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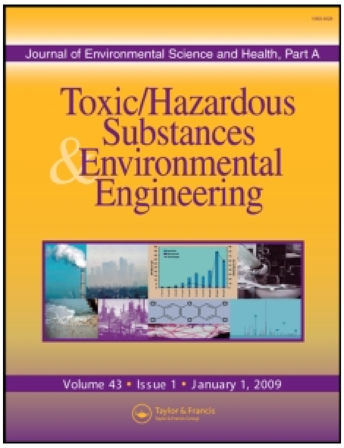

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CHARACTERIZATION OF MIXING PATTERNS IN A VENTILATED AIRSPACE WITH A MULTIPLE AIRFLOW REGIONS GAMMA MODEL

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CHARACTERIZATION OF MIXING PATTERNS IN A VENTILATED AIRSPACE WITH A MULTIPLE AIRFLOW REGIONS GAMMA MODEL

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ABSTRACT

A model called the multiple airflow regions gamma model (MARGM) was developed based on a continuous distribution of residence time for predicting the mixing behavior in a ventilated airspace in that data interpretation and mean residence time calculation for a specified output concentration profile can also be evaluated. The MARGM takes the form of the two-parameter gamma distribution and accounts for different mixing types: complete mixing, no mixing (piston flow), incomplete mixing, and various combinations of the above types. In these combinations, the different mixing types simulated by the MARGM conceptually represent airflow regions in series. The mixing efficiency was introduced to characterize the extent or degree of mixing in a ventilation system. Mixing efficiency equals zero for piston flow (no mixing), unity for complete mixing, and a value in between these two extremes for incomplete mixing. The MARGM simulates the combinations of complete mixing, incomplete mixing, and piston flow. Therefore, seven models are introduced in this effort: complete mixing model, piston flow model, complete-piston flow model, complete-incomplete-piston flow model (the general model), complete-incomplete mixing model, incomplete-piston flow model, and incomplete mixing model. The applicability of models was tested by several case studies. Results show that combination models give better fitting than other simpler models. The MARGM enables building

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microenvironment designers reconsider the possibilities and consequences of various forms of incomplete mixing in investigating indoor air quality problems.

Key Words: Mixing; Indoor air quality; Residence time distribution; Ventilation

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INTRODUCTION

Knowledge of the mixing patterns in a ventilated airspace is valuable at both the design stage and during operation to determine the effectively mixed proportion of the air volumes and existing contaminants and to assess the values of different operational procedures as a means of optimizing process performance in indoor air quality control problems. Liao and Feddes (1990, 1991, 1992), Gao and Feddes (1993) and Liao and Bundy (1995) have been presented a multiple airflow regions model (MARM) for predicting airborne dust and gaseous pollutant concentrations in a ventilated airspace. The well-established MARM can be used confidently to model the local dynamic transport behavior of airborne dust and gaseous pollutants in a ventilated airspace (Hoff and Bundy, 1996). The quantitative interpretation of gaseous and particulate contaminant data in ventilation systems could also be evaluated. The MARM can also be used to simulate the combination of complete mixing, incomplete mixing and piston flow where the mixing types occupy tanks-in-series or tanks-in-parallel (Himmelblau and Bishoff, 1968; Naunamm and Buffham, 1983; Levenspiel, 1999).

Barber and Ogilvie (1982) suggested that departure from complete mixing might be caused by the formation of multiple flow regions within the airspace or by short-circuiting of supply air to exhaust outlet. In a work presented by Chen et al. (1999) regarding the methods to measure dust production and deposition rates in buildings indicated that the assumption of complete mixing was not valid during tests. Their experiments showed that tanks-in-series flow, i.e., different flow regions behaving as a number of mixed tanks connected in series dominated the overall mixing process within the ventilated airspace. Their work also suggested that a more complicated multi-zone mixing model might be needed to account for incomplete mixing to better understand the behavior of dust local transport mechanisms.

Himmelbau and Bischoff (1968) indicated that mixing in a continuous flow system could range from piston flow (plug flow or no mixing) as a lower limit, to complete mixing as an upper limit. Both complete mixing and piston flow represents the limiting cases that seldom occur in ventilation systems. It is most probable that the type of mixing encountered in ventilation systems in general, lies somewhere between these two extremes. We call this type of mixing incomplete mixing or partial mixing.

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The most widely used analytical models for simulating the airflow patterns and convection-diffusion distribution of a contaminant in livestock buildings are those that utilize the dispersivity concept such as the dispersive model in a distributed scheme (Choi et al., 1988; Hoff et al., 1992; Maghirang and Manbeck, 1993; Liu et al., 1996). Reliable estimates the dispersivity that may be scale-dependent, however, are very difficult to obtain. The dispersive model is also characterized by other limitations. Liu et al. (1996) and Hoff and Bundy (1996) pointed that the theoretical foundation for the dispersive model in its multiple airflow regions is not sound for systems other than air jet with turbulent or laminar flow.

The main parameter determined from the MARM is the mean residence time (or mean age) of air in the ventilation system (Liao and Feddes, 1990; Liao, 1997). For time-invariant systems, the mean age T is defined as (Naunamm and Buffham, 1983):

$$T = \int_0^\infty tf(t)dt, \tag{1}$$

where $f(t)$ is the residence time distribution (RTD) function which describes the exit time distribution of fluid element (air) entered the system at a given time $t = 0$. The RTD approach can also be used to describe aerodynamic transport in a ventilated airspace. The methodology is general in the sense that the heterogeneity in transport properties may also be incorporated using various RTDs of specific interest.

Mathematically, $f(t)$ is the probability density function, for the time t. Grot and Lagus (1991), Sandberg (1993), Freeman et al. (1982), Skarret and Mathisen (1981), and among others pointed out that $f(t)$ could be obtained by a tracer gas (e.g., SF_6) experiment or by measuring the existing gaseous pollutant concentrations (e.g., CO_2, NH_3, *p*-cresol) presented in the ventilated airspace. A parsimonious representation of $f(t)$ can be obtained by a simple unimodel and continuous mathematical function. A convenient and flexible is the gamma density function. The gamma distribution is a distribution of the Pearson's Type III in statistics. The unique feature of the gamma distribution is that one end of the distribution is bound to a fixed value, whereas the other end is distributed over a large scale of the variate. The overall shape of the gamma distribution is not balanced as a normal distribution. The other reason to utilize the gamma distribution is that this approach may reduce the mathematical terms in the analytical solution.

The three most common models used in a continuous flow system are the complete mixing model, piston flow model, and the complete-piston flow model that simulates the combination of complete mixing and piston flow (Naunamm and Buffham, 1983). The new models will be derived in this study are the complete-incomplete-piston flow model (or referred to as the general model), complete-incomplete mixing model, incomplete-piston flow model, and incomplete mixing model. It is obvious that incomplete mixing alone or in combination with the other mixing types is more likely to occur in a general ventilation system than piston flow alone, complete mixing alone, or the combination of complete mixing-piston flow.

A multiple airflow regions gamma model (MARGM) is developed, which accounts for different types of mixing in general ventilation systems: no mixing (piston flow), incomplete mixing, complete mixing and their combinations which represent the simultaneous existence of the three or any two of the mixing types in the system. The proposed models are free from the limitations of the dispersive model (i.e., the dispersivity problem) that makes the new models an effective tool for investigating indoor air quality problems. The MARM is particularly useful for investigating systems that lack detailed ventilation data, i.e., systems with unknown distribution of parameters. Considering that ventilation systems in many animal housing are characterized by little or no ventilation data, analysis of tracer data by MARM may be the only available approach for solving air supply problems.

The MARGM accounts for the mixing types of complete mixing, incomplete mixing, piston flow, and their combinations with one mathematical formula. The significance of the MARGM has two effects: (1) this study will alert designers of building environment to the possibilities and consequences of various forms of incomplete mixing, and (2) the model can be used to quantify mixing characteristics to replace the complete mixing assumption in ventilation design calculation.

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MARGM STRUCTURE

Model Assumptions

There are two premises must be met before the model can be applied. The capable model accounting for the effects of molecular diffusion is only the dispersive model. The MARM assumes that molecular diffusion is negligible, i.e., the RTD function of the tracer or existing gaseous contaminant and air are identical and both are related to the distribution of air velocities.

Furthermore, the application of these models requires that an ideal tracer be injected into and measured in the flux concentration. The proposed model thus assumes that injected and measured in the flux has the same response function as the tracer material.

Theses conditions, however, are met that restrictive for two reasons: (1) molecular diffusion is negligible compared with mechanical dispersion in most field situations, especially in large-scale systems, and (2) flux concentration samples are easy to obtain from typical occupied airspace.

Mixing Efficiency

The general ventilation systems are very complex. The complexities arise from field-scale heterogeneity that create significant variability in the air velocity. The complexities make characterization of mixing in ventilation system a very difficult task. The task becomes even more difficult when the systems are viewed as distributed-parameter systems. An alternative approach for considering spatially distributed parameter or processes, is to quantify the lumped response of mixing in the system, i.e., to adopt the multiple airflow regions approach to characterizing the overall mixing degree in the system. The concept of mixing efficiency therefore is adopted for the purpose of this study.

Cholette and Cloutier (1959) and Jennings and Armstrong (1971) pointed out that incomplete mixing describes the type of mixing between piston flow and complete mixing in that the mixing efficiency is introduced and defined. The approaches presented by Cholette and Cloutier (1959) and Jennings and Armstrong (1971) for characterizing the overall decay rate when mixing is not complete is to fit an expression of the type in terms of measurement data as

$$C(t) \approx C(0)\exp(-\mu t/T_n), \tag{2}$$

in which $C(t)$ is time-dependent tracer concentration, $C(0)$ is the initial condition of tracer concentration, $T_n = V/Q$ is the inverse mean residence time of ventilation system; Q is the ventilation rate, V is the air volume, parameter μ is referred to as mixing efficiency of air in a ventilation system. The mixing efficiency describes the portion of ventilation airflow that has mixed with room air.

Incomplete mixing can range from near complete mixing to near piston flow, but never reaches these extremes. We introduce the concept of mixing efficiency to specify the exact location of incomplete mixing in the range between piston and complete mixing. In general ventilation systems, mixing is due to mechanical dispersion and molecular diffusion. The proposed model is applicable to systems where molecular diffusion is negligible. Accordingly, we define the mixing efficiency μ to describe the extent of mixing resulting from factors other than molecular diffusion:

$$\mu = \begin{cases} 1 & \textit{for complete mixing} \\ 0 & \textit{for piston flow} \end{cases} \tag{3}$$

and incomplete mixing is characterized by $0 < \mu < 1$.

The major difference between MARM and dispersive model therefore is that the MARM employs the concept of mixing efficiency, whereas the dispersive model utilizes the dispersivity/diffusivity that may be scale-dependent. Therefore, the proposed models simulate incomplete free from the dispersivity/diffusivity limitation.

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Two-Parameter Gamma Density Function

Our approach is to use the gamma distribution to describe the MARM of incomplete mixing in a ventilated airspace. Because mixing air complexity, their RTD is not expected to follow an normal distribution. The gamma distribution, thus, has a greater chance to describe the mixing behavior than does the normal distribution. The presented analytical solution demonstrates that it is possible to construct a systematic way of lumping parameters and the incomplete mixing behavior of an MARM can be analyzed. Buffham and Gibilaro [21] also suggested that the simplest extension of the tanks-in-series model is the gamma model.

The two-parameter gamma density distribution may be written as

$$f(t;\alpha,\beta) = \frac{1}{\beta^{\alpha}\Gamma(\alpha)} e^{-t/\beta} t^{\alpha-1}, \; t > 0, \tag{4}$$

with the two parameters $\alpha > 0$, and $b > 0$. The normalization factor

$$\Gamma(\alpha) = \int_0^{\infty} x^{\alpha-1} e^{-x} dx, \; \alpha > 0,$$

is the gamma function of mathematical statistics. The gamma distribution also plays an important role in queuing theory and reliability problems (Wadpole and Meyers, 1985), and heterogeneous sorption processes in contaminated soil (Ahn et al., 1999; Culver et al., 1997). In Eq. (4) β is the scale parameter and α is the shape parameter. With the gamma model, the mean, $E(t)$ and standard deviation, $sd(t)$ are

$$E(t) = \alpha\beta; \quad sd(t) = \alpha^{1/2}\beta, \tag{5}$$

or rearranging Eq. (2) we have

$$\alpha = E(t)\beta^{-1}; \quad \beta = sd(t)\alpha^{-1/2}, \tag{6}$$

If $\alpha\beta$ is held constant, then as α approaches infinity the gamma distribution function approaches Gaussian distribution, the variance of which decreases as α increases. The gamma distribution also satisfies the physical requirement that residence time must be nonnegative. Eq. (6) shows β serves to determine both the mean and standard deviation of the t's. Using Eq. (3) we can also calculate α and β. The mean and variance of each gamma distribution governs the characteristic properties of multiple airflow regions.

The effect of α and β is illustrated in Fig. 1. In Fig. 1, $\alpha\beta = 1$, all the curves have a mean of 1. For $0 < \alpha < 1$, much of the air corresponds to very small values of residence time t near 0; for $\alpha = 1$, an exponential distribution is obtained, for which $t = 0$ is still the most likely value. As α increases above 1 the density function becomes increasingly symmetric with small values of t occurring less frequently. In Eq. (4) we define $g(t) \equiv f(t;\alpha,\beta)$ is the RTD function, also called the weighting function or the system response function.

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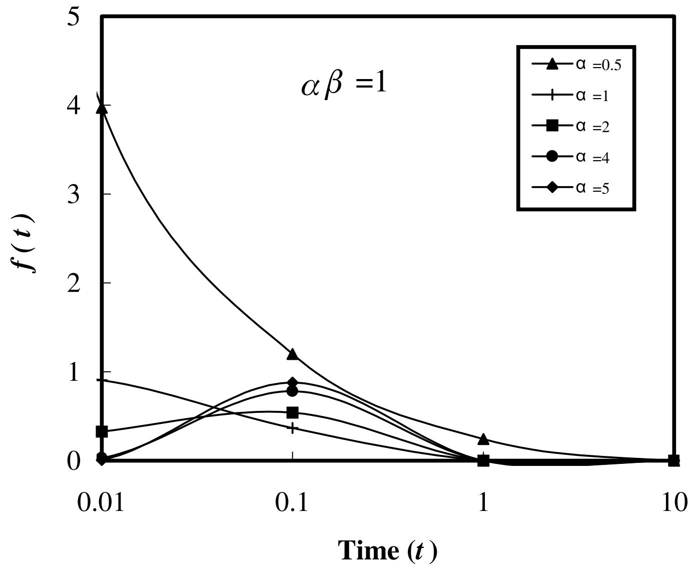

Figure 1. The effects of α and β on the two-parameter gamma density function $f(t) = \beta^{-\alpha}e^{-t/\beta}t^{\alpha-1}/\Gamma(\alpha)$.

Eq. (1) also referred to as the general model. Firstly, we are taking the complete-piston flow model as an example to get insight into the concept of α and β.

The occupied air volume relationship among the total volume (V_T), complete mixing volume (V_c), and piston flow volume (V_p) in the complete-piston flow model is

$$V_T = V_c + V_p. \tag{7}$$

For $\alpha = 1$, Eq. (4) reduces to

$$g(t) = \frac{1}{\beta}e^{-t/\beta},\ t > 0. \tag{8}$$

Based on Eqs. (7) and (8), the complete-piston flow model is mathematically defined as

$$g(t) = \begin{cases} \dfrac{\eta}{T}e^{-t\eta/T}, & t \geq 0, \\ 0, & t < 0, \end{cases} \tag{9}$$

where

$$\beta = \frac{T}{\eta}; \quad \eta = \frac{V_c + V_p}{V_c}. \tag{10}$$

The complete-piston flow model simulates a completely mixed tanks-in-series with a region where airflow moves in piston flow. Therefore, the complete-piston flow model is a special case of the two-parameter gamma distribution when β is defined by Eq. (10) and $\alpha = 1$. Substituting Eq. (10) and $\alpha = 1$ in Eq. (5) gives $E(t) = sd(t) = T/\eta$ which are the mean and standard deviation of the complete-piston flow model.

The complete-piston flow model combines the complete mixing model with the piston flow model. For $\eta = 1$, the complete-piston flow model gives the complete mixing model, and for $\eta \to \infty$, it reduces to the piston flow model. Substituting $\eta = 1$ or letting $\eta \to \infty$ in Eq. (10) yields $\beta = T$ for the complete mixing model, and $\beta = 0$ for the piston flow model. In both complete mixing and piston flow, the mean is T and variance is T^2 for complete mixing and 0 for piston flow.

These values can also be obtained from the mean and variance of the two-parameter gamma distribution. In other words, complete mixing, piston flow, and their combinations (which represent airflow regions in series of the different mixing types) are all accounted for by the two-parameter gamma distribution. For this reason, we propose the general model, which takes the functional form of the two-parameter gamma distribution, for simulating mean residence times in ventilation systems, and consequently the additional fitting parameters will give some information about the structure of the system.

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The General Model

The air volume relations for the general model in a time-invariant systems is

$$V_T = V_c + V_p + V_i \tag{11}$$

where V_i is the air volume of incomplete mixing region (in a time-varying system, $V_T(t) = V_c(t) + V_p(t) + V_i(t)$). Based on Eqs. (10) and (11), after some mathematical manipulation, the shape and scale parameters α and β can be obtained as,

$$\alpha = \begin{cases} \dfrac{V_T}{V_p + V_c + \mu V_i} = \dfrac{1}{\dfrac{1}{\theta} + \mu\left(1 - \dfrac{1}{\theta}\right)}, & \mu \geq 0.5, \\[2ex] \dfrac{V_T}{V_p + V_c + (1-\mu) V_i} = \dfrac{1}{\dfrac{1}{\theta} + (1-\mu)\left(1 - \dfrac{1}{\theta}\right)}, & \mu \leq 0.5, \end{cases} \tag{12}$$

$$\beta = \frac{T}{\alpha\left(\dfrac{V_T}{V_c + \mu V_i}\right)} = \frac{T}{\alpha}\left(\frac{1}{\eta\theta} + \mu - \frac{\mu}{\theta}\right), \tag{13}$$

in which

$$\theta = \frac{V_T}{V_c + V_p}, \tag{14}$$

and

$$\eta = \frac{V_T}{\theta V_c}, \tag{15}$$

where θ and η may be referred to as the mixing volume factor I and II, respectively. The MARGM may also treat time-varying systems, however, such data are generally difficult to obtain.

Table 1 gives the six models that can be obtained as special cases from the general model in Eq. (4). The first three cases in Table 1 correspond to the models of complete-piston flow, complete mixing, and piston flow, respectively. The models corresponding to cases 4, 5, and 6 are designated as the incomplete-piston flow model, complete-incomplete mixing, and incomplete

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Table 1. The RTD Functions and $U(T)$ of the Mixing Models Based on MARGM[a]

Model	RTD function $g(t)$, $t \geq 0$	$U(T) = C_{out}/C_0$
1. Complete-piston	$\frac{\eta}{T}\exp(-t\eta/T)$	$\frac{1}{\left[\frac{\lambda T}{\eta} + 1\right]}$
2. Complete mixing	$\frac{1}{T}\exp(-t/T)$	$\frac{1}{[\lambda T + 1]}$
3. Piston flow	$\delta(t - T)$	$\exp(-\lambda T)$
4. Incomplete-piston	$\frac{t^{\alpha-1}}{\left[\frac{T}{\alpha}\left(\mu - \frac{\mu}{\theta}\right)\right]^{\alpha}\Gamma(\alpha)}\exp\left\{\frac{-t}{\frac{T}{\alpha}\left(\mu - \frac{\mu}{\theta}\right)}\right\}$	$\left[\frac{\lambda T}{\alpha}\left(\mu - \frac{\mu}{\theta}\right) + 1\right]^{-\alpha}$
5. Complete-incomplete	$\frac{t^{\alpha-1}}{\left[\frac{T}{\alpha}\left(\frac{1}{\theta} + \mu - \frac{\mu}{\theta}\right)\right]^{\alpha}\Gamma(\alpha)}\exp\left\{\frac{-t}{\frac{T}{\alpha}\left(\frac{1}{\theta} + \mu - \frac{\mu}{\theta}\right)}\right\}$	$\left[\frac{\lambda T}{\alpha}\left(\frac{1}{\theta} + \mu - \frac{\mu}{\theta}\right) + 1\right]^{-\alpha}$
6. Incomplete mixing	$\frac{t^{\alpha-1}}{\left[\frac{T\mu}{\alpha}\right]^{\alpha}\Gamma(\alpha)}\exp\left\{\frac{-t}{\frac{T\mu}{\alpha}}\right\}$	$\left[\frac{\lambda T\mu}{\alpha} + 1\right]^{-\alpha}$

[a] The general model (complete-incomplete-piston):

$$g(t) = \frac{t^{\alpha-1}}{\left[\frac{T}{\alpha}\left(\frac{1}{\eta\theta} + \mu - \frac{\mu}{\theta}\right)\right]^{\alpha}\Gamma(\alpha)}\exp\left\{\frac{-t}{\frac{T}{\alpha}\left(\frac{1}{\eta\theta} + \mu - \frac{\mu}{\theta}\right)}\right\}$$

$$U(T) = \left[\frac{\lambda T}{\alpha}\left(\frac{1}{\eta\theta} + \mu - \frac{\mu}{\theta}\right) + 1\right]^{-\alpha}$$

mixing models, respectively. These six models and the general model can give other models, as special cases, by changing the types of mixing or mixing efficiency.

As an example of changing the mixing efficiency, consider now the complete-incomplete mixing model. If μ is changed from $0 < \mu < 1$ to 0, then the complete-incomplete mixing model will give the complete-piston flow model; if μ is changed to 1, then the resulting model will be the complete mixing model. As an example of changing the mixing type, again consider the complete-incomplete mixing model. Suppose the mixing type is changed from complete-incomplete mixing to just complete mixing, then the complete-incomplete mixing model will give the complete mixing model.

Cases 4 and 5 that shown in Table 1 are more likely to occur in actual ventilation systems than piston flow, complete mixing, or the combination of complete-piston flow; and case 6 may modeled differently by the dispersive model.

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System Responses

Consider now in the case of time-invariant systems with constant input, the output concentration can be determined analytically using the convolution integral and the appropriate RTD function that best suits the investigated systems. For time-invariant systems, the convolution integral is

$$C_{out}(t) = \int_0^\infty C_{in}(t-\tau)g(\tau)d\tau = \int_\infty^t C_{in}(\tau)g(t-\tau)d\tau = C_{in}(t) * g(t), \tag{16}$$

where $C_{out}(t)$ is the output concentration, $C_{in}(t)$ is the input concentration, $g(t)$ is the RTD function, and * denotes the convolution operation defined by the integrals in Eq. (16). In Eq. (16a), t is the calendar time, and τ is the integration variable which represents the residence time of airflow; whereas in Eq. (16b), t is the time of entry, t is the time of exit and $(t-\tau)$ is the residence time.

The Laplace transform of a convolution integral is the product of the Laplace transforms of the functions involved. Thus

$$\hat{C}_{out}(s) = \hat{C}_{in}(s)\hat{g}(s), \tag{17}$$

provided the system is said to be initially relaxed, i.e., $C_{in}(t) = 0, \forall t < 0$. The ratio $\hat{C}_{out}(s)/\hat{C}_{in}(s)$ is known as the transfer function and is seen to be identical to $\hat{g}(s)$.

In the case of the constant tracer material or existing gaseous contaminant input (e.g., odor causing volatile organic compounds emitted from stored swine manure such as p-cresol and p-xylene) with a constant decay rate λ, the fraction remaining after t, is simply $\exp(-\lambda t)$. Thus for total initial input concentration, the output concentration remaining after t is

$$C_{out}(t) = \int_0^\infty C_{in}(t-\tau)e^{-\lambda\tau}g(\tau)d\tau = \int_\infty^t C_{in}(\tau)e^{-\lambda(t-\tau)}g(t-\tau)d\tau. \quad (18)$$

Analytical determination of the mean age in case of constant tracer input requires no additional information if the selected model is an one-parameter gamma model, such as piston flow or complete mixing model. For two-parameter gamma models, a priori knowledge of the parameters other than the mean age, the unknown, is necessary. Practically, however, a priori knowledge of these parameters is very difficult to obtain.

Substituting Eq. (4) in Eq. (18) for a constant $C_{in} = C_o$ gives an output and input relations as a function of T as,

$$\frac{C_{out}(T)}{C_o} = \frac{1}{(\lambda\beta + 1)^\alpha}, \quad (19)$$

and define now a relative concentration $U(T)$ as

$$U(T) = \frac{C_{out}(T)}{C_o}. \quad (20)$$

Substituting the parameters of α and β of any of the models into Eq. (18) gives the U of that particular model. In case of the general model in Eq. (4),

$$U(T) = \frac{1}{\left[\frac{\lambda T}{\alpha}\left(\frac{1}{\eta\theta} + \mu - \frac{\mu}{\theta}\right) + 1\right]^\alpha}. \quad (21)$$

Knowledge of U and λ ($\lambda = \ln 2/t_{1/2}$ where $t_{1/2}$ is the half-life of tracer), and the other parameters in Eq. (21), allows the determination of mean age of air T. The characteristics of the mixing behavior can be illustrated in Fig. 2, where U is plotted against relative mean residence time λT for different mixing types for a constant input concentration and constant θ and η with different μ ranged from 0.4-0.7. Fig. 2 shows that, for instance, for $U = 0.01$ the piston flow mean age is $T = 4.5/\lambda$, whereas in incomplete mixing flow, mean age is $T = 100/\lambda$. The real age lies somewhere between the extreme values of Fig. 2. Its better estimate depends on the choice of an adequate model.

Table 1 lists the relationship of U for six other models with a constant decay rate λ. The relationship for the incomplete mixing model reduces to those of the complete mixing model for $\mu = 1$ and $\alpha = 1$, and the piston flow model for $\mu = 0$ and $\alpha = 1$.

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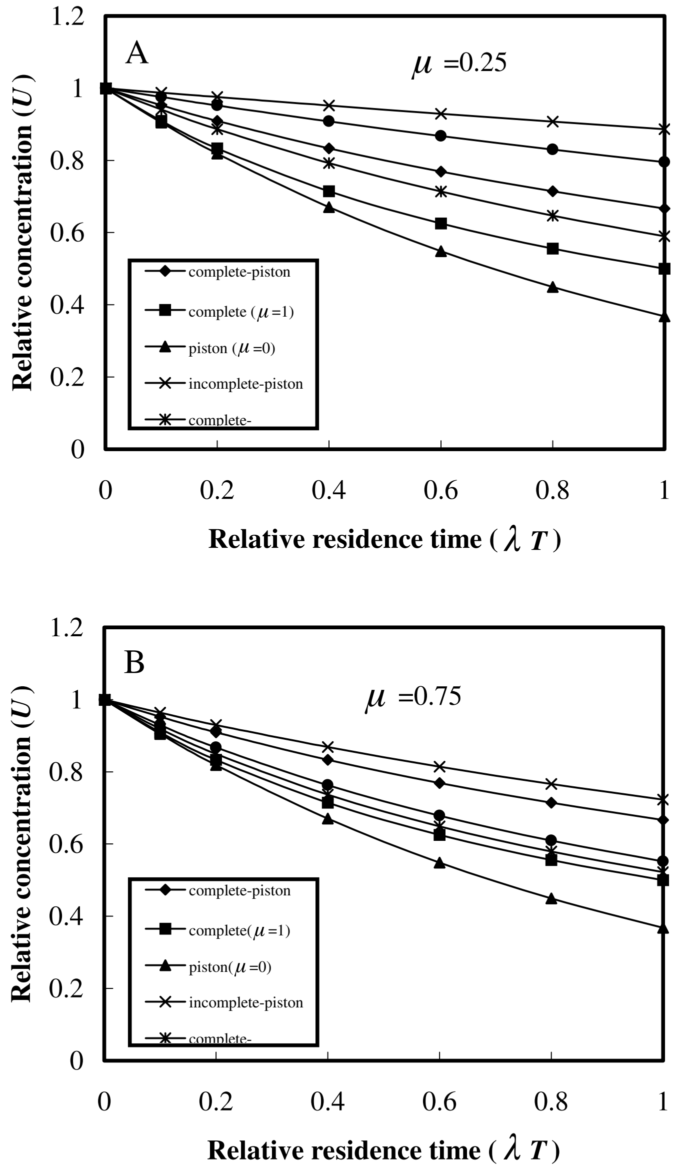

Figure 2. Relative concentration as a function of the relative residence time for a constant input concentration and for different mixing modesl: (A) $\mu = 0.25$, and (B) $\mu = 0.75$.

MODEL IMPLEMENTATION

A Numerical Example

The RTD functions can assumed any shape, depending on the values of the shape parameter α. It is obvious that the flexibility of the RTD functions

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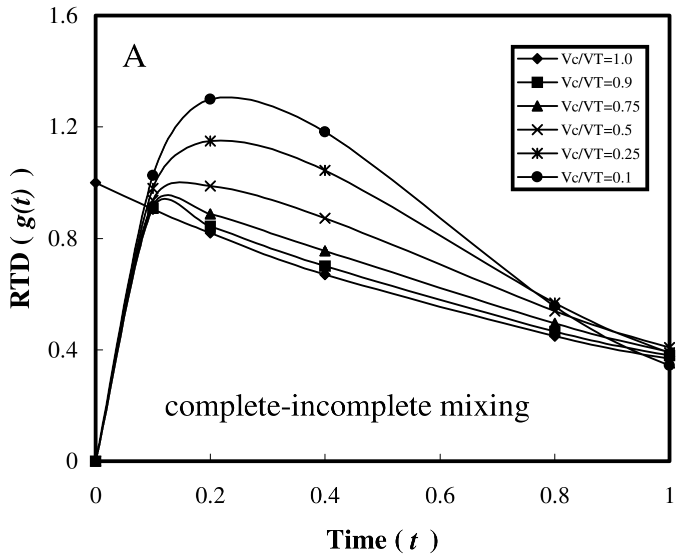

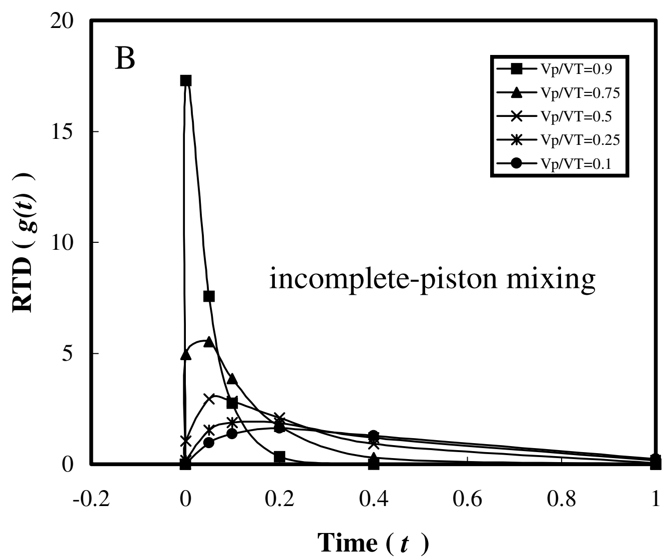

Figure 3. The RTD function of different shapes constructed for two of mixing models: (A) complete-incomplete mixing, and (B) incomplete-piston flow.

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is very important if a realistic model is derived to describe the complex mixing behavior of the general ventilation systems.

The minimum number of fitting parameters requires to model the combinations of complete-incomplete or incomplete-piston flow is three: the mean age of air (T), the mixing efficiency (μ), and mixing volume factor I

(θ) in the systems. On the other hand, the minimum number of parameters required to model the general case is four: T, μ, θ, and mixing volume factor II, η.

Figs. 3A and B illustrate the RTD function of different shapes constructed for two of the models developed in the study: the complete-incomplete mixing and the incomplete-piston flow models. Fig. 3A shows that the graph with $V_c = 0.9V_T$ (90% of the system is completely mixed), closely approximates that of the complete mixing model. Fig. 3B shows that as the volume occupied by piston flow increases, the variances of the graph decrease and their peaks increase, i.e., the graphs approach piston flow resulting from increasing V_p and decreasing μ imply piston flow.

The numerical example reveals that the development of the general model with one set of parameters to account for the three mixing types (complete, incomplete, and piston flow) and their combinations.

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Fitting MARGM to Measurement Data

Systems that tend to highly mixed can be modeled with the incomplete mixing model with a highly mixing efficiency (i.e., the modeler has to specify a high value for μ), or the complete-incomplete mixing model with high μ and/or high V_c, or the general model with high μ and/or high V_c.

Systems characterized by a low degree of mixing, can be simulated by the incomplete mixing model with a low μ, or the incomplete-piston flow model with low μ and/or high V_p with low μ and/or high V_p, or the general model with low μ and/or high V_p. In case of the general model, the incomplete mixing volume (V_i) will provide a realistic transitional mixing zone between the highly or complete mixing volume (V_c) and the piston flow volume (V_p).

Of three case studies were given to illustrate the application of the proposed models for the case of variable tracer input. The examples are limited to systems in steady state since few data exist for transient ventilation systems. To select the propor models for interpreting the transient times, we use knowledge of the gaseous and airborne particulate output concentrations as well as the nature of ventilation systems. Of three examples designated as Systems I, II, and III were chosen to our model simulation. Table 2 gives the physical properties for Systems I, II, and III. Table 3 summarizes the simulation results of fitting MARGM to the measurement data.

The fitting procedure in this work is preformed by guess work and no objective criteria are applied. A low number of experimental data as well as a limited significance of the residence time of the investigated systems did not justify a great effort needed for a more rigorous reinterpretation. Whenever the fitting is performed by trial and error, the term ‘‘quantitative’’ is rather an arbitrary one.

Table 2. Physical Characteristics of Systems I, II, and III

	System		
	I[a]	II[b]	III[c]
Target gas	CO_2	CO_2	CO_2
Ventilation system	Passive	Slot-inlet	Slot-inlet
Building type	Portland federal building	Test chamber	Test chamber
Dimension (m^3)	NA[d]	$4.9 \times 2.4 \times 2.4$	$1.5 \times 0.92 \times 0.95$
Ventilation rate range	0.5-1.6 m^3h^{-1}	281-995 m^3h^{-1}	371-1070 m^3h^{-1}

[a] Adapted from Grot and Lagus (1991).
[b] Adapted from Liao and Bundy (1995).
[c] Adapted from Liao et al. (1991).
[d] Not available.

Table 3. Simulation Results of Fitting MARGM to Measure Data

	System			
	I	II	III(A)	III(B)
T(h)	6.34 ± 3.45[a]	2.44 ± 0.37	2.80 ± 0.22	3.50 ± 0.20
μ	0.34 ± 0.18[a]	0.53 ± 0.02	0.48 ± 0.05	0.56 ± 0.01
% error	57.79	7.92	34.10	28.59
Mixing pattern	complete-incomplete-piston	complete-incomplete	complete-piston	incomplete

[a] Mean $\pm$ standard deviation.

Based on the above information, we chose the incomplete mixing model with relatively short T and moderate μ for the interpretation of System I. Using this model, the best fit is obtained for $T = 6.34 \pm 3.45\,\mathrm{h}$, $\mu = 0.34 \pm 0.18$. The best fit of the simulations to the measurements is determined using the maximum likelihood method for parameter estimation where the error is assumed to be distributed log-normally and the sum of the relative squared errors (SRSE) was minimized,

$$SRSE = \sum_{n=1}^{N} \left(\frac{C_{m,n} - C_{s,n}}{C_{s,n}} \right)^2, \tag{22}$$

where N is the number of data points, $C_{m,n}$ is the measurement data, and $C_{s,n}$ is the simulation result corresponding to data point n. The percent error then was defined as follows,

$$\%error = \sqrt{SRSE} \times 100. \tag{23}$$

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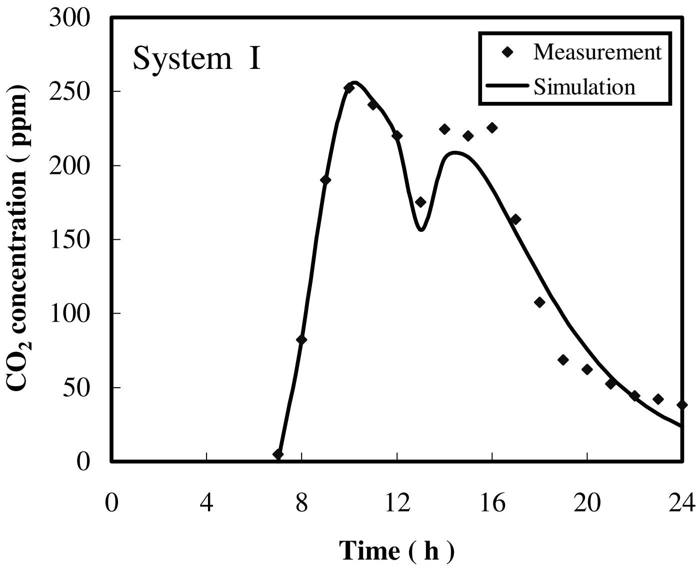

Figure 4. Fitting MARGM to measurement data from System I of a Portland federal building as calculated from the general model with $\mu = 0.34 \pm 0.18$ and $T = 6.34 \pm 3.45\,\mathrm{h}$.

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The predicted and observed output concentration profiles for System I are shown in Fig. 4 in that the steady-state is never obtained. This non-attainment of equilibrium leads to a large over estimate of the ventilation rate occurring in the building. On comparison to the MARGM, the percent error in the estimates as defined by Eq. (23) was 57%, indicating that the numerics of the MARGM were performing unfavorable well in the 13:00 to 18:00 h shown in Fig. 4. The relatively large scatter of the measurement data in 13:00 to 18:00 h was probably caused by the variable airflow rates. Therefore, more data would be necessary to interpret properly the mean residence time of this case. The present interpretation gives an approximation of the mean residence time and mixing efficiency.

For System II, the complete-incomplete flow model with relatively low T and moderate μ was selected. The best fit is obtained for $T = 2.44 \pm 0.37\,\mathrm{h}$, $\mu = 0.53 \pm 0.02$, and it is given in Fig. 5 in that the percent error is less than 7.9% indicating that the MARGM were performing well. The interpretation of the data in this case confirms the use of more refined model gives more confidence and additional details concerning the physical parameters of the investigated systems.

For System III(A), the mean transient time of air in the system with the complete-piston mixing model was selected owing to the general tendency of such system to be highly mixed. Fig. 6A shows the best fit obtained $T = 2.80 \pm 0.0.22\,\mathrm{h}$, $\mu = 0.48 \pm 0.05$. In this case, the obtained fit is found to be favorable good at the percent error 34%.

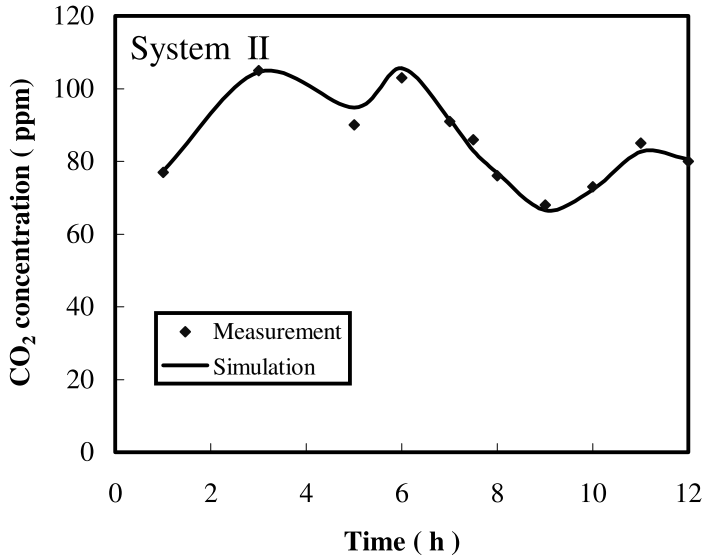

Figure 5. Fitting of MARGM to measurement data from System II of a test chamber as calculated from the complete-incomplete mixing model with $\mu = 0.53 \pm 0.02$ and $T = 2.44 \pm 0.37\,\mathrm{h}$.

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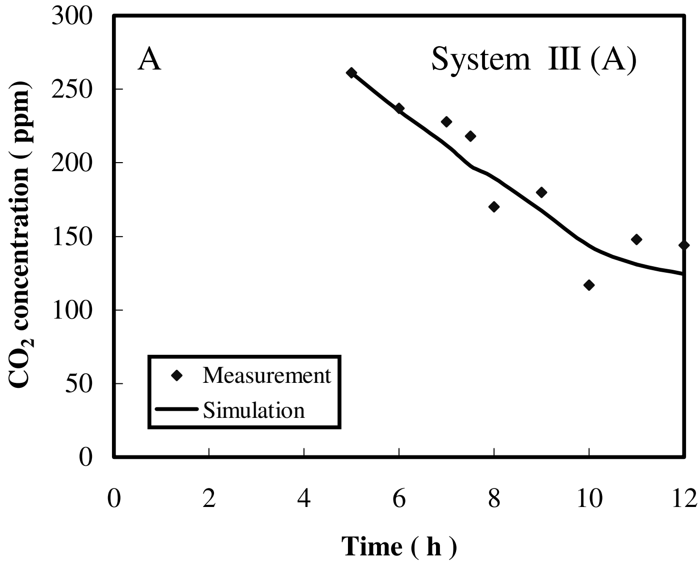
For System II(B), the mean transient time of air in the system with the incomplete mixing model was selected owing to the general tendency of such system to be highly mixed. Fig. 6B shows the best fit obtained $T = 3.5 \pm 0.2\,\mathrm{h}$, and $\mu = 0.56 \pm 0.2$. The percent error is found to be 28% indicating that the deviation is well within normal experimental accuracy.

From the above examples it is clear that two-parameter MARGM gave a good fit and would be considered as more reliable in those cases. In addition they supply additional physical information on the ventilation system (T or μ). The MARGM do not require any physical knowledge priori to the data interpretation, but the obtained weighting function supply physical information. If a given type of the g(t) function is unacceptable, the model can be rejected and another model sought.

CONCLUSIONS

A multiple airflow regions gamma model (MARGM) that capable of interpreting concentration profiles and simulating a variety of mixing regimes was presented. The gamma distribution was chosen because it is characterized by only two parameters. The mixing types of piston (no mixing), incomplete mixing, and complete mixing can be obtained from the MARGM. Incomplete mixing can be modeled separately or in combination



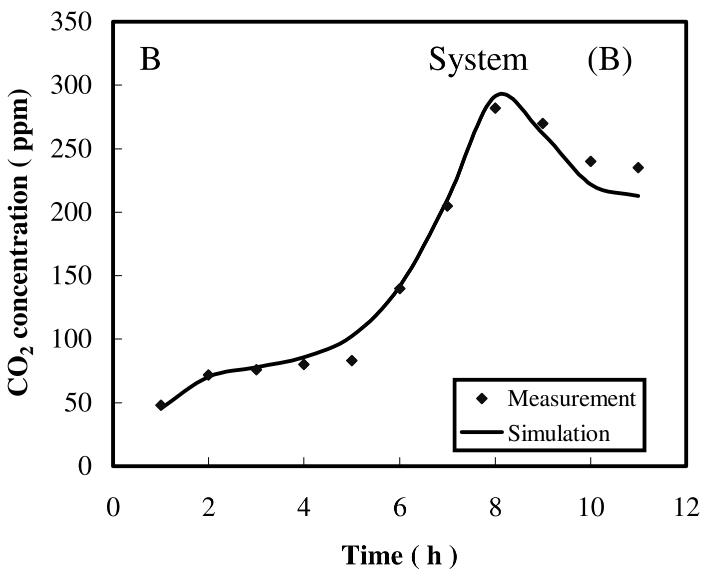


Figure 6. Fitting of MARGM to two different measurement data from System III of a test chamber as calculated from the complete-piston flow model with $\mu = 0.48 \pm 0.05$ and $T = 2.80 \pm 0.05\,\mathrm{h}$ for III(A) and from the incomplete mixing model with $\mu = 0.56 \pm 0.01$ and $T = 3.50 \pm 0.20\,\mathrm{h}$ for III(B).

with complete mixing or piston mixing with the complete-incomplete and the incomplete-piston flow model, respectively.

Heterogeneous mixing is simulating free from the dispersivity limitations (scale-dependence). The shape flexibility of the weighting functions presented in this paper re comparable to those of the dispersive model.

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The MARGM have the potential for estimating mean residence time in ventilation systems characterized by different mixing regimes.

APPENDIX A: LIST OF SYMBOLS

$C_{in}(t)$	time-dependent input concentration (mg m^{-3})
$C_{out}(t)$	time-dependent output concentration (mg m^{-3})
$C_{out}(T)$	output concentration as a function of T (mg m^{-3})
C_o	constant input concentration (mg m^{-3})
$C_{m,n}$	measurement data point n in error analysis
$C_{s,n}$	simulation data point n in error analysis
$E(t)$	mean of the gamma distribution
$f(t)$	residence time distribution (h^{-1})
$g(t)$	residence time distribution, also called weighting function (h^{-1})
N	number of data point
Q	volumetric airflow rate through the system (m^3 h^{-1})
$sd(t)$	standard deviation of the gamma distribution
T	mean residence time (h)
T_n	normal mean residence time (h)
t	time variable (h)
$t_{1/2}$	tracer half-life (h)
$U(T)$	relative concentration as a function of T (dimensionless)
V	air volume (m^3)
V_T	total air volume in the system (m^3)
V_c	air volume occupied by complete mixing (m^3)
V_i	air volume occupied by incomplete mixing (m^3)
V_p	air volume occupied by piston flow (m^3)

Greek letters

α, β	shape and scale parameters respectively in the gamma distribution
$\Gamma(\cdot)$	gamma function
θ	mixing volume factor I ()
η	mixing volume factor II ()
λ	tracer decay rate (h^{-1})
λT	relative residence time (dimensionless)
μ	mixing efficiency
τ	residence time variable (h)

Abbreviation

MARM	Multiple airflow region model
MARGM	Multiple airflow region gamma model
RTD	Residence time distribution
SRSE	Sum of the relative squared errors

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