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Symmetries in Atomic Collisions

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General kinematic analyses of various atomic collision processes by using the density-matrix and helicity formulations are reviewed. The rotation, parity, and time-reversal symmetries of the colliding systems are considered. In addition to polarization and angular correlations in each collision process, correspondences among different collision processes are also studied. As an example, the time-reversal symmetry between photoionization and electron-capture fluorescence are demonstrated explicitly, and there is a one-to-one correspondence between angular correlation functions for the two processes. The symmetry study in atomic collisions provides a great deal of valuable information about the colliding systems and enhances our knowledge on the collision dynamics.

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I. Introduction

Atomic collisions have been the most fruitful probes of elementary interactions and atomic structure. Besides fundamental interests, atomic collisions have also important applications in astrophysics, plasma physics, and radiation physics. Most studies on atomic collisions are about cross sections, and not much attention has been paid to polarization and angular correlations in the collisions. As a consequence, significant achievements in understanding of the collision dynamics have been elusive. Polarization and angular correlations in atomic collisions can reveal a wealth of information about the collision dynamics and coherence. A notable application, for example, is the observation of parity nonconservation in inelastic scattering of longitudinal polarized electrons from unpolarized targets [1].

In this paper, we present a brief account of polarization and angular correlations in atomic collisions. A relativistic helicity formulation is adopted such that the spin polarization of the electron and fine structure of the atom are built in from the outset. This approach is particularly suitable when colliding systems involve highly ionized ions.

Photon and electron helicity states are presented in Section II. Density matrix descriptions of polarization for the photon, electron, and atom are given in Section III. Section IV outlines the helicity formulation of atomic collisions employed in the present application.

A general collision equation is introduced for collision processes in Section V. Photoionization and electron-capture fluorescence are treated respectively in Sections VI and VII. Roles played by the symmetry principle are demonstrated in Section VIII. Cconclusions are made in Section IX.

II. Relativistic helicity states

The angular-momentum helicity state (or spherical helicity state) $|k\lambda; jm\rangle$ with helicity λ can be constructed from the linear-momentum helicity state (or linear helicity state) $|\boldsymbol{k}\lambda\rangle$, and vice versa. They are related by the following relations [2-4]:

$$|k\lambda; jm\rangle = \left(\frac{2j+1}{4\pi}\right)^{1/2} \int d\hat{k} D^{(j)}_{m\lambda}(\hat{k})^* |\boldsymbol{k}\lambda\rangle, \tag{2.1}$$

and

$$|\boldsymbol{k}\lambda\rangle = \sum_{jm} \left(\frac{2j+1}{4\pi}\right)^{1/2} D^{(j)}_{m\lambda}(\hat{k}) |k\lambda; jm\rangle, \tag{2.2}$$

where $D^{(j)}_{m\lambda}(\hat{k})$ are rotation matrices [5]. Here the helicity states are normalized such that

$$\langle \boldsymbol{k}\lambda | \boldsymbol{k}'\lambda' \rangle = \delta^3(\boldsymbol{k} - \boldsymbol{k}')\delta_{\lambda\lambda'}, \tag{2.3}$$

$$\langle k\lambda; jm | k'\lambda'; j'm' \rangle = \frac{1}{k^2}\delta(k - k')\delta_{\lambda\lambda'}\delta_{jj'}\delta_{mm'}. \tag{2.4}$$

II-1 Photon helicity states

The linear helicity state of the photon is given as

$$\boldsymbol{A}_{\boldsymbol{k}q} = (2\pi)^{-3/2} \hat{e}_q e^{i\boldsymbol{k}\cdot\boldsymbol{r}}, \tag{2.5}$$

where $\hat{e}_q$ denotes the spherical unit vector [5]. The spherical helicity state can be obtained by

$$\boldsymbol{A}_{kq;jm} = \left(\frac{2j+1}{4\pi}\right)^{1/2} \int d\hat{k} D^{(j)}_{mq}(\hat{k})^* \boldsymbol{A}_{\boldsymbol{k}q}. \tag{2.6}$$

In terms of the normalized electric, magnetic, and longitudinal multipole potentials we get [6]

$$\boldsymbol{A}_{k\pm1;jm} = \mp\left(1/\sqrt{2}\right)\left(\boldsymbol{A}^{(M)}_{jm} \pm i\boldsymbol{A}^{(E)}_{jm}\right), \tag{2.7}$$

$$\boldsymbol{A}_{k0;jm} = \boldsymbol{A}^{(L)}_{jm}, \tag{2.8}$$

where $A_{k\pm1;jm}$ correspond respectively to the positive and negative helicity states of the photon, and $A_{k0;jm}$, however, is not a physical state of the photon.

11-2. Electron helicity states

The linear helicity state of the electron is given by

$$\langle \boldsymbol{r}|\boldsymbol{k}\mu\rangle = (2\pi)^{-3/2}\sqrt{\frac{E+c^2}{2E}}\begin{pmatrix} 1 \\ \dfrac{c\vec{\sigma}\cdot\boldsymbol{k}}{E^2+c^2}\end{pmatrix}\chi_\mu e^{i\boldsymbol{k}\cdot\boldsymbol{r}}, \tag{2.9}$$

where χ_μ is the spin eigenfunction with the quantization axis chosen in the $\boldsymbol{k}$ direction, $s_{\mathbf{k}}=\mu$. The spherical helicity state can be obtained as [6]

$$\begin{aligned}\langle \boldsymbol{r}|k\mu;jm\rangle &= \left[\frac{2j+1}{4\pi}\right]^{1/2}\int d\hat{k}D^{(j)}_{m\mu}(\hat{k})^*\langle \boldsymbol{r}|\boldsymbol{k}\mu\rangle \\ &= \sum_l i^l\left[\frac{(2l+1)c^2}{kE(2j+1)}\right]^{1/2}\langle l0s\mu|j\mu\rangle\langle \boldsymbol{r}|\kappa m\rangle,\end{aligned} \tag{2.10}$$

where $\langle \boldsymbol{r}|\kappa m\rangle$ is the usual angular-momentum eigenstate normalized on the energy scale,

$$\langle \boldsymbol{r}|\kappa m\rangle \equiv \psi_{\kappa m}(\boldsymbol{r}) \equiv \frac{1}{r}\begin{pmatrix} G_{\kappa m}(r)\Omega_{\kappa m} \\ iF_{\kappa m}(r)\Omega_{-\kappa m}\end{pmatrix}. \tag{2.11}$$

In the Pauli approximation, we have

$$\begin{aligned}\langle \boldsymbol{r}|\kappa m\rangle &\equiv \langle r|(ls)jm\rangle \\ &= \sum_{\mathbf{MU}}\langle lM s\nu|jm\rangle\langle \tilde{\boldsymbol{r}}|lM\rangle\chi_\nu,\end{aligned} \tag{2.12}$$

where χ_ν is the spin eigenfunction with $s_z=\nu$.

III. Density matrix description of polarization

In the description of a physical system involved in the collision, a linear superposition of helicity states will suffice when the system is in a pure state. However, when the system is in a mixed state, the density matrix description is necessary.

III-1. Photon

One convenient description of the photon polarization is the density matrix of the form [6]

$$\begin{aligned}P &\equiv (\rho_{q'q}) \equiv \begin{pmatrix} \rho_{11} & \rho_{1-1} \\ P\text{-}11 & \text{P-1-1}\end{pmatrix} \\ &= \frac{I}{2}\begin{pmatrix} 1+p\sin 2\alpha & -pe^{-i2\gamma}\cos 2\alpha \\ -pe^{i2\gamma}\cos 2\alpha & 1-p\sin 2\alpha\end{pmatrix},\end{aligned} \tag{3.1}$$

where we have

I : intensity,

p : $0 \leq p \leq 1$, degree of polarization,

α : $-\pi/4 \leq \alpha \leq \pi/4$, type of polarization,

γ : $0 \leq \gamma < 2\pi$, orientation of polarization.

An equivalent description of the photon polarization is provided by Stokes parameters S = (S_1, S_2, S_3) which are related to (p, α, γ) as

$$S_1 = -p \cos 2\alpha \cos 2\gamma, \tag{3.2}$$

$$S_2 = -p \cos 2\alpha \sin 2\gamma, \tag{3.3}$$

$$S_3 = p \sin 2\alpha. \tag{3.4}$$

Mathematically, we may expand the density matrix ρ in the complete basis set for 2 x 2 matrices: the 2 x 2 identity matrix together with the Pauli matrices a' · The expansion is written as

$$\rho = \frac{1}{2} I[1 + \boldsymbol{S} \cdot \vec{\sigma}], \tag{3.5}$$

where the intensity I and Stokes vector S are thus defined as

$$I = tr\{\rho\}, \tag{3.6}$$

$$\boldsymbol{S} = \frac{tr\{\rho\vec{\sigma}\}}{tr\{\rho\}}. \tag{3.7}$$

For later convenience, we shall define a four-component tensor S_μ and a 1 x 4 matrix S as

$$S \equiv (S_\mu) \equiv \begin{pmatrix} S_0 \\ S_1 \\ S_2 \\ S_3 \end{pmatrix} \equiv \begin{pmatrix} 1 \\ S_x \\ S_y \\ S_z \end{pmatrix}. \tag{3.8}$$

111-2. Electron

The density matrix for the electron polarization is given by

$$\rho_e \equiv (\rho_{\mu'\mu}) \equiv \begin{pmatrix} \rho_{\frac{1}{2}\frac{1}{2}} & \rho_{\frac{1}{2}-\frac{1}{2}} \\ \rho_{-\frac{1}{2}\frac{1}{2}} & \rho_{-\frac{1}{2}-\frac{1}{2}} \end{pmatrix}. \tag{3.9}$$

In terms of the intensity I_e and the polarization vector $\boldsymbol{P}$, we have [7]

$$\rho_e = \frac{1}{2} I_e [1 + \boldsymbol{P} \cdot \vec{\sigma}], \tag{3.10}$$

where

$$I_e = tr\{\rho_e\}, \tag{3.11}$$

$$\boldsymbol{P} = \frac{tr\{\rho_e \vec{\sigma}\}}{tr\{\rho_e\}}. \tag{3.12}$$

Here we also define a four-component tensor P_μ and a 1 x 4 matrix P *as*

$$P \equiv (P_\mu) \equiv \begin{pmatrix} P_0 \\ P_1 \\ P_2 \\ P_3 \end{pmatrix} \equiv \begin{pmatrix} 1 \\ P_x \\ P_y \\ P_z \end{pmatrix}. \tag{3.13}$$

111-3. Atom

The density matrix for the atom polarization may be expressed in the basis of total angular momentum eigenstates $|JM\rangle$ as

$$\rho_A \equiv (\rho_{J'M',JM}) \equiv (\langle J'M'|\rho_A|JM\rangle). \tag{3.14}$$

It takes, in general, $4J(J+1)$ independent measurements to specify the polarization of state ρ_A with $J' = J$. Define the spherical matrices, which may be considered as the generalization of Pauli matrices, as

$$T_{lm}(J'M', JM) = \sqrt{2l+1} \begin{pmatrix} J' & m & M \\ M' & l & J \end{pmatrix}. \tag{3.15}$$

$-$ j coefficient in the covariant notation, called the 3 $-$ jm coefficient [4,8]. By expanding the density matrix for the atom polarization in terms of spherical matrices, we obtain

$$\rho_{J'M',JM} = \sum_{lm} Q_{lm}(J'J) T_{lm}(J'M', JM), \tag{3.16}$$

where the expansion coefficients $Q_{lm}(J'J)$ are called the state multipole (tensor) of the atom [9].

IV. Helicity formulation of atomic collisions

The helicity formulation of collision processes [2,3] has the merits of being elegant in form for collisions involving spin, applicable as well to particles of zero mass, such as photon, and having simple transformation properties under Lorentz transformations. Consequently

in the formulation of atomic collision processes, we shall describe physical systems generally by density matrices with relativistic helicity states as the basis.

Consider the general collision process where we have initially two colliding systems A and B, each of which may be either an elementary particle or a composite particle, like atom or ion. After the collision, we have finally two departing systems C and D, each of which may also be either an elementary particle or a composite particle. An elementary collision process, in which all systems involved are in linear helicity states in the laboratory (LAB) frame, is shown in Fig. 1. The scattering matrix (S-matrix) of this elementary collision process is expressed as

$$\langle \boldsymbol{p}_C \lambda_C; \boldsymbol{p}_D \lambda_D | S | \boldsymbol{p}_A \lambda_A; \boldsymbol{p}_B \lambda_B \rangle, \tag{4.1}$$

where $\boldsymbol{p}_\alpha$ and λ_α are the momentum and helicity, respectively, of system α. Define the total 4-momenta of the colliding and departing systems as P and P', respectively,

$$P = \boldsymbol{p}_A + p_B, \tag{4.2}$$

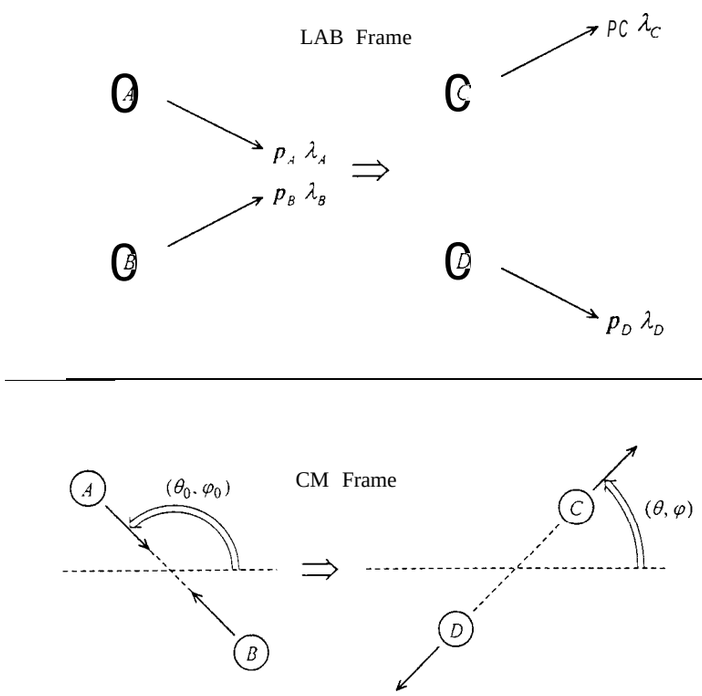


FIG. 1. The schematic diagram for the reaction: $A + B \rightarrow C + D$. Here $\boldsymbol{p}_\alpha$ and λ_α are respectively the momentum and helicity of system α in the LAB frame, and (θ_0, φ_0) and (θ, φ) are respectively scattering angles before and after the collision in the CM frame.

$$P' = p_C + p_D, \tag{4.3}$$

where p_α denotes the 4-momentum of system α, and $\alpha = A, B, C, D$. The same collision process in the center-of-momentum (CM) frame is also shown in Fig. 1, where (θ_0, φ_0) and (θ, φ) are scattering angles before and after the collision, respectively. The S-matrix $\langle \theta\varphi\lambda_C\lambda_D|S(P)|\theta_0\varphi_0\lambda_A\lambda_B\rangle$ in the CM frame is related to the S-matrix in the LAB frame as

$$\begin{aligned}&\langle \boldsymbol{p}_C\lambda_C;\boldsymbol{p}_D\lambda_D|S|\boldsymbol{p}_A\lambda_A;\boldsymbol{p}_B\lambda_B\rangle \\ &= (2\pi)^6\frac{1}{pp'}\sqrt{vv'}\,\delta^4(P-P')\langle\theta\varphi\lambda_C\lambda_D|S(P)|\theta_0\varphi_0\lambda_A\lambda_B\rangle.\end{aligned} \tag{4.4}$$

Here p and v are the relative momentum and velocity, respectively, for the colliding systems, and p' and v' for the departing systems, defined as

$$p = |\boldsymbol{p}_A - \boldsymbol{p}_B|, \qquad v = |\boldsymbol{v}_A - \boldsymbol{v}_B|, \tag{4.5}$$

$$p' = |\boldsymbol{p}_C - \boldsymbol{p}_D|, \qquad v' = |\boldsymbol{v}_C - \boldsymbol{v}_D|, \tag{4.6}$$

All the information about the collision dynamics is contained in the S-matrix in the CM frame, or equivalently in the transition matrix (T-matrix), which are related to the S-matrix as

$$S_{fi} = \delta_{fi} - 2\pi i\delta(P_0' - P_0)T_{fi}. \tag{4.7}$$

where P_0' and P_0 denote the zeroth components of the 4-momenta P' and P, respectively.

V. The collision equation

Denote symbolically the density matrices of the initial and final states of the colliding systems as ρ_i and ρ_f, respectively. We have therefore the collision equation [6],

$$\rho_f = S_{fi}\rho_i S_{fi}^\dagger, \tag{5.1}$$

where S_{fi} represents the appropriate S-matrix.

As a simple illustration, we consider photoemission from an excited atom. The elementary collision amplitude is given by [10]

$$S(q, J_fM_f; J_iM_i) \equiv \sqrt{\frac{\omega}{2\pi c}}\left\langle J_fM_f\left|\sum_{i=1}^{N}\vec{\alpha}_i\cdot\hat{\varepsilon}_q^* e^{-i\mathbf{k}\cdot\mathbf{r}_i}\right|J_iM_i\right\rangle, \tag{5.2}$$

where J_fM_f are the angular momentum quantum numbers of the de-excited atom, N is the number of electrons in the atom, $\vec{\alpha}_i$ are the Dirac matrices, and $\hat{\varepsilon}_q$ and k are respectively the polarization vector and wavenumber vector of the emitted photon.

If the excited atom is initially described by the density matrix $\rho_{J_i'M_i',J_iM_i}$, then the density matrix of the final state of the colliding systems, i.e., the departing systems, is given by

$$\rho_{q'q,J'_fM'_fJ_fM_f} = \sum_{\substack{J'_iM'_i \\ J_iM_i}} S(q', J'_fM'_f; J'_iM'_i)\rho_{J'_iM'_iJ_iM_i} S^*(q, J_fM_f; J_iM_i). \tag{5.3}$$

Kinematic and dynamic effects are entangled together in the S-matrix, especially when the collision processes involve composite particles, such as atoms or ions. The complexities in angular momentum couplings may be dealt with by a graphical approach [8].

The angular distribution and polarization of the photon, which leaves the atom in a definite angular momentum J_f state, would be described by

$$\rho_{q'q} = \sum_{M_f} \rho_{q'q,J_fM'_fJ_fM_f}. \tag{5.4}$$

Other atomic collision processes may be treated in a similar manner, and we will give only a few examples in the subsequent sections.

VI. Photoionization

Consider the photoionization process in which initially a polarized photon with linear momentum $\boldsymbol{q}$ is incident on an unpolarized target atom with total angular momentum J_α. After the collision, an electron with linear momentum $\boldsymbol{k}$ is emitted, and the residual ion is left with total angular momentum J_β. Two coordiante frames XYZ and xyz, shown in Fig. 2, are employed for the initial and final states, respectively, with unit vectors defined as

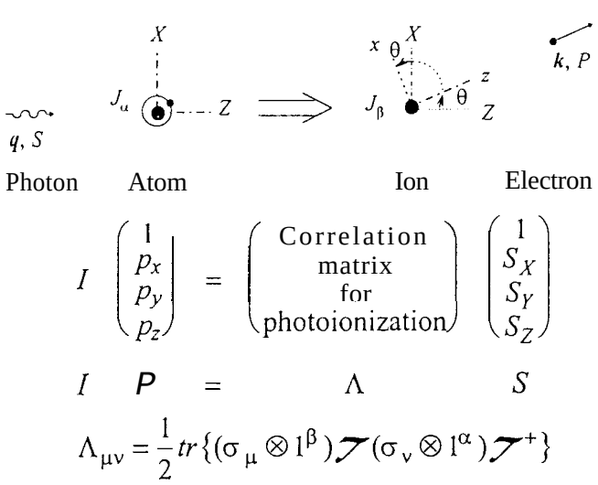


FIG. 2. The schematic diagram of the photoionization processes.

$$\hat{X},\hat{Y},\hat{Z}=\hat{Y}\times\hat{Z},\frac{\boldsymbol{q}\times\boldsymbol{k}}{|\boldsymbol{q}\times\boldsymbol{k}|},\frac{\boldsymbol{q}}{q}, \tag{6.1}$$

$$\hat{x},\hat{y},\hat{z}=\hat{y}\times\hat{z},\frac{\boldsymbol{q}\times\boldsymbol{k}}{|\boldsymbol{q}\times\boldsymbol{k}|},\frac{\boldsymbol{k}}{k}, \tag{6.2}$$

where $q=|\boldsymbol{q}|$ and $k=|\boldsymbol{k}|$. When the polarization of the residual ion is not observed, the density matrix $\rho^{(e)}$ of the emerging electron is related to the density matrix $\rho^{(\gamma)}$ of the incident photon by the collision equation as

$$\rho^{(e)}=\frac{1}{2}tr_\beta\left\{\mathcal{T}(\rho^{(\gamma)}\otimes 1^\alpha)\mathcal{T}^\dagger\right\}, \tag{6.3}$$

where

$$\mathcal{T}\equiv\mathcal{T}_{fi}\equiv V_i+V_f\frac{1}{E-H+i\varepsilon}V_i, \tag{6.4}$$

is the transition operator with i and f denoting, respectively, the entrance and exit channels of photoionization, $1^\alpha\equiv 1_{(2J_\alpha+1)\times(2J_\alpha+1)}$ is the identity matrix in the subspace of the target atom, $tr_\beta\{\}$ denotes the trace of the quantity within the curly brackets taken in the subspace of the residual ion.

In terms of the polarization vector of the emitted electron and the Stokes parameters of the incident photon, we may rewrite the collision equation as

$$IP_\mu=\sum_{\nu=0}^{3}\Lambda^{PI}_{\mu\nu}S_\nu, \tag{6.5}$$

where we have defined the 4 x 4 correlation matrix $\Lambda^{PI}\equiv\left(\Lambda^{PI}_{\mu\nu}\right)$ of photoionization by

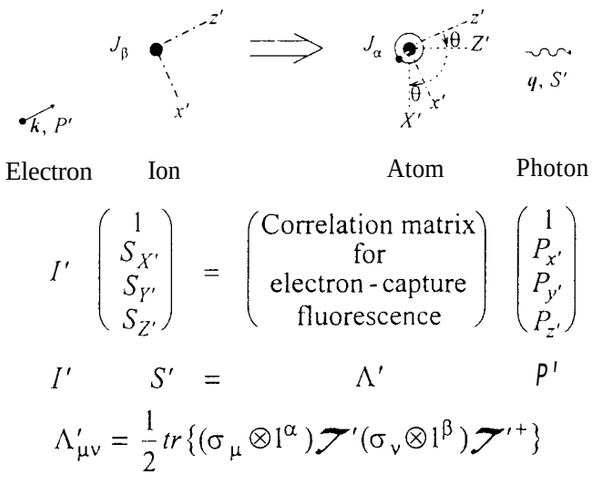
$$\Lambda^{PI}_{\mu\nu}=\frac{1}{2}tr\left\{(\sigma_\mu\otimes 1^\beta)\mathcal{T}(\sigma_\nu\otimes 1^\alpha)\mathcal{T}^\dagger\right\}. \tag{6.6}$$

Here the trace is now taken both in the spin space of the emitted electron and in the subspace of the residual ion. Explicit formulas have been given previously [6].

VII. Electron-capture fluorescence

Electron-capture fluorescence is the time-reversed process of photoionization. Consider initially an electron with linear momentum k is incident upon an unpolarized ion with total angular momentum J_β and is captured by the ion. After the collision, a photon with linear momentum $\boldsymbol{q}$ is emitted while the atom system is left with total angular momentum J_α. Two coordiante frames $x'y'z'$ and X'Y'Z', shown in Fig. 3, are employed for the initial and final states, respectively, with unit vectors defined as

$$\hat{x}',\hat{y}',\hat{z}'=\hat{y}'\times\hat{z}',\frac{k\times q}{|\boldsymbol{k}\times\boldsymbol{q}|},\frac{k}{k}, \tag{7.1}$$



$$\Lambda'_{\mu\nu} = \frac{1}{2} tr\{(\sigma_\mu \otimes 1^\alpha)\mathcal{T}'(\sigma_\nu \otimes 1^\beta)\mathcal{T}'^{+}\}$$

FIG. 3. The schematic diagram of the electron-capture fluorescence.

$$\hat{X}', \hat{Y}', \hat{Z}' = \hat{Y}' \times \hat{Z}', \frac{\boldsymbol{k} \times \boldsymbol{q}}{|\boldsymbol{k} \times \boldsymbol{q}|}, \frac{\boldsymbol{q}}{q}. \tag{7.2}$$

Here a similar convention is adopted as the unprimed coordinate systems for photoionization. When the polarization of the final atomic state is not observed, we can derive the collision equation as

$$\rho'^{(\gamma)} = \frac{1}{2} tr_\alpha \left\{ \mathcal{T}'(\rho'^{(e)} \otimes 1^\alpha)\mathcal{T}'^{\dagger} \right\}, \tag{7.3}$$

where $\mathcal{T}'$ denotes the transition operator of the electron-capture fluorescence process, similarly defined as in the photoionization case. Again, this collision equation may be rewritten as

$$\Gamma' S'_\mu = \sum_{\nu=0}^{3} \Lambda^{ECF}_{\mu\nu} P'_\nu, \tag{7.4}$$

where we have defined the 4 x 4 correlation matrix $\Lambda^{ECF} \equiv \left(\Lambda^{ECF}_{\mu\nu}\right)$ of electron-capture fluorescence by

$$\Lambda^{ECF}_{\mu\nu} = \frac{1}{2} tr \left\{ (\sigma_\mu \otimes 1^\alpha)\mathcal{T}'(\sigma_\nu \otimes 1^\beta)\mathcal{T}'^{\dagger} \right\}. \tag{7.5}$$

Similar processes have been studied previously [11-13].

VIII. Rotation, space inversion, and time-reversal symmetries

By symmetry consideration, we may reduce the correlation matrix to a simpler form. We shall assume all interactions involved in the collision processes are invariant under rotation, space-inversion, and time-reversal transformations.

The time-reversal symmetry between photoabsorption and photoemission processes, as shown in Fig. 4, is one of the simple cases studied previously. A more interesting case is provided by the symmetry between photoionization and electron-capture fluorescence. We can show that the rotation plus space-inversion invariance implies

$$\Lambda^{PI} = M^{(e)}\Lambda^{PI}M^{(\gamma)}, \tag{8.1}$$

$$\Lambda^{ECF} = M^{(\gamma)}\Lambda^{ECF}M^{(e)}, \tag{8.2}$$

where the transformation matrices $M^{(e)}$ and $M^{(\gamma)}$ are

$$M^{(e)} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}, \tag{8.3}$$

$$M^{(\gamma)} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}, \tag{8.4}$$

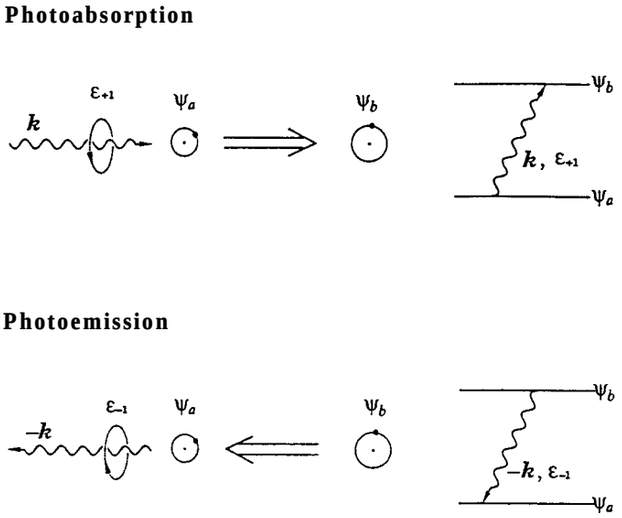


FIG. 4. The absorption and emission of a photon by an atom making a transition between states ψ_a and ψ_b.

Consequently, the correlation matrices have the following simpler forms

$$\Lambda^{PI} = \begin{pmatrix} \Lambda_{00}^{PI} & \Lambda_{01}^{PI} & 0 & 0 \\ 0 & 0 & \Lambda_{12}^{PI} & \Lambda_{13}^{PI} \\ \Lambda_{20}^{PI} & \Lambda_{21}^{PI} & 0 & 0 \\ 0 & 0 & \Lambda_{32}^{PI} & \Lambda_{33}^{PI} \end{pmatrix}, \tag{8.5}$$

$$\Lambda^{ECF} = \begin{pmatrix} \Lambda_{00}^{ECF} & 0 & \Lambda_{02}^{ECF} & 0 \\ \Lambda_{10}^{ECF} & 0 & \Lambda_{12}^{ECF} & 0 \\ 0 & \Lambda_{21}^{ECF} & 0 & \Lambda_{23}^{ECF} \\ 0 & \Lambda_{31}^{ECF} & 0 & \Lambda_{33}^{ECF} \end{pmatrix}, \tag{8.6}$$

The rotation plus time-reversal invariance implies the symmetry between photoionization and electron-capture fluorescence,

$$\Lambda^{PI} = N^{(e)} \tilde{\Lambda}^{ECF} N^{(\gamma)}, \tag{8.7}$$

or, equivalently

$$\Lambda^{ECF} = N^{(\gamma)} \tilde{\Lambda}^{PI} N^{(e)}, \tag{8.8}$$

where the tilt above a matrix denotes the transpose of that matrix, and the transformation matrices $N^{(e)}$ and $N^{(\gamma)}$ are

$$N^{(e)} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{8.9}$$

$$N^{(\gamma)} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{8.10}$$

We have explicitly the relation between the correlation matrices Λ^{PI} and Λ^{ECF},

$$\Lambda^{PI} = \begin{pmatrix} \Lambda_{00}^{ECF} & \Lambda_{10}^{ECF} & 0 & 0 \\ 0 & 0 & & - \\ \Lambda_{02}^{ECF} & \Lambda_{12}^{ECF} & 0 & 0 \\ 0 & 0 & -\Lambda_{21}^{ECF} & \Lambda_{33}^{ECF} \end{pmatrix}, \tag{8.11}$$

$$\Lambda^{ECF} = \begin{pmatrix} \Lambda_{00}^{PI} & 0 & \Lambda_{20}^{PI} & 0 \\ \Lambda_{01}^{PI} & 0 & \Lambda_{21}^{PI} & 0 \\ 0 & \Lambda_{12}^{PI} & 0 & -\Lambda_{32}^{PI} \\ 0 & -\Lambda_{13}^{PI} & 0 & \Lambda_{33}^{PI} \end{pmatrix}, \tag{8.12}$$

IX. Conclusions

We have considered four atomic collision processes to illustrate applications of the symmetry principle and the relativistic helicity formulation of atomic collisions. A detailed kinematic analysis has been presented previously for each process. Polarization and angular correlations are given in concise parametrized forms in terms of known functions of angles. Angle-independent dynamic parameters in the formulas are expressible as sums of reduced matrix elements which reflect the interactions and target structure. Photoionization of polarized targets [14], coherent fluorescence radiation [15] and Auger transition [16] following photoexcitation and photoionization as well as other collision processes can also be treated similarly. These analyses bring to light the coherence and overall symmetry of the processes and facilitate comparison between theory and experiment.

The unraveling of all polarization and angular correlations in atomic collision processes would provide critical tests of many-body dynamic theories. The symmetry principle facilitates a comparison study of different collision processes. Because of difficulties in obtaining polarized electron beams and in detecting the polarization of outgoing electrons, only a limited number of experiments concern the polarization. Both theoretical and experimental efforts in this respect are needed toward a better understanding of the collision dynamics.

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