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Modeling dust-borne odor dynamics in swine housing based on age and size distributions of airborne dust

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Abstract

In this paper, we derive a mathematical model characterizing the adsorption of odor on the surface of airborne dust in swine housing based on the concept of homogeneous surface diffusion of a complete mixing airflow system. The philosophy of the paper is to incorporate the age and size distributions of airborne dust into the diffusion model for evaluating the dust-borne odor dynamics in a ventilated airspace. A closed-form solution is presented here to allow a series of numerical experiments for investigating the effects of adsorption characteristics, the mean age of airborne dust, surface effective diffusivity, and dust particle size on the adsorption of odor to the existing aerosol. Results obtained show that the most favorable performance of a ventilation system in reducing odor concentrations is when the system model is operated under $r_p/\sqrt{D_s\bar{\tau}} < 1$, in which r_p is the radius of an airborne dust, D_s the effective diffusivity of bulk odor in air, and $\bar{\tau}$ is the mean residence time of airborne dusts in ventilated airspace. The model enables engineers to evaluate the performance of the ventilation systems in reducing the odor emitted from stored manure in swine housing.

Keywords: Airborne dust; Dust-borne odor; Diffusion; Age distribution; Particle size distribution

1. Introduction

Odor is one of the major environmental concerns for the livestock industries. Complaints and lawsuits are filed because of odors generated from livestock facilities. The offensive smell produced by decomposition is a product of complex interactions of many individual odorous components mixed in the air [1–3]. The majority of odor problems are related to the storage and spreading of manure [2,4]. A number of studies were carried out to study odor dispersion and the area downwind that may be affected by a given odor source under a given weather condition [5,6].

The effects of odors on animals have not yet been determined. The presence and detection of olfactory components from boars in piggery air is a vital requirement for early attainment of puberty in female pigs [7]. Pheromones play an important role in the breeding behavior of animals, and odor plays a large part in the establishment of the maternal/offspring bond [7]. The

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toxicity of odors is a complex and delicate topic. An odorous compound can, like many other substances, be toxic if present in excessive concentrations.

Ghaly and Bully [2] stated that the concentration of odors in the exhaust air is proportional to the ventilation air. High ventilation rates dilute the gases, and therefore the odor has a less objectionable strength. Animal housing requires minimum ventilation during winter, thus increasing odor problem. Odors exhausted from farm buildings were often associated with dust particles [8,9]. Hammond et al. [10] identified 10 volatile organic compounds in the air from swine buildings which were the suspended sources of odor. Hammond et al. [10] also found that odors at points remote from a lagoon were mediated by dust particles. According to Hangartner [11], filtering dust from the exhaust air has proved to reduce odor emission from swine housing by up to 65%. As about 85% of the dust in livestock housing originates from the feed, wet feeding is recommended to reduce the odor problem. Therefore, there is a high correlation between odor and suspended particulate matter concentrations in animal housing.

Presently researchers have identified more than 150 specific volatile compounds in solid manure, liquid manure, gases emitted from manure and dust in swine housing [8,12–16]. The most intense odors in filtered swine housing air are attributable to tri- and tetra-methylpyrazine, *p*-cresol, butyric acid, diacetyl, and hexanal [17]. Yasuhara et al. [18], Yasuhara and Fuwa [14] and Hammond et al. [17] also agreed that odor is formed by mixing phenols and carboxylic acids. Miner [8] also reported that acids and phenols caused offensive odors in swine housing air. Odor enters the ventilated airspace from manure pit and a substantial fraction of it is adsorbed to the existing aerosol to yield a size distribution of the dust-borne odor.

Air contaminants may be removed from an air stream by absorption, by physical adsorption (or, van der Waals adsorption), by chemisorption, by catalysis, and by combustion [19–21]. Catalysis is closely related to chemisorption. Chemisorption is similar in many ways to physical adsorption. Assuming no chemical reaction takes place on the surface of dust particles, absorption is negligible. Thus, adsorption becomes the main removal mechanism for odor from an air stream to the surface of airborne dust based on the surface diffusion.

The adsorption of odor-causing compounds on the surface of ambient aerosol therefore plays an important role in reducing odor from ventilation air in animal housing. A dust particle may have a finite adsorption capacity. Because intraparticle diffusive resistance frequently controls the overall adsorption rate, the performance of the ventilation systems directly depends on the size and residence time (or called age, i.e., the time allowed for adsorption) distributions of ambient aerosol. The use of residence time distribution for evaluation of gaseous contaminants emitted from stored swine manure has been discussed by Liao [22]. Evaluation of dust-borne odor removal performance of ventilation systems can be achieved by fixing odor residence time and dust residence time.

In this paper, we derive a mathematical model characterizing the adsorption of odor on the surface of airborne dust in animal housing based on the concept of homogeneous surface diffusion of a complete mixing airflow system with no external limitation. The philosophy of the scheme is to incorporate the effects of age and size distributions of airborne dust into the model for evaluating the dust-borne odor dynamics in a ventilated airspace. A sensitivity analysis of model parameters on the system performance is also presented based on an operational data in a typical pig unit. The input parameters to the model were varied to correspond to performance factors and the model was applied to evaluate the range of possible conditions. The present work is not an analysis of the experimental data but complements it.

2. Model development

2.1. Model assumptions

Before deriving the system model to describe the adsorption of odor onto the surface of airborne dust, the following assumptions were made.

1. The ventilation airflow system is assumed to be a complete mixing system i.e., a uniform mixing of air and odor concentration.
2. All odor ingredients are treated as one gas (i.e., bulk odor concentration).
3. A dust particle is treated as an aerodynamic equivalent sphere and is electrically neutral.
4. Mass transfer resistance from bulk odor to outer surface of dust particles is negligible.
5. Bulk odor concentration is in equilibrium with the adsorbed phase concentration at the surface.
6. Odor is assumed to be adsorbed on the outer surface first, then enter the dust particle through surface diffusion, and eventually occupy adsorption sites in the abundant inner surface.
7. The rate controlling step is the transport of adsorbed odors within dust particles.

2.2. Mass balance on bulk odor phase

When applying a steady-state mass balance equation on the bulk odor phase (Fig. 1), we have,

$$\dot{q}(C_0 - C_\mathrm{b}) = Q\bar{w} \tag{1}$$

in which $\dot{q}$ is the odor emission rate (m^3 s^{-1}), C_0 the initial odor concentration (kg m^{-3}), C_b is the exhausted odor concentration (kg m^{-3}), Q the dust particle exchange rate (kg s^{-1}), $Q = \dot{V}\rho_\mathrm{p}$ where $\dot{V}$ is the ventilation rate (m^3 s^{-1}), ρ_p the particle density (kg m^{-3}), and $\bar{w}$ the dust-borne odor concentration which is the average amount of odor adsorbed per unit mass of dust particles (kg kg^{-1}).

2.3. Mass balance over a single dust particle

If the initial conditions are such that the dust-borne odor can be viewed as a function of time and of the spherical space coordinate, we obtain from mass conservation describing the phenomenon of surface diffusion of the odor concentration on a single, spherical dust particle,

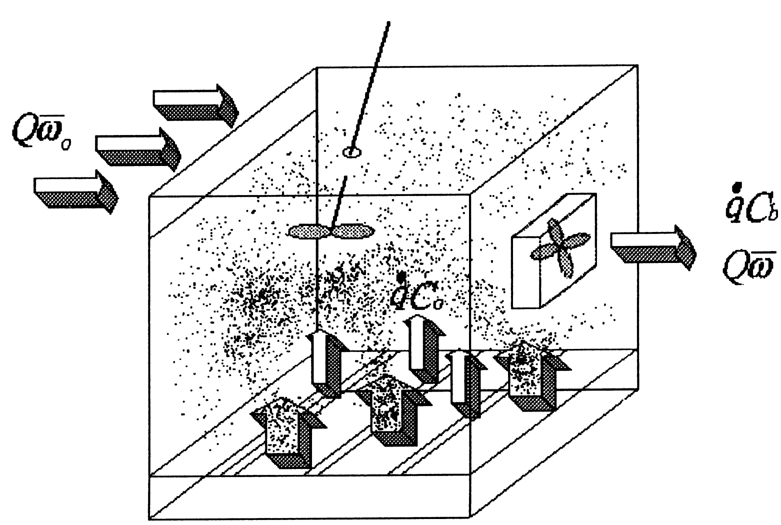


Fig. 1. Schematic of a slot-inlet ventilated chamber with a totally slatted floor shows the mass balance of bulk odor and dust-borne odor phases of a complete mixing airflow system.

$$\frac{\partial w}{\partial \tau} = D_s \left(\frac{\partial^2 w}{\partial r^2} + \frac{2}{r} \frac{\partial w}{\partial r} \right) \tag{2}$$

subject to the following initial and boundary conditions,

$$w = 0, \quad \tau = 0, \quad 0 \leqslant r \leqslant r_p,$$
$$\frac{\partial w}{\partial r} = 0, \quad \tau \geqslant 0, \quad r = 0,$$
$$w = w_e, \quad \tau \geqslant 0, \quad r = r_p,$$

where τ is the residence time of the dust particles in a ventilated airspace (min), r a position variable representing the distance from the center of the particle (μm), w the amount of odor adsorbed per unit mass of a dust particle at r (kg kg^{-1}), D_s the surface effective diffusivity of odor in air (cm^2 s^{-1}), r_p the radius of a dust particle (μm), and w_e the amount adsorbed in equilibrium with the bulk odor concentration (kg kg^{-1}).

The analytical solution of Eq. (2), i.e., the total amount of odor adsorbed on the surface of a dust sphere is given by [23],

$$\frac{\bar{w}(\tau)}{w_e} = 1 - \frac{6}{\pi^2} \sum_{n=1}^{\infty} \frac{1}{n^2} \exp\left(\frac{-D_s n^2 \pi^2 \tau}{r_p^2} \right), \tag{3}$$

where $\bar{w}(\tau)$ is the particle-averaged time varying mass of odor absorbed per unit mass of dust particle (kg kg^{-1}).

2.4. Incorporation of an age distribution

Considering a ventilated animal unit, air entering and odor emitting from stored manure into the ventilated airspace will spend some time in the airspace before leaving. It is obvious that the exit time of one dust-borne odor is different from that of another not only because of the circulation of airflow but because of the internal mixing in each airspace.

Therefore, there is an exit age distribution in the leaving airflow. The definition of $\bar{w}(\tau)$ shown in Eq. (3) in that all dust particles have the same age τ would not apply in a complete mixing system where a portion of the airborne dust has a range of ages.

If we consider a step input to a perfect mixing system (Fig. 1), a lumped mass balance on the dust-borne odor phase gives,

$$Q\bar{w}_0 = Q\bar{w} + M \frac{d\bar{w}(\tau)}{d\tau}. \tag{4}$$

This simple differential equation can be rewritten as,

$$\bar{\tau} \frac{dw(\tau)}{d\tau} + \bar{w} = \bar{w}_0, \tag{5}$$

where $\bar{\tau} \equiv M/Q$ is the mean holding time or mean age of dust particles inside the system (min), and M the total mass of dust particles (kg).

Eq. (5) has the solution for zero initial condition,

$$\frac{\bar{w}(\tau)}{w_0} = 1 - \exp\left(\frac{-\tau}{\bar{\tau}} \right). \tag{6}$$

Incorporating the definitions of $\bar{w}(\tau)/w_0 \equiv F(\tau/\bar{\tau})$ and the exit age distribution function $E(\tau) \equiv dF(\tau/\bar{\tau})/d\tau$ into Eq. (6) [21], the exit age distribution of dust particles in a complete mixing system can be obtained as,

$$E(\tau) = \frac{1}{\bar{\tau}} \exp\left(\frac{-\tau}{\bar{\tau}}\right). \tag{7}$$

Eq. (7) reveals that about 63% of the dust particles have an age less than the mean age $\bar{\tau}$ and that the most probable age for dust particles in a complete mixing system is at $\tau = 0$.

The experimental determination of the age distribution function defined in Eq. (7) would be accomplished for a particular vessel by a stimulus–response technique using some sort of tracer material in the inlet-fluid stream [21,24]. Eq. (7) is not the only possible distribution in a real system. Analytical solution, however, may not be possible for all distributions.

Incorporating the age distribution function into Eq. (3), the age and particle-averaged amount of dust-borne odor is given as,

$$\frac{\bar{w}(\bar{\tau})}{w_e} = \int_0^\infty \left[1 - \frac{6}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{n^2}\exp\left(\frac{-D_s n^2\pi^2\tau}{r_p^2}\right)\right]E(\tau)\,d\tau = 1 - \frac{6}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{n^2}\frac{1}{1+\left(\frac{n^2\pi^2 D_s\bar{\tau}}{r_p^2}\right)}. \tag{8}$$

Here we define an effectiveness factor (η) as

$$\eta \equiv \frac{\bar{w}(\bar{\tau})}{w_e}. \tag{9}$$

Effectiveness factor may be expressed as a function of a dimensionless parameter (Φ) as $\eta(\Phi)$, in which Φ is defined as [25]

$$\Phi \equiv \frac{r_p}{\sqrt{D_s\bar{\tau}}} \tag{10}$$

and is referred to as a diffusion length modulus (DLM). Therefore, Eq. (8) becomes,

$$\eta(\Phi) = 1 - \frac{6}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{n^2}\frac{\Phi^2}{\Phi^2 + n^2\pi^2}. \tag{11}$$

The effectiveness factor indicates the degree of saturation in adsorption capacity [20]. $\eta = 1$ represents the complete exposure of the dust particle surface, thus the complete use of adsorption capacity (i.e., an equilibrium case). $\eta = 0$ represents no exposure, thus fresh airborne dust. DLM represents the ratio of the airborne dust exchange rate to that of diffusion. As the DLM becomes much greater than 1, for instance, the adsorption capacity of the airborne dust is not fully used because the diffusion rate is too low or airborne dust exchange rate is too high. Eq. (11) also reveals that as the effectiveness factor converges to 1, the DLM becomes less than 1.

2.5. Incorporation of a dust particle size distribution

Airborne dust particles in animal housing are usually not homogeneous in size [26,27]. Sieve analysis is often performed to determine a particle size distribution in terms of various size fractions [28]. For instance, a typical particle size distribution of the swine dust is shown in Fig. 2 [28]. Therefore, incorporating particle size distribution by applying the model to each dust particle size fraction, the overall effectiveness factor becomes,

$$\eta \equiv \frac{\bar{w}(\bar{\tau})}{w_e} = \sum_{j=1}^{n}\eta_j(\Phi_j)f_j, \tag{12}$$

where N is the number of size fractions, η_j the effectiveness factor of the jth fraction, Φ_j the DLM of the jth fraction, and f_j the weight fraction of the jth fraction.

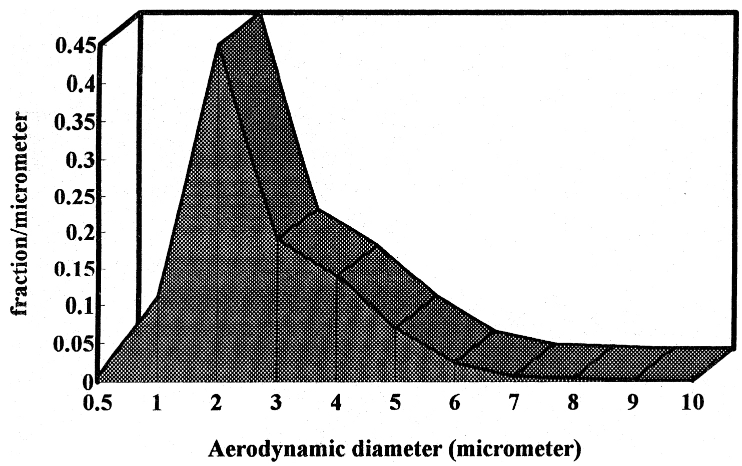


Fig. 2. A typical size distribution of the swine dust.

The lognormal distribution is the most common used distribution for characterizing aerosol particle size [29]. The mathematical form of a particle size distribution thus can be obtained using a lognormal distribution model [29],

$$p(d_{\mathrm{p}}) = \frac{1}{\sqrt{2\pi} d_{\mathrm{p}} \ln \sigma_{\mathrm{g}}} \exp\left(-\frac{(\ln d_{\mathrm{p}} - \ln \mathrm{CMD})^2}{2(\ln \sigma_{\mathrm{g}})^2} \right), \tag{13}$$

where d_{p} is the diameter of a dust particle (μm), CMD the count median diameter (μm), and σ_{g} the geometric standard deviation (μm). These parameters are varied to correspond to typical aerosol profiles found in animal housing.

The conversion of CMD to any type of average diameter could be calculated directly using the Hatch–Choate equation [29],

$$d_{\mathrm{a}} = \mathrm{CMD} \exp(b \ln^2 \sigma_{\mathrm{g}}), \tag{14}$$

where d_{a} is the average diameter of a dust particle (μm), and b a constant that depends on the type of conversion, that is, on which type of average diameter d_{a} is.

Consequently, the size-averaged effectiveness factor has the form,

$$\eta(\Phi) = \frac{\bar{w}(\bar{\tau})}{w_{\mathrm{e}}} = \int_0^\infty \eta\left[\Phi(d_{\mathrm{p}})\right] p(d_{\mathrm{p}})\, \mathrm{d}d_{\mathrm{p}}. \tag{15}$$

There is no known analytical solution for Eq. (15).

Rather than using a computer for the iterative summation of the terms in the series shown in Eqs. (3) and (11) with a criterion for convergence, it is preferable to convert Eqs. (3) and (11) to a closed-form equation by using some identified mathematical formulas.

Thus after some mathematical manipulations, Eq. (11) can be converted to a closed-form equation as (see Appendix A):

$$\eta(\Phi) = 1 - \frac{6}{\pi^2} \sum_{n=1}^{\infty} \frac{1}{n^2} \frac{\Phi^2}{\Phi^2 + n^2\pi^2} = \frac{3}{\Phi^2} (\Phi \coth \Phi - 1). \tag{16}$$

Therefore, it may be possible to find equivalent particle size ($d_{\mathrm{p,eq}}$) result in an effectiveness factor from Eq. (16) based on equivalent DLM (Φ_{eq}) which is the same one that can be obtained by integrating Eq. (15) numerically.

In doing so, Eq. (16) becomes:

$$\eta(\Phi_{\mathrm{eq}}) = \int_0^\infty \eta[\Phi(d_\mathrm{p})]p(d_\mathrm{p})\,\mathrm{d}d_\mathrm{p} = \frac{3}{\Phi_{\mathrm{eq}}^2(d_{\mathrm{p,eq}})}[\Phi_{\mathrm{eq}}(d_{\mathrm{p,eq}})\coth\Phi_{\mathrm{eq}}(d_{\mathrm{p,eq}}) - 1]. \tag{17}$$

2.6. Overall mass balance with dimensionless variables

The steady-state exhausted odor concentration can be calculated using Eqs. (1) and (12) as follows,

$$\dot{q}(C_0 - C_\mathrm{b}) = w_\mathrm{c}\eta(\Phi)Q. \tag{18}$$

The adsorption reaction of the equilibrium odor concentration at the outer surface of a dust particle is described by means of a Freundlich isotherm, that is [30],

$$w_\mathrm{c} = KC_\mathrm{b}^n, \quad (n > 0), \tag{19}$$

where K ((m^3 kg^{-1})^n) and n are constants and could be determined experimentally.

The Freundlich isotherm is widely used to describe sorption equilibrium in environmental systems with heterogeneous surfaces. Although, airborne dust is superficially assumed as homogeneous materials, significant heterogeneity likely. Additionally, over time, airborne dust surface slowly reacted by means of physicochemical or biological manners, which may further contribute to surface heterogeneity.

The Freundlich type isotherm is sometimes used to correlate data for hydrocarbon gases on activated carbon [30,31]. Often one finds $0 < n \leqslant 1$ for Freundlich exponent in Eq. (19). For example, Geankoplis [30] shows that a batch test performed in the laboratory using the solutions of phenol (the most intense odor causing compound) in water and particles of granular activated carbon followed the Freundlich isotherm and gave the values of $n = 0.229$ and $K = 0.199$.

Combining Eqs. (18) and (19) in terms of two dimensionless variables yields a dimensionless system equation,

$$C_\mathrm{b}^* = 1 - D_\mathrm{g}(C_\mathrm{b}^*)^n\eta(\Phi), \tag{20}$$

where the dimensionless variables of C_b^* and D_g in Eq. (20) can be defined as,

$$C_\mathrm{b}^* \equiv \frac{C_\mathrm{b}}{C_0}, \tag{21}$$

$$D_\mathrm{g} \equiv \frac{QK}{\dot{q}C_0^{1-n}}, \tag{22}$$

where D_g is referred to as a dust-borne odor distribution parameter.

If considering the equivalent DLM and an equivalent particle size, Eq. (20) becomes,

$$C_\mathrm{b}^* = 1 - D_\mathrm{g}(C_\mathrm{b}^*)^n\eta(\Phi_{\mathrm{eq}}) = 1 - D_\mathrm{g}(C_\mathrm{b}^*)^n\frac{3}{\Phi^2(d_{\mathrm{p,eq}})}[\Phi_{\mathrm{eq}}(d_{\mathrm{p,eq}})\coth\Phi_{\mathrm{eq}}(d_{\mathrm{p,eq}}) - 1]. \tag{23}$$

Relationship between C_b^* and Φ_{eq} therefore would be obtained from Eq. (23).

3. Model analysis

3.1. Input parameters

A typical pig unit measuring 24 m × 9.5 m × 3 m and consisting of six compartments with a totally slatted floor is chosen for illustrative purposes. This unit has one negative pressure ventilation system of three high endwall exhaust fans with a continuous slot inlet. Each compartment contains six pigs. The body weight of each growing pig was estimated to be 70 kg.

To simulate the steady-state performance of the model, the values of the surface effective diffusivity (D_s) and isotherm parameters (K, n) have to be known. Ventilation rate ($\dot{V}$), odor emission rate ($\dot{q}$) , airborne dust particle size (r_p), and mean age of airborne dust ($\bar{\tau}$) are operating variables. Airborne dust exchange rate, $Q = \dot{V}\rho_p$, in which $\rho_p = 1$ kg m^{-3} for an aerodynamic equivalent particle, thus, $Q = \dot{V}$.

The value of D_s for this case is taken from the mean value for three diffusivities in air (at temperature $= 25°C$) of butyric acid (0.081 cm^2 s^{-1}), *p*-cresol (0.0777 cm^2 s^{-1}), and hexane (0.080 cm^2 s^{-1}),[31] thus, $D_s = 0.0796$ cm^2 s^{-1}. The values of K and n are 0.199, and 0.23, respectively [30]. The values of C_0 and $\dot{V}$ used for this illustration were 5 mg l^{-1} and 0.05 m^3 s^{-1} per 70 kg pig, respectively [32].

It is difficult, expensive and not of particularly great value to try to quantify a wide range of odor compounds. Exceptions include NH_3, because it is so easy to measure almost down to its threshold odor concentration, therefore, odor emission rate was chosen to be 0.03 g min^{-1} of NH_3 which is adapted from an experimental data in a controlled laboratory performed by Liao and Bundy [33].

3.2. Effect of dust-borne odor distribution parameter

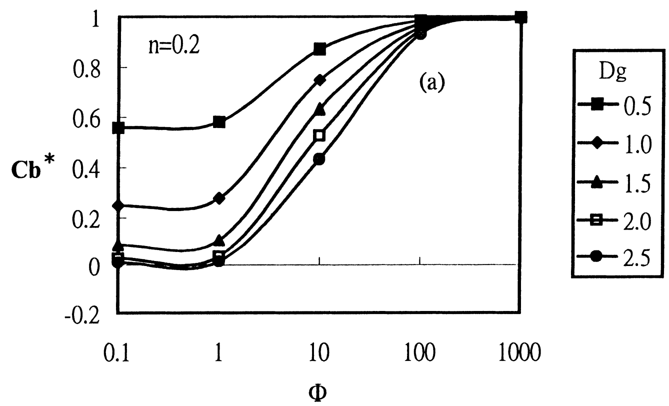
Fig. 3 shows the dimensionless exhausted odor concentration C_b^* in Eq. (20) is a function of Φ varying with different values of dust-borne odor distribution parameter (D_g) for favorable isotherms ($n = 0.2$ and 0.5). A linear isotherm ($n = 1$) is shown in Fig. 4, whereas Fig. 5 shows an unfavorable isotherm ($n = 2$). As shown in these figures, the exhausted odor concentration converges to a constant value as Φ becomes less than 1, indicating that the adsorption approaches equilibrium.

Figs. 3–5 indicate that the removal rate of dust-borne odor concentration can only be increased (i.e., the bulk odor concentration lowered) by increasing the dust-borne odor distribution parameter (D_g) or increasing the airborne dust exchange rate (Q). For values of Φ larger than 1, the adsorption reaction becomes diffusion limited and only a fraction of adsorptive capacity of airborne dusts is used.

In view of Figs. 3–5, the most favorable performance of a ventilation system in reducing odor concentration is when the system model is operated under $\Phi < 1$ (i.e., effectiveness factor (η) converges to 1). The results given in Figs. 3–5 also indicate that the airborne dust with the favorable isotherms yielded the lowest odor concentration in the exhaust air.

3.3. Effect of airborne dust size

Fig. 6 shows the dimensionless odor concentration as a function of mean age of airborne dusts in a range of dust particle sizes. Reducing the particle size by one order of magnitude reduces the required mean age by two orders of magnitude to achieve approximately the same exhaust air quality. This quadratic effect is shown in the definition of the Φ (Eq. (10)), in which r_p^2 and $\bar{\tau}$ are directly proportional. Fig. 6 also reveals that at the condition of the same mean age of airborne dust, the smaller the dust particle size the lower the bulk odor concentration. When the mean age of airborne dust tends to infinity, as shown in Fig. 6, the exhausted odor concentration converges to a constant value. This effect indicates that dust particles become fully saturated with bulk odor as the mean age of airborne dust reaches infinity.



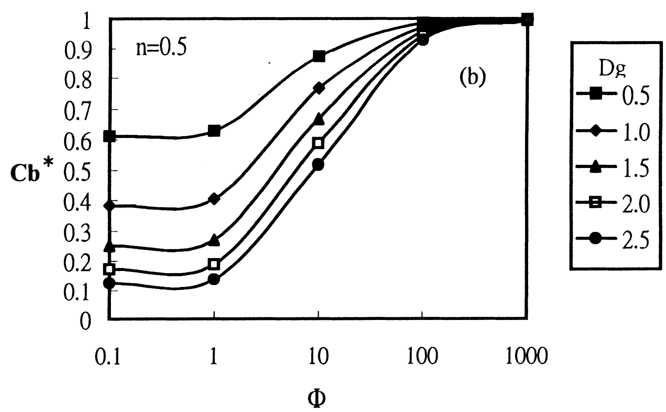


Fig. 3. Dimensionless exhausted odor concentration (C_b^*) as a function of Φ varying with different values of D_g for favorable isotherms of: (a) $n=0.2$ and (b) $n=0.5$.

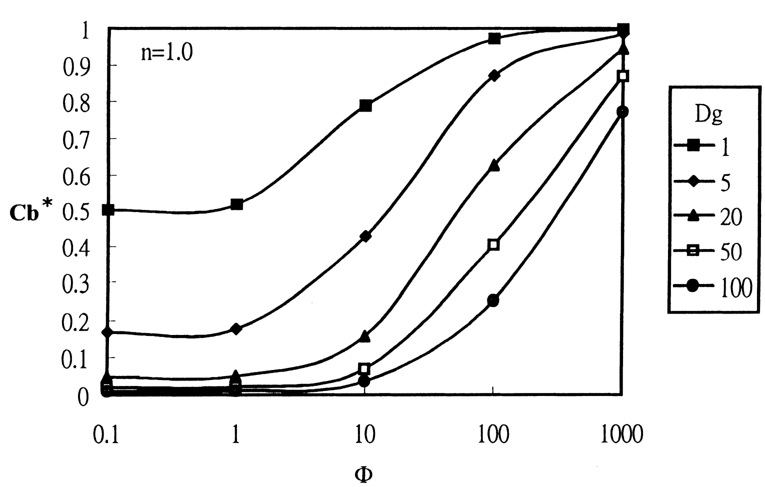


Fig. 4. Dimensionless exhausted odor concentration (C_b^*) as a function of Φ varying with different values of D_g for a linear isotherms ($n=1$).

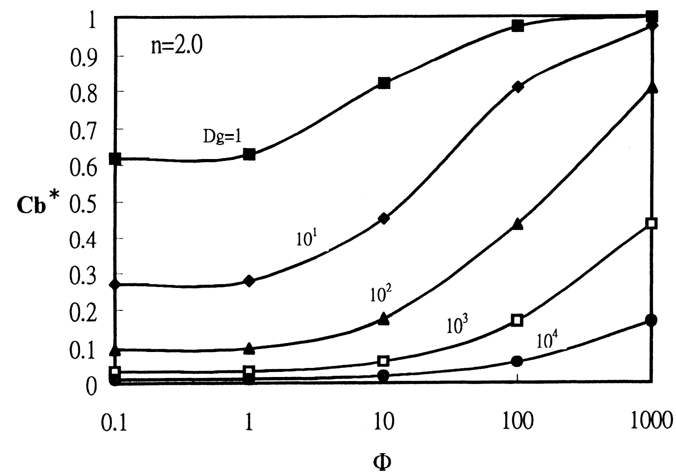


Fig. 5. Dimensionless exhausted odor concentration (C_b^*) as a function of Φ varying with different values of D_g for an unfavorable isotherms ($n = 2$).

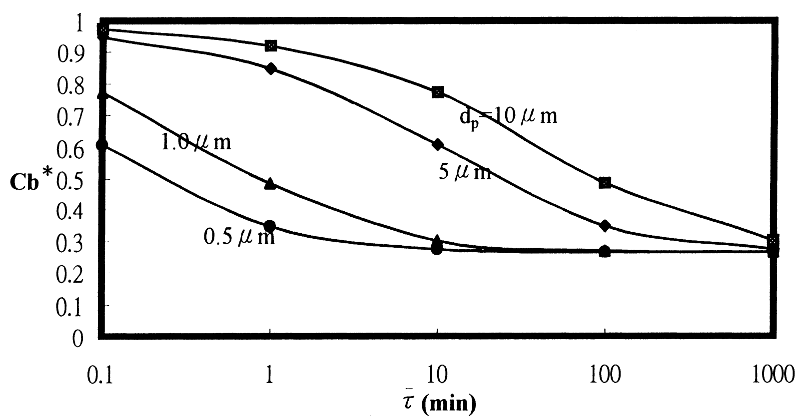


Fig. 6. Dimensionless exhausted odor concentration (C_b^*) as a function of mean age of airborne dust ($\bar{\tau}$) for a range of dust particle sizes.

3.4. Effect of effective diffusivity

Fig. 7 shows the similar profiles for two dust particle sizes and two effective diffusivities. An increase in the effective diffusivity by one order of magnitude is equivalent to a decrease of the mean age of airborne dust by approximately the same order of magnitude. This relationship is shown again in Eq. (10) in which D_s and $\bar{\tau}$ are inversely proportional. Fig. 7 shows that at the same mean age and the same particle size of airborne dust, increasing the surface effective diffusivity of odor in air lowers the bulk odor concentration.

3.5. Effect of particle size distribution

The definition in Eq. (15) of incorporating the particle size distribution into the system model were simulated using a size distribution of swine dust given in Eq. (13). The results are shown as solid lines in Fig. 8 for three dust generation rates in a wide range of the conditions.

To evaluate the possibility of using an equivalent particle size to represent the particle size distribution, Eq. (23) associated with Eq. (17) was solved by trial and error to find an equivalent

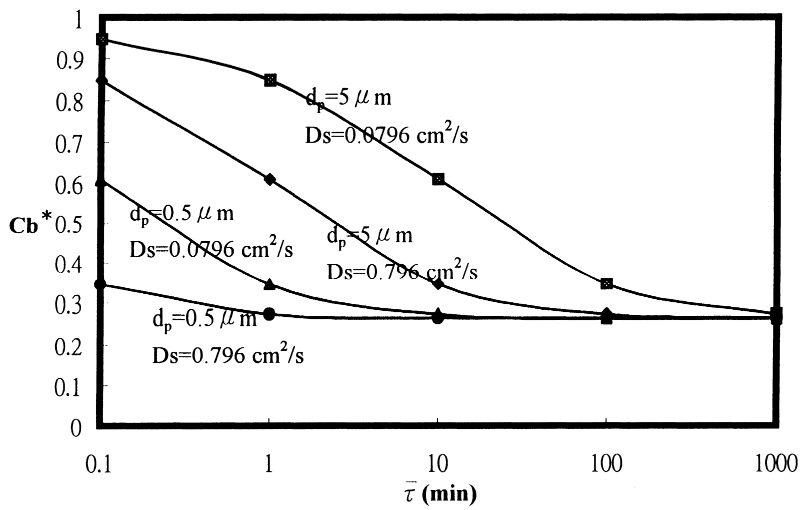


Fig. 7. Dimensionless exhausted odor concentration (C_b^*) as a function of mean age of airborne dust ($\bar{\tau}$) for two different particle sizes and effective diffusivities.

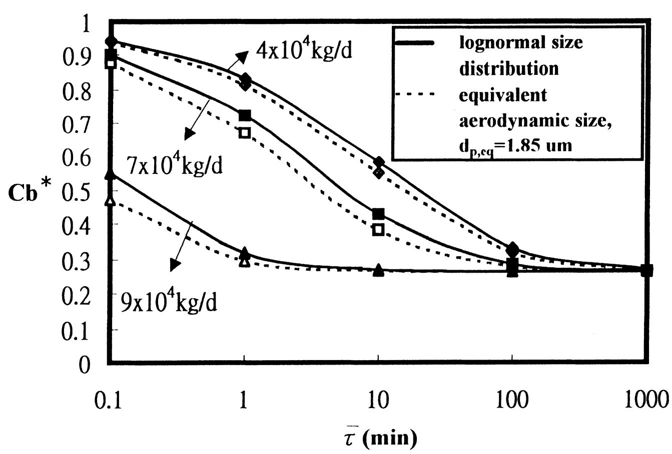


Fig. 8. Dimensionless exhausted odor concentration (C_b^*) as a function of mean age of airborne dust ($\bar{\tau}$) relative to a lognormal particle size distribution and one equivalent particle size for a range of dust generation rates.

particle size in order to match the solid line histories shown in Fig. 8. The results are shown as dashed line in Fig. 8 with an equivalent particle size, $d_{p,eq} = 1.85$ μm. Fig. 8 indicates that the deviation becomes larger at the lower dust generation rates and disappears at low and high mean ages. Therefore, there is no unique equivalent single particle diameter that exactly represents a particle size distribution. The error, however, is bounded and the use of an equivalent particle size could be an acceptable and practical way of evaluating the performance of the model.

4. Summary and conclusions

We introduced an analytical model characterizing the adsorption of odor on the surface of airborne dust in animal housing based on the concept of homogeneous surface diffusion of a complete mixing airflow system. The philosophy of the analysis is to incorporate the age and size distributions of airborne dust into the model for evaluating the dust-borne odor dynamics in a ventilated airspace.

The resulting solution of the model has a closed-form expression which gives more reliable answers with less computational effort. A series of numerical experiments were made to illustrate the effects of dust particle size, surface effective diffusivity, mean age of airborne dust, odor emission rate and dust generation rate on the steady-state exhausted odor concentration and to demonstrate the simplicity and usefulness of the model for evaluating the steady-state performance of a ventilation system for swine housing.

Results obtained show that the most favorable performance of a ventilation system in reducing odor concentrations is when the system is operated under $r_p/\sqrt{D_s\bar{\tau}} < 1$, in which r_p is the radius of an airborne dust, D_s the effective diffusivity of bulk odor in air, and $\bar{\tau}$ the mean residence time of airborne dust in ventilated airspace.

There is no single, equivalent particle size that can exactly represent an airborne dust particle size distribution, however, the use of a carefully selected equivalent particle size could lead to a quite accurate result for evaluating the performance of the model. The most attractive feature of the model is that it is able to take into account not only the diffusion in a sphere and adsorption process but also the residence time/particle size distributions and their influence on the dust-borne odor dynamics.

We conclude that this model should give a new impulse to the analysis of odor reduction technologies, such as the theoretical study of the deodorization processes of a biofilter used to clean the ventilation air in swine housing.

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Appendix A. Derivation of a closed-form solution of Eq. (11)

Series for hyperbolic function of coth x can be expressed as follows [34],

$$\coth x = \frac{1}{x} + 2x\left[\frac{1}{\pi^2 + x^2} + \frac{1}{(2\pi)^2 + x^2} + \frac{1}{(3\pi)^2 + x^2} + \cdots\right] = \frac{1}{x} + 2x\sum_{n=1}^{\infty}\frac{1}{(n\pi)^2 + x^2}. \quad \text{(A.1)}$$

Therefore,

$$\coth \Phi = \frac{1}{\Phi} + 2\Phi\sum_{n=1}^{\infty}\frac{1}{n^2\pi^2 + \Phi^2}. \quad \text{(A.2)}$$

From the formulas for Fourier series, the following relations is given [34],

$$\sum_{n=1}^{\infty}\frac{1}{n^2} = \frac{\pi^2}{6}. \quad \text{(A.3)}$$

The definition of effectiveness factor in Eq. (11) may be rewritten as:

$$\eta(\Phi) = 1 - \frac{6}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{n^2}\frac{\Phi^2}{\Phi^2 + n^2\pi^2} = 1 - \frac{6}{\pi^2}\sum_{\pi^2}^{\infty}\left(\frac{1}{n^2} - \frac{\pi^2}{\Phi^2 + n^2\pi^2}\right). \quad \text{(A.4)}$$

Eq. (A.4) can be rearranged by using the relations of Eqs. (A.1) and (A.3) as:

$$\eta(\Phi) = \frac{3}{\Phi^2}[\Phi \coth \Phi - 1]. \tag{A.5}$$

Therefore, a closed-form solution for Eq. (11) is obtained.

Appendix B. List of symbols

b	parameter in the Hatch–Choate equation (dimensionless)
C_b	exhaust odor concentration (kg m^{-3})
C_b^*	dimensionless exhaust odor concentration
C_0	initial odor concentration (kg m^{-3})
D_g	dust-borne odor distribution parameter (dimensionless)
D_s	surface effective diffusivity of odor in air (cm^2 s^{-1})
d_a	average diameter of a dust particle (μm)
d_p	diameter of a dust particle (μm)
$d_{p,eq}$	equivalent diameter of a dust particle (μm)
$E(\tau)$	exit age distribution of airborne dust (h^{-1})
$F(\tau/\overline{\tau})$	cumulative age distribution of airborne dust (dimensionless)
f_j	weight fraction of the jth size fraction (dimensionless)
K	Freundlich isotherm parameter ((m^3 kg^{-1})^n)
M	total mass of dust particles (kg)
n	Freundlich isotherm parameter (dimensionless)
$P(d_p)$	dust particle size distribution (μm^{-1})
Q	airborne dust exchange rate (kg s^{-1})
$\dot{q}$	odor emission rate (m^3 s^{-1})
$\dot{V}$	ventilation rate (m^3 s^{-1})
w	dust-borne odor concentration (kg kg^{-1})
$\overline{w}$	average amount of odor adsorbed per unit mass of dust (kg kg^{-1})
w_e	amount adsorbed in equilibrium with the bulk odor concentration (kg kg^{-1})
r	a position variable representing distance from center of a particle (μm)
r_p	radius of a dust particle (μm)
Φ	diffusion length modulus (DLM) (dimensionless)
Φ_{eq}	equivalent diffusion length modulus (dimensionless)
η	effectiveness factor (dimensionless)
τ	age or residence time of airborne dust (h)
$\overline{\tau}$	mean age of airborne dust (h)
ρ_p	density of an aerodynamic dust particle (kg m^{-3})
σ_g	geometric standard deviation (μm)

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