQR with external constant disturbances as defined in equations (5) and (6). The optimal control can be expressed as a combination of a linear function of the state vector and the first time integral of a certain other linear function of the state vector. Thus, this procedure can be seen as a modern control theory method for designing the proportional plus integral feedback controller (PI controller).

4. Implementation

Controller performances

The objective of this section is to evaluate the performance of the optimal PI feedback controller in a typical shallow aquaculture pond located at Tungkang Marine Laboratory of Taiwan Fisheries Research Institute, Pingtung, Taiwan (designated as the TML pond). The weight of the cultured biomass (milk fish [*Chanos chanos*]) was estimated to be 150 kg (total weight). *Figure 2* shows the structures of an optimal PI controller for the system considered. The performance investigation will illustrate the procedure of the approach and the improvement in performance when these controllers are compared with each other. A four-water volume element (surface, mid-depth, bottom, and sediment) aquaculture pond model (*Figure 1*) will be considered. Measured meteorological data (8:00–7:00 a.m., 24 hr) for solar irradiance, air temperature, relative humidity, and wind speed profiles are shown in *Figure 3*. Average measured values for solar irradiance, air temperature, relative humidity, and wind speed are 150 W/m^2, 28.8°C, 78.2% and 2.93 m/sec, respectively. The geometric and system parameters used in this performance stimulation are listed in *Table 1*. The simulations were done in compiled FORTRAN 77 on a personal computer.

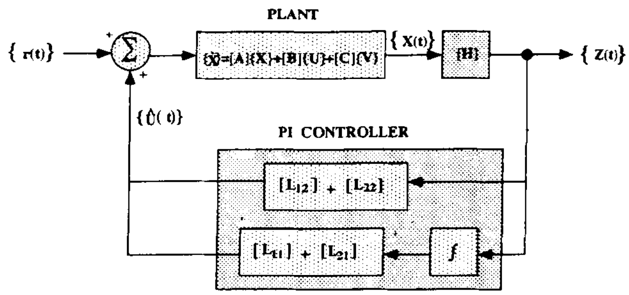

Figure 2. Structure of optimal PI controller for the thermal destratification control system

The optimal PI controller can be expressed appropriately by equation (15) with $\{\tilde{U}_0\} = \{0\}$ as:

$$\begin{aligned}\{\hat{U}(t)\} &= \begin{bmatrix}[L_{11}] & [L_{12}] \\ [L_{21}] & [L_{22}]\end{bmatrix}\left\{\int_0^t \{\hat{Z}(\tau)\}d\tau \,|\, \{\hat{Z}(t)\}\right\}^T \\ &= [K_{\mathrm{PI}}]\left\{\int_0^t \{\hat{Z}(\tau)\}d\tau \,|\, \{\hat{Z}(t)\}\right\}^T\end{aligned} \tag{16}$$

where $[K_{\mathrm{PI}}]$ may be referred to as a feedback PI controller gain matrix:

$$[K_{\mathrm{PI}}] = \begin{bmatrix} k_{11}k_{12}\cdots k_{18} \\ k_{21}k_{22}\cdots k_{28} \\ k_{31}k_{32}\cdots k_{38} \\ k_{41}k_{42}\cdots k_{48}\end{bmatrix}$$

Thus for the optimal PI controller, there are 32 feedback gain elements ($k_{11}, k_{12}, \ldots, k_{48}$) that need to be determined.

The first step in the performance investigation is to determine matrices $[\mathbf{A}]$, $[\mathbf{B}]$, and $[\mathbf{H}]$. The determinations of $[\mathbf{A}]$ and $[\mathbf{B}]$ are followed by the definitions already developed in the system control model. Matrix $[\mathbf{H}]$ in the output relationship ($\{\mathbf{Z}\} = [\mathbf{H}]\{\mathbf{X}\}$) is selected as a diagonal matrix: $[\mathbf{H}] = \mathrm{diag}[h_{11}, h_{22}, h_{33}, h_{44}]$. The

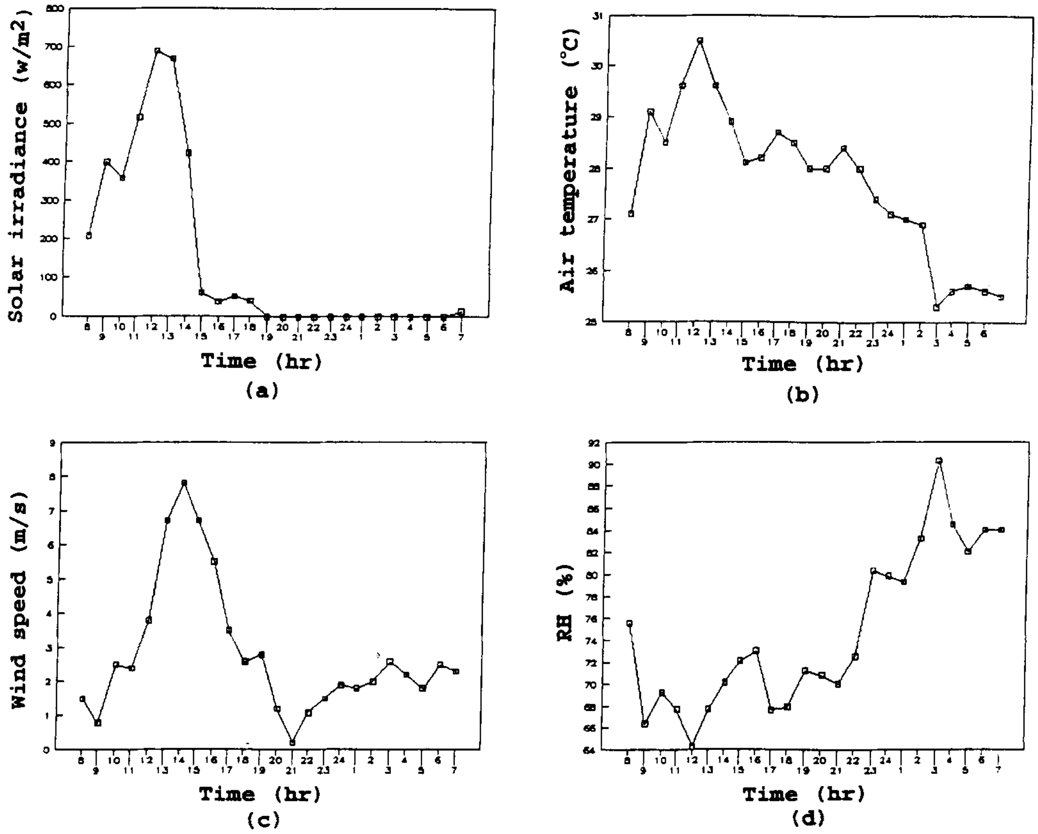

Figure 3. Measured meteorological profiles for (a) solar irradiance, (b) air temperature, (c) wind speed, and (d) relative humidity (8:00–7:00 a.m., 24 hr)

entries of [H] may be chosen from the results of model verification between model predictions from the system control model (equation [4]) and measurements based on an experiment implemented in the TML pond.[15] These values are $h_{11} = 1.0181$, $h_{22} = 1.011$, $h_{33} = 1.014$, and $h_{44} = 1.004$, respectively.

Having defined the matrices [A], [B], and [H], the state controllability and observability of the systems can then be verified. Next, the effect of weighting matrices [Q] and [R] in the quadratic cost functions were investigated. Usually [Q] and [R] are selected to be diagonal, i.e., $[\mathbf{Q}] = \mathrm{diag}[q_{11}, q_{22}, q_{33}, q_{44}]$, and $[\mathbf{R}] = \mathrm{diag}[r_{11}, r_{22}, r_{33}, r_{44}]$; so that specific components of the state and control vectors can be penalized individually if their response is undesirable. Moreover, if diagonal [Q] and [R] are used, the choice of [Q] and [R] entries is more readily reflective of physical insights, and then the system's robustness properties can be secured.[12]

The corresponding scalar expressions of the quadratic cost function for a four-water volume element model in an optimal PI control system can be attained by equation (6):

$$J = 1/2\int_0^\infty (q_{11}Z_1^2 + q_{22}Z_2^2 + q_{33}Z_3^2 + q_{44}Z_4^2 + r_{11}\dot{U}_1^2 + r_{22}\dot{U}_2^2 + r_{33}\dot{U}_3^2 + r_{44}\dot{U}_4^2)dt \tag{17}$$

Weighting matrices may be chosen in the following way to achieve specified performance bounds:[16]

$$\int_0^\infty Z_i^2(t)dt \le \sigma_i^2$$

and (18)

$$\int_0^\infty U_i^2(t)dt \le \mu_i^2, \qquad i = 1, 2, 3, 4$$

Table 1. Input parameters used in performance simulation.

Geometric parameters
Altitude at Tungkang: 21.5°N
Pond area: 30 × 20 m
Pond depth: 110 cm
System parameters
Measured meteorological data (solar irradiance, air temperature, relative humidity, and wind speed): Figure 3
Secchi disk depth (SDD) = 47 cm
Fraction of light absorbed near pond surface (β) = 0.4
Thermal conductivity for sediment and earth (k_{sed}, k_e) = 2.53 kJ m^{-1} hr^{-1} °C^{-1}
Ground water temperature (T_{gw}) = 20°C
Specific heat of water = 4.18 kJ kg^{-1} °C^{-1}
Air density = 1.1988 kg m^{-3}
Water density = 997–998 kg m^{-3}
Initial input pond temperature = 28°C
Optimal fish growth temperature = 29°C

The root mean squared (RMS) values of the multiple outputs and inputs can be used to determine the performance bounds:

$$Z_{i\text{RMS}} = \left[\int_0^\infty Z_i^2(t)dt\right]^{1/2},$$
$$U_{i\text{RMS}} = \left[\int_0^\infty U_i^2(t)dt\right]^{1/2}, \qquad i = 1, 2, 3, 4 \tag{19}$$

Because $\mathbf{Z}_i$, $\mathbf{U}_i$, and $\dot{U}_i$, are of different orders of magnitude, some approximate scaling factors need to be used in selection of q_{ii} and r_{ii}.[12] These scaling factors are selected so that all terms representing derivations in state variables in the integrands of objective equations are of the same order of magnitude. For simplicity, all terms representing derivations in controls variables (or their time derivatives) are the same order of magnitude. Because no general criterion could be found, it is assumed that U_i^2 and $\dot{U}_i^2$ are of the same magnitude; thus, $U_{i\text{RMS}}^2 \cong \dot{U}_{i\text{RMS}}^2$, $i = 1, 2, 3, 4$.

In this particular case, it may be necessary to simultaneously limit the RMS values of output state variable as, $Z_{1\text{RMS}}$ and $Z_{2\text{RMS}} \leq 2°\text{C}$, $Z_{3\text{RMS}}$ and $Z_{4\text{RMS}} \leq 1°\text{C}$, and the RMS values of control input

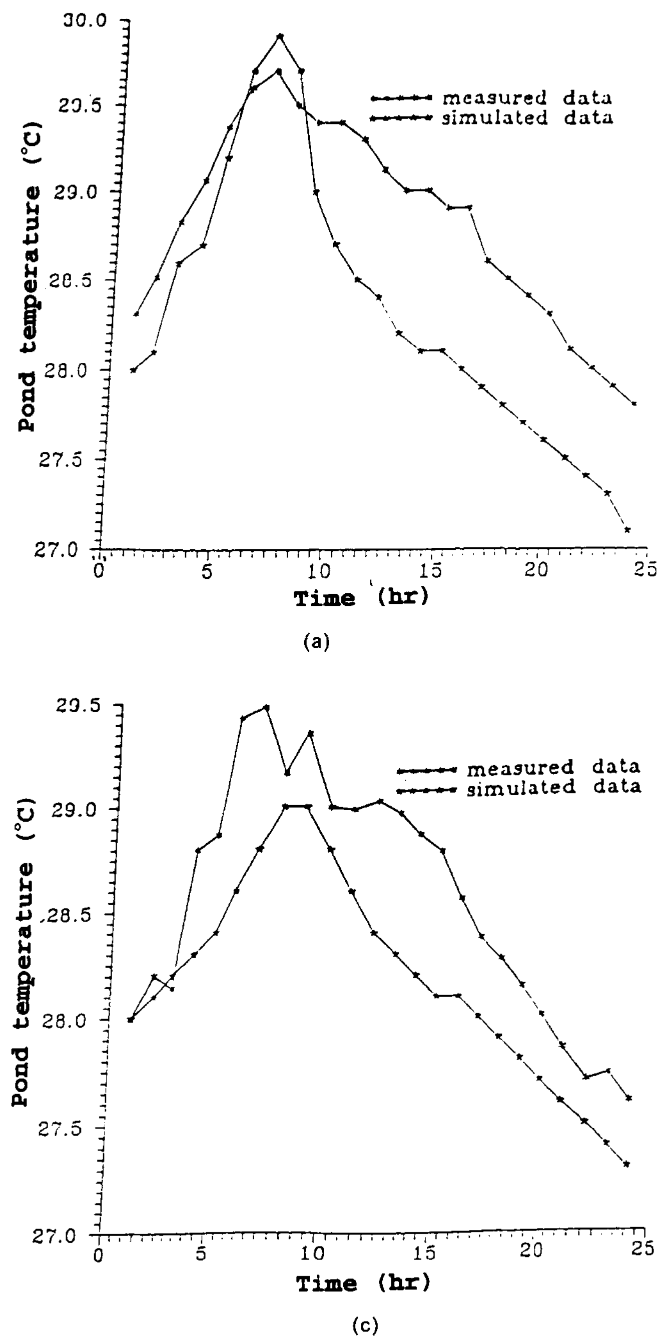

Figure 4. Measured and simulated pond temperature responses under no control action for four different pond depths: (a) 10, (b) 35, (c) 65, and (d) 95 cm

variables as U_{1RMS} and $U_{2RMS} \le 400$ W/m^2 and U_{3RMS} and $U_{4RMS} \le 300$ W/m^2.

Having determined the values of Z_{iRMS}, and U_{iRMS}, the elements in the weighting matrices can be calculated as follows by a scaling method. Therefore, in equation (17), let:

$$q_{11}Z_{1RMS}^2 = q_{22}Z_{2RMS}^2 = q_{33}Z_{2RMS}^2 = q_{44}Z_{4RMS}^2$$

then, $q_{11}/q_{22} = Z_{2RMS}^2/Z_{1RMS}^2$, $q_{22}/q_{33} = Z_{3RMS}^2/Z_{2RMS}^2$, and $q_{33}/q_{44} = Z_{4RMS}^2/Z_{3RMS}^2$. As a result, $q_{44} = q_{33} = 4q_{11}$, $q_{22} = q_{11}$; and similarly $r_{44} = r_{33} = 0.56\, r_{11}, r_{22} = r_{11}$. Thus in this performance investigation, it is only necessary to adjust q_{11} and r_{11} while q_{22}, q_{33}, q_{44}, r_{22}, r_{33}, and r_{44} are fixed by the above scaling factors. In this work, r_{11} is kept constant throughout at 1, and only q_{11} is varied. The approach to the numerical solution of the nonlinear algebraic Riccati equation is followed by a simple nonrecursive method,[17] which is applicable to the case of nonsingularity of matrix [A] in the plant.

Figure 4 shows the comparison between experimental data and simulated pond temperature responses under no control action for four different pond depths (10, 35, 65, and 95 cm). *Figure 4* indicates that simulated results of upper layers of water column compared favorably well (deviations are within 0.5°C), while for sediment element, the deviations are up to 1.5°C. Errors of standard deviation between measured and calculated data is 0.31. The simulated transient temperature responses under the optimal PI controller are illustrated in *Figure 5*. Therefore, when comparing *Figures 4* and *5*, the PI control action is recognized as preferable to that of the no control condition. In *Figure 5*, the responses were governed by the parameter q_{11}. The success of these manipulations on control efforts can be achieved by adjusting the specified performance bounds for output and input variables to properly select the weighting matrices.

It is concluded that the designed optimal feedback controllers, when suitably tuned, give satisfactory control of thermal stratification in an aquaculture pond. Numerical results from the simulation show that the optimal choice of parameter q_{11} and the resulting costs vary with desired equilibrium state.

Sensitivity analysis

For every dynamic model of the forms in equation (5), there exist model error vectors, say $\{e_x\}$ and $\{e_z\}$ which represent corrections to the state equation and output measurement equation, respectively, such that $\{\mathbf{Z}\}$ evolving from a dynamic model matches the measurements from the physical plant. Furthermore, the sum of four kinds of errors are always present: parameter errors, errors in model order, neglected disturbances, and nonlinearities. Thus, after having designed a control system, to duplicate precisely the nominal performance characteristics of the system parameters cannot be expected.

Moreover in using any number of sensors and actuators, the physical plant will not be completely observable or controllable.[18] Therefore, there are always manufacturing tolerances in the components used to construct the controllers.

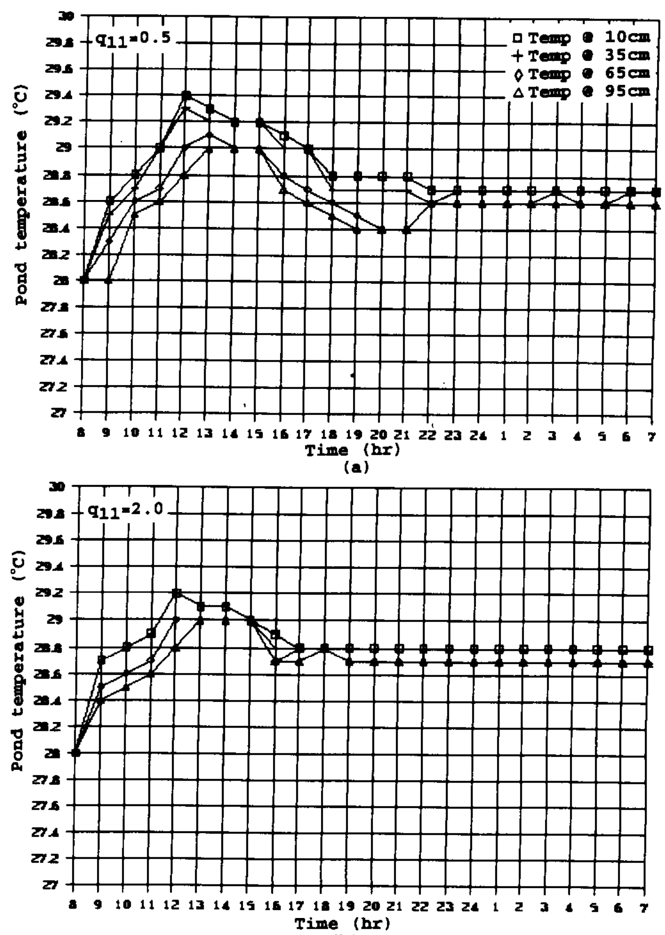

Figure 5. The simulated transient responses of pond temperature under optimal PI controller: (a) $q_{11} = 0.5$ and (b) $q_{11} = 2.0$

There are two ways in which the variation in parameters can affect the objective function: variation in the controller parameters and variation in the parameters of the controlled system. This paper is only limited in determining sensitivity with respect to controller parameters, and it is assumed that the system parameters are not subject to any variation.

One of the goals of a sensitivity is to assign accuracy requirements for system parameters consistent with sensitivity significance in a system model. The work done by Pagurek[19] can be used to analyze the sensitivity of the feedback gains of the optimal PI controllers. Based on the results of such analysis, some insight into the accuracy requirements of the various feedback gains during the actual design of the individual controllers can be obtained.

Equations (16) is valid for the optimal PI control. The contoller parameters that shall be concentrated on are the 32 feedback gain elements $(k_{11}, k_{12}, \ldots, k_{48})$, which will be redesignated as $k_1, k_2, \ldots, k_{32}$, respectively.

Table 2 summarizes the results of the sensitivity analysis ($q_{11} = 0.5$ and 2.0) of the optimal PI controller. The nominal values of k_i shown are the optimal values

Table 2. Results of sensitivity analysis for PI controller.

$q_{11} = 0.5$	Nominal value, K_0
$k_1 = -0.07(2.045)$[a]	$k_{17} = -5.54(5.34)$
$k_2 = 10.01(0.0657)$	$k_{18} = -11.59(0.148)$
$k_3 = 34.67(0.00023)$	$k_{19} = 76.89(0.0032)$
$k_4 = -0.62(1.632)$	$k_{20} = -4.46(0.765)$
$k_5 = -0.0002(0.0031)$	$k_{21} = 0.0004(1.123)$
$k_6 = -0.164(0.0065)$	$k_{22} = -0.65(0.009)$
$k_7 = -0.083(0.0076)$	$k_{23} = -0.0015(2.85)$
$k_8 = -0.042(0.0082)$	$k_{24} = -0.00036(0.675)$
$k_9 = 9.7(167304)$	$k_{25} = -6.43(0.132)$
$k_{10} = -0.86(2172.6)$	$k_{26} = -0.78(6.654)$
$k_{11} = 30.56(2061.2)$	$k_{27} = 234.78(0.005)$
$k_{12} = 16.34(10876.4)$	$k_{28} = -9.67(0.087)$
$k_{13} = 24.36(120675.3)$	$k_{29} = 75.007(0.0054)$
$k_{14} = 10.86(1328.7)$	$k_{30} = -7.435(0.41)$
$k_{15} = -0.786(4238.4)$	$k_{31} = -2.36(1.65)$
$k_{16} = 26.7(156011.8)$	$k_{32} = 46.73(0.0043)$
$q_{11} = 2.0$	Nominal value, K_0
$k_1 = -5.6(0.0005)$[a]	$k_{17} = -5.45(5.06)$
$k_2 = 97.6(0.006)$	$k_{18} = -10.64(0.143)$
$k_3 = 167.8(0.00032)$	$k_{19} = 84.67(0.0065)$
$k_4 = -8.3(0.054)$	$k_{20} = -4.46(0.76)$
$k_5 = 190.6(0.00043)$	$k_{21} = -9.65(0.065)$
$k_6 = 235.1(0.004)$	$k_{22} = 234.6(0.001)$
$k_7 = -0.02(0.108)$	$k_{23} = -0.98(6.43)$
$k_8 = -0.97(0.09)$	$k_{24} = -6.43(0.121)$
$k_9 = -0.002(347986.5)$	$k_{25} = -2.65(1.66)$
$k_{10} = -0.08(33467.9)$	$k_{26} = -0.008(2.006)$
$k_{11} = -0.97(3421.4)$	$k_{27} = 9.96(0.074)$
$k_{12} = -0.00097(167078.3)$	$k_{28} = 40.56(0.0002)$
$k_{13} = -0.0008(151608.7)$	$k_{29} = -0.54(1.146)$
$k_{14} = -0.079(4536.3)$	$k_{30} = -0.003(1.132)$
$k_{15} = -2.07(3146.5)$	$k_{31} = 10.87(0.076)$
$k_{16} = -0.05(1706105.3)$	$k_{32} = -0.0078(0.0035)$

[a] Magnitude of $[\mathbf{M}_{ii}]$.

determined via the development of feedback control synthesis for the system considered. The sensitivity of the objective function with respect to the feedback gain elements may be represented as the magnitude of a symmetry matrix, $[\mathbf{M}_{ii}]$. Matrix $[\mathbf{M}_{ii}]$ may be referred to as a relative sensitivity coefficient matrix. The magnitude of $[\mathbf{M}_{ii}]$ can be defined by the maximum eigenvalue of $[\mathbf{M}_{ii}]$: $\|[\mathbf{M}_{ii}]\| = \max(|\lambda_1|, |\lambda_2|, \ldots, |\lambda_n|)$, where, λ_i, $i = 1, 2, \ldots, n$ are the eigenvalues of $[\mathbf{M}_{ii}]$. Matrix $[\mathbf{M}_{ii}]$ and $\|[\mathbf{M}_{ii}]\|$ are determined according to the work developed by Pagurek.[19]

Table 2 shows that the objective function is 10^6 times more sensitive to the feedback gains associated with $\mathbf{U}_3$ ($\Delta\phi_{\text{sn},4}$) than that to those associated with $\mathbf{U}_1$ ($\Delta\phi_{\text{net}}$), $\mathbf{U}_2$ ($\Delta\phi_{\text{sn},2}$), and $\mathbf{U}_4$ ($\Delta\phi_{\text{sn},6}$). This indicates that great care has to be exercised in maintaining the feedback gains of temperature at a water depth zone of Δz_2 (*Figure 1*) at their nominal values because the optimal performance of the overall control scheme depends more on these nominal values than those of the $\mathbf{U}_1$, $\mathbf{U}_2$, and $\mathbf{U}_4$.

The results from the sensitivity analysis provide useful information to install properly the submersible mixers or pumps at an optimum mixing depth.

5. Summary and conclusions

A linear dynamic model describing the behavior of temperature and thermal stratification in aquaculture ponds is used in developing the optimal feedback controllers for a thermal destratification control system. The optimal PI feedback controller is devised from the viewpoint of modern control theory. The controllers employ microcomputers not only to perform control calculations but also for other monitoring and recording operations.

The optimization and suboptimization of the linear quadratic regulators (LQR's) with output feedback of a linear-invariant system are defined to determine an output feedback control loop such that the integral quadratic cost function meets its minimum value. Both initial the condition and constant disturbances are formulated, and the optimal PI controller is synthesized. The optimal PI control has been simulated in a typical shallow aquaculture pond application.

The results from the sensitivity analysis with respect to the controller gains show that feedback gains associated with pond temperature at mid depth are shown to be far more sensitive as compared with that of temperature at other water depths. This result provides useful information to install the control devices at an optimum mixing depth.

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Appendix A: The L-P Model

For the purpose of describing the L-P Model, the heat transfer processes are subdivided into surface, water column, and bottom heat transfer (*Figure 1*). The equation representations of the heat flux vector ($\{\phi_1\}$) are described as follows.[9]

Surface energy balance

$$\phi_{net} = \phi_s - \phi_r + \phi_{at} - \phi_{ws} - \phi_e \pm \phi_c \tag{A1}$$

where:

$$\phi_r = \phi_s R,$$
$$R = (2.2(180\lambda/\pi)^{0.97})(1 - 0.08W_Z)),$$
$$\phi_{at} = 2.44 \times 10^{-14}(T_{ak})^{6.148},$$
$$\phi_{ws} = 2T_{wk}^4,$$
$$\phi_e = 5.093W_2(e_s - e_a),$$
$$e_s = 25.37\exp(17.62 - 5271/T_{wc}),$$
$$e_a = 25.374RH\exp(17.62 - 5271/T_{ac}),$$
$$\phi_c = 1.5701W_2(T_{wc} - T_{ac}).$$

Water column energy transport

(1) Shortwave solar irradiance-derived heat flux:

$$\phi_{sn,i} = (\phi_s - \phi_r)(1 - R)(1 - \beta)\exp(-\eta_e z_i) \tag{A2}$$

where:

$$\beta = 0.4 \sim 0.5,$$
$$\eta_e = 1.7/SDD.$$

(2) Turbulent diffusion-derived heat flux:

$$\phi_{d,i} = \rho_w C_{pw} E_{z,i}(T_{i-1} - T_i)/\Delta z \tag{A3}$$

where:

$$E_{z,i} = E_{\theta,i}(1 + 0.05R_{i,z_i})^{-1}$$
$$E_{\theta,i} = (6.32 \times 10^{-3}(\rho_a/\rho_w)^{1/2}W_z^{-0.84}) \times \exp(-6W_z^{-1.84}z_i),$$
$$R_{i,z_i} = 1.0 \times 10^{-3}a_v g z_i^2/(W_z)^2(\rho_w/\rho_a)(\Delta T/\Delta z),$$
$$a_w = 1.5 \times 10^{-5}(T_{av} - 277) - 2.0 \times 10^{-7}(T_{av} - 277)^2.$$
$$\rho_w = \{(0.99987 + [(0.69 \times 10^{-5})(T_{wc})] + [(8.89 \times 10^{-6})(T_{wc})^2] + [(7.4 \times 10^{-8})(T_{wc})^3]\}1{,}000$$

Sediment energy exchange

$$\phi_{sed} = k_{sed}[(T_3 - T_4)/\Delta z] \tag{A4}$$

$$\phi_{gw} = k_e[(T_4 - T_{gw})/\Delta z] \tag{A5}$$

Appendix B: Notations for the L-P Model

C_{pw}	= specific heat of water, kJ kg^{-1} °C^{-1}
$E_{z,i}$	= effective diffusion coefficient at depth z_i, m^2 hr^{-1}
$E_{\theta,i}$	= neutrally buoyant effective diffusion coefficient at any depth z_i, m^2 hr^{-1}
e_a	= water vapor pressure above pond surface, mm Hg
e_s	= saturated vapor pressure, mm Hg
g	= gravitational acceleration, m sec^{-2}
k_{sed}	= thermal conductivity for sediment, kJ m^{-1} hr^{-1} °C^{-1}
k_e	= thermal conductivity for earth, kJ m^{-1} hr^{-1} °C^{-1}
R	= reflectivity
RH	= air relative humidity, %
R_{i,z_i}	= Richardson number at depth z_i
SDD	= Secchi disk depth, m
T_{ac}	= air temperature, °C
T_{WC}	= water temperature, °C
T_{ak}	= absolute air temperature, °K
T_{wk}	= absolute water tempeature, °K
T_{av}	= average water temperature of two adjacent volume elements, °C
W_z	= wind velocity at a height z above pond surface, km hr^{-1}
W_2	= wind velocity at a reference height 2 m above pond surface, km hr^{-1}
ϕ_{net}	= net heat flux passing the air/water interface, kJ m^{-2} hr^{-1}
ϕ_s	= shortwave solar irradiance
ϕ_r	= reflected shortwave solar irradiance
ϕ_{at}	= net atmospheric radiation
ϕ_{ws}	= water surface back radiation
ϕ_e	= evaporative heat transfer
ϕ_c	= sensible heat transfer
$\phi_{sn,i}$	= solar irradiance at depth z_i
$\phi_{d,i}$	= effective diffusion of heat at depth z_i
ϕ_{sed}	= heat transfer between bottom and sediment
ϕ_{gw}	= heat loss from sediment to ground water
β	= fraction of light absorbed near pond surface
η_e	= effective light extinction coefficient, m^{-1}
ρ_a	= air density, kg m^{-3}
ρ_w	= water density, kg m^{-3}
λ	= solar altitude angle, radians
ΔT	= temperature difference, °C
Δz	= distance between center of volume element, m