

行政院國家科學委員會補助專題研究計畫成果報告

情況惡化下受限於先後順序關係之單機排序

Single Machine Sequencing under Deteriorating Situations  
with Precedence Relations

計畫類別：個別型計畫

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## 一、摘要

本計畫探討情況惡化下且工作間有先後順序關係下之單機排序問題。在情況惡化下，一工作之處理時間與該工作何時開始處理有關且愈等待愈長。本計畫證明，如果先後順序關係乃一鏈狀結構、一樹狀結構、一森林結構、或一串-並列結構，該問題可可在多項式時間內求解。在一般性之結構下，證明該問題係一 NP-難度問題，但如果處理時間呈等比例關係，則該問題之求解時間為二次式。

關鍵詞：單機，排序，情況惡化，先後順序關係，作業全程，複雜度，NP-難度，可多項式時間內求解，最佳演算法。

## Abstract

This research project considers the single machine sequencing in the presence of precedence relations under deteriorating situations with an objective to minimize the makespan. In a deteriorating situation, the processing time of a job is not only increasing while waiting but also dependent on the time at which the job begins processing. It is proved that the problem can be solved in polynomial time when precedence constraints are in the form of a set of chains, a tree, a forest or series-parallel graph. The general case, when precedence constraints are given by arbitrary directed a-cyclic graph and processing times are linear, is proven to be NP-hard. It is also shown that if in the latter case the processing times are proportional functions, the problem can be solved in quadratic time.

Keywords: *single machine, sequencing, deteriorating situations, precedence relations, makespan, complexity, NP-hard, polynomial solvable, optimal algorithms*

## 二、緣由與目的

The single machine sequencing problem considered in this research can be described as follows. A set of  $n$  jobs are available for processing at a specific time  $t_0$ . There exists a set of precedence relations among these  $n$  jobs. If a job  $j$  starts processing at time  $t$ , then the processing time of job  $j$  is  $a(j)+b(j)t$ , where  $a(j) > 0$  and  $b(j) > 0$ . The objective is to finish processing these  $n$  jobs as soon as possible, i.e., minimize the makespan.

This problem arises naturally in real-world scheduling scenarios, e.g., steel

rolling mills, controlling of queuing and communication systems, repayment of loans, scheduling of emergency medical response teams, telephone inquiry service, fire fighting, scheduling of resources to control epidemic activities, shaping metals in metallurgical processes, etc. See, e.g., Gupta and Gupta (1988), Browne and Yechiali (1990), Mosheiov (1991, 1996), Kunnathur and Gupta (1990), Sundararaghavan and Kunnathur (1994) for citing the applications.

Scheduling under deteriorating situations was first introduced independently by Tanaev et al. (1984), Gupta and Gupta (1988), and Browne and Yechiali (1990). Subsequent works include Mosheiov (1994, 1995, 1996), Gawiejnowicz and Pankowska (1995), Kononov (1996, 1997), Chen (1995, 1996), Woeginger (1995), Alidaee and Landram (1996), Cheng and Ding (1998a, 1998b), Cai et al. (1998), Kovalyov and Kubiak (1998), and Kubiak and Velde (1998).

To the best knowledge of the author, there are no works on scheduling under deteriorating situations with precedence relations. This research project aims to contribute in this direction.

### 三、成果與討論

It is shown in this project that if the precedence relations are in the form of chains, a tree (an out-tree or in-tree), a forest or series-parallel, the problem can be solved in polynomial time. It is shown that in the general case, when the precedence relations are given by an arbitrary directed acyclic graph, the problem is NP-hard in the ordinary sense. In the latter case, it is shown to be solvable in quadratic time if the processing times are proportional.

There are two further research avenues. First, one can seek good polynomial-time approximation algorithm for the general case, which is NP-hard. Second, one can seek other forms of deterioration for which simple precedence relations, such as chains or tree, still preserve the property of being polynomial solvable.

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