

行政院國家科學委員會補助專題研究計畫成果報告

作業管理若干最適化問題之預先處理

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## 一：中文摘要

作者與 Professor James Orlin (MIT) 共同引進一類新問題用來預先處理一最適化問題，其中該最適化問題目標函數之成本係數的值可落於一上下界區間。該類問題來自於實際情境，提出新的挑戰，並要求新的求解技巧。第一部分結果包含證明若干該類問題係 NP-complete 並包括若干該類問題之一般性結果。第二部分針對十個問題(例子)分別提出可於多項式時間內解答之演算出。這十個問題來自於作業管理:包含 matroid, 網路最適化, 排序, 位置選擇。

**關鍵字：**最適化, 作業管理, 預先處理, NP-complete, 可於多項式時間內解答

## Abstract

We introduce a new class of yes-no decision problems to preprocess an optimization problem in which each cost coefficient in the objective function can assume any value between a lower and upper bound. This new yet rich class of decision problems arises naturally in real-world scenarios, poses new challenges and demands new types of solution techniques. In the first part we establish NP-completeness results for some problems in this class. We also establish some general results for this new class of problems. In the second part we present ten problems (examples) among this new class of problems along with their polynomial-time algorithms to demonstrate some types of new solution techniques for those that are polynomially solvable. These ten problems arising in operations management include matroid, network optimization, sequencing, and location.

**Key Words:** Optimization, Operations Management, Preprocessing, NP-complete, Polynomially Solvable.

## 二：緣由與目的

Sun Tzu, an ancient Chinese philosopher and expert on the art of warfare, once said, “Do not depend on the enemy not coming; depend rather on being ready for him. Do not depend on the enemy not attacking; depend rather on having a position that cannot be attacked.” In managerial settings, the uncertain and uncooperative environment in planning and scheduling operations, it is important to ask ourselves, as planners, the question “Are we ready for him?”.

In real-world scenarios, the input data available to an optimization model is typically less precise at the planning or scheduling stage while the input data may become more precise as the planned or scheduled process evolves. On the other hand, we have more time to conduct a deep preliminary analysis at the control stage, especially in real-time control. It is, therefore, meaningful to preprocess an optimization problem under imprecise data beforehand at the planning or scheduling stage. In particular, it is significant as well as interesting to foretell at the planning or scheduling stage whether a specific decision variable (set of decision variables) will ever be a part (parts) of an optimal solution for all possible realization costs.

We introduce a new class of yes-no decision problems to deal with an optimization problem in which each cost coefficient in the objective function can assume any value between a lower and upper bound. This new class of decision problems arises naturally, say, in a scenario in which at the planning or scheduling stage each cost coefficient is less precise while the cost coefficient becomes more precise as the planned or scheduled process evolves. We assume that at the planning or scheduling stage each cost coefficient may fall between a lower and upper bound while the uncertain cost coefficient becomes realized at the control stage immediately before an optimal solution is resolved and implemented. In particular, we address the type of yes-no decision problem (question) whether there exists a realization of the uncertain cost coefficients in the objective function such that a set of elements (decision variables) are parts of an optimal solution.

There are some managerial implications in knowing beforehand whether a specific decision variable can ever be part of an optimal solution for all possible realizations of the uncertain cost coefficients. First, if one knows beforehand that a specific decision variable will never be part of an optimal solution for all possible realizations, the decision variable can be dropped and thus the optimization problem at the control stage will be simplified. Hence, it can alleviate our decision-making burdens at the control stage during which time is highly limited as well as very precious. Second, knowing beforehand whether a specific decision variable can ever

be part of an optimal solution helps rationalize our anticipations of the future as well as helps us prepare beforehand for the uncertain environments. Technically speaking, this new class of decision problems poses new challenges and demands new types of solution techniques. For technical results, the interested readers may refer to the two working papers by Lai and Orlin (2000), which are available on request to Tsung-Chyan Lai (email: tclai@ccms.ntu.edu.tw).

### *Reference*

1. Lai, Tsung-Chyan and Jim Orlin (2000). "Preprocessing some optimization problems under imprecise data. Part I: Complexity analysis", working paper.
2. Lai, Tsung-Chyan and Jim Orlin (2000). "Preprocessing some optimization problems under imprecise data. Part II: Polynomailly solvable cases", working paper.