行政院國家科學委員會專題研究計畫　成果報告

層變流域中內重力波與可滲透性邊界之互制作用研究

計畫類別：個別型計畫
計畫編號：NSC92-2611-E-002-031-
執行期間：92 年 08 月 01 日至 93 年 07 月 31 日
執行單位：國立臺灣大學工程科學及海洋工程學系暨研究所

計畫主持人：孔慶華

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一、中文摘要

瞭解層變流域(stratified flow)中內重力波與具滲透性(permeability)結構物或邊界的互制作用對海洋工程的發展有極大幫助。藉由研究內重力波並正確估算海中固體邊界受波浪作用之動力變化情形，進而瞭解海域生態、週遭設施的構建基礎之穩定性，再做最安全的設計考量。而其解析模式可藉波動力學理論及固體與流體相互作用的二相理論配合數學解析模式進行分析研究，以期對可滲透邊界與內重力波間複雜的固體流體交互作用現象能有清晰的瞭解並能提出合理的說明，提供海洋環保污染防制工程及水利工程之相關領域方面的參考。

理論解析部份將以布氏方程式(Boussinesq equation)針對雙層流及層變流中的內重力波求解，並推廣應用於較複雜的情況及邊界條件，即更吻合自然界的真實海況，並應用 Biot 理論以簡化求解水波作用下多孔彈性底床之孔隙水壓力(pore water pressure)及有效剪應力(effective shear stress)，建立一套分析此物理現象的基本機制。本研究計畫藉由理論解析及數值分析的交叉比較，以期建立一套完整的解析模式。

關鍵詞：*內重力波，滲透性、層變流*

Abstract

The interaction of internal waves and permeable boundaries in a stratified flow is the most important factor both on coastal engineering. The complicated results come from the consideration of the interaction of the two-phrase dynamic mechanisms which contain a series of water waves in a porous elastic bed. Accordingly, in recent years this oceanic topic has drawn the attention of various investigations using analytic, experimental or numerical methods. The main purpose of this study is to explain the complicated phenomenon of coexistence of liquid and solid and to get some useful data for engineering design.

The method for solving analytic solution will make use of Boussinesq and Korteweg de-Vries(KdV) equations to a forcing disturbance moving steadily in a two-layer flow at different boundary conditions. Comparing the result and the relation of the above two equations, we can expand the application to stratified flow. For the purpose of increasing the applied ability of this study, the moving disturbance modeled by different pressure distribution or section area as forcing function under water also be discussed. There is influence of permeability on the distribution of pore pressure in the problem of nonlinear waves in a channel. The general solution is obtained from the application of the theory of Biot. A numerical analysis is also attempted in simulating this flow problem by using numerical analysis scheme. The computed results should be compared to the analytical solution in order to increase the reliability of the result. We also want to setup an experimental frame to get experimental data in this work. After all, it is expected that the efforts of this project would be

helpful in the designing of these engineering problems.

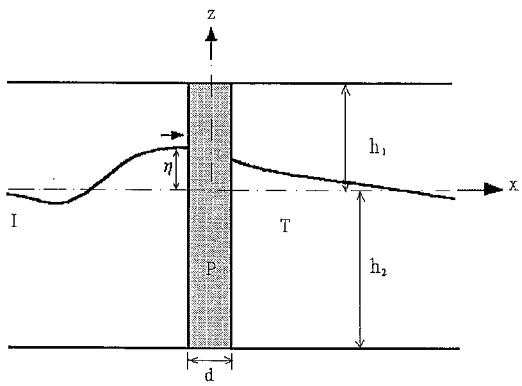
Keywords: *Internal wave, Permeability, Stratified flow*

二、控制方程式與邊界條件

首先將解析區域分為入射區(I)、透水區(P)及穿透區(T)，則(P)區之控制方程式為 (i=1 表上層 i=2 則表示下層流域)

$$\nabla\left(\hat{v}_p\right)_i = 0 (i = 1,2) \qquad (1)$$

$$\rho_i \frac{\partial\left(\hat{v}_p\right)_i}{\partial \hat{t}} = -\nabla\left(\hat{p}_p\right)_i + \rho_i g - \frac{\mu_i n_0 \hat{w}\hat{d}}{k_s \hat{A}_w}\left(\hat{v}_p\right)_i \qquad (i = 1,2) \qquad (2)$$



圖一 解析區域圖

其中 n_0 表示孔隙率，k_s 為穿透係數，d 透水牆厚度，我們可得

$$\left(\hat{v}_p\right)_i = -\frac{\rho_i k_s \hat{A}_w}{\mu_i n_0 \hat{d}}\nabla\left(\phi_p\right)_i \qquad (i = 1,2) \qquad (3)$$

將(3)代入(1)及(2)，我們可以得到

$$\nabla\left(\phi_p\right)_i = 0 (i = 1,2) \qquad (4)$$

$$\frac{\left(p_p\right)_i}{\rho_i g} + z = \frac{w}{g}\left(\phi_p\right)_i + \frac{\rho_i k_s A_w}{\mu_i n_0 g d}\frac{\partial\left(\phi_p\right)_i}{\partial t} + h_1 \qquad (5)$$

至於(I)區之控制方程式則表示如下

$$\nabla^2\left(\phi_I\right)_i = \nabla^2\left(\phi_T\right)_i = 0 \qquad (i = 1,2) \qquad (6)$$

$$\frac{\left(p_I\right)_i}{\rho_i g} + z = -\frac{1}{g}\frac{\partial\left(\phi_I\right)_i}{\partial t} + h_1 \qquad (i = 1,2) \qquad (7)$$

$$\frac{\left(p_T\right)_i}{\rho_i g} + z = -\frac{1}{g}\frac{\partial\left(\phi_T\right)_i}{\partial t} + h_1 \qquad (i = 1,2) \qquad (8)$$

我們假設上下表面均為固定的，故可得邊界條件為

$$\frac{\partial\left(\phi_I\right)_1}{\partial z} = \frac{\partial\left(\phi_T\right)_1}{\partial z} = \frac{\partial\left(\phi_p\right)_1}{\partial z} = 0 \qquad \left(z = h_1\right) \quad (9)$$

$$\frac{\partial\left(\phi_I\right)_2}{\partial z} = \frac{\partial\left(\phi_T\right)_2}{\partial z} = \frac{\partial\left(\phi_p\right)_2}{\partial z} = 0 \qquad \left(z = -h_2\right) (10)$$

而兩層流之交界面邊界條件為

$$\frac{\partial\left(\phi_I\right)_i}{\partial z} = \frac{\partial\left(\eta_I\right)_i}{\partial t} \qquad \left(i = 1,2\right) \qquad (11)$$

$$\left(\eta_I\right)_i = -\frac{1}{g}\frac{\partial\left(\phi_I\right)_i}{\partial t} \qquad \left(i = 1,2\right) \qquad (12)$$

(11)及(12)可以合併為

$$\frac{\partial^2\left(\phi_I\right)_i}{\partial t^2} + g\frac{\partial\left(\phi_I\right)_i}{\partial z} = 0 \qquad \left(i = 1,2\right) \quad (13)$$

(T)區之邊界條件為

$$\left(\eta_T\right)_i = -\frac{1}{g}\frac{\partial\left(\phi_T\right)_i}{\partial t} \qquad \left(i = 1,2\right) \qquad (14)$$

$$\frac{\partial^2\left(\phi_T\right)_i}{\partial t^2} + g\frac{\partial\left(\phi_T\right)_i}{\partial z} = 0 \qquad \left(i = 1,2\right) \qquad (15)$$

透水牆之條件為

$$-\frac{\rho_i k_s A_w}{\mu_i n_0 d}\frac{\partial\left(\phi_p\right)_i}{\partial z} = \frac{\partial\left(\eta_p\right)_i}{\partial t} \qquad \left(i = 1,2\right) \quad (16)$$

$$\left(\eta_p\right)_i = \frac{w}{g}\left(\phi_p\right)_i + -\frac{\rho_i k_s A_w}{\mu_i n_0 d}\frac{\partial\left(\phi_p\right)_i}{\partial t} \qquad \left(i = 1,2\right) \quad (17)$$

故

$$\frac{\partial^2\left(\phi_p\right)_i}{\partial t^2} + \frac{\rho_i k_s A_w}{\mu_i n_0 d}\frac{\partial\left(\phi_p\right)_i}{\partial t} + g\frac{\partial\left(\phi_p\right)_i}{\partial z} = 0 \qquad (18)$$

而每一區之交界面必須滿足 $\left(i = 1,2, x = \pm\frac{d}{2}\right)$

$$-\frac{\rho_i k_s A_w}{\mu_i n_0 d}\frac{\partial\left(\phi_p\right)_i}{\partial x} = \frac{\partial\left(\phi_I\right)_i}{\partial x} \qquad (19)$$

$$-\frac{\rho_i k_s A_w}{\mu_i n_0 d}\frac{\partial\left(\phi_p\right)_i}{\partial x} = \frac{\partial\left(\phi_T\right)_i}{\partial x} \qquad (20)$$

$$\left(\phi_p\right)_i + \frac{\rho_i k_s A_w}{\mu_i n_0 d}\frac{\partial\left(\phi_p\right)_i}{\partial t} = -\frac{1}{g}\frac{\partial\left(\phi_I\right)_i}{\partial t} \quad (21)$$

$$\left(\phi_p\right)_i + \frac{\rho_i k_s A_w}{\mu_i n_0 d}\frac{\partial\left(\phi_p\right)_i}{\partial t} = -\frac{1}{g}\frac{\partial\left(\phi_T\right)_i}{\partial t} \quad (22)$$

最後，Sommerfeld 輻射邊界條件可表示為

$$\left(\phi_I\right)_i\left(x \to -\infty\right) \Rightarrow outgoing \quad (23)$$

$$\left(\phi_T\right)_i\left(x \to \infty\right) \Rightarrow outgoing \qquad (24)$$

三、解析方式

首先定義透水雷諾數為 $R = \frac{\rho k k_s A_w}{\mu_i n_0 d}$。則

上述之控制方程式及邊界條件便成為
(I)區

$$\nabla^2(\phi_I)_1 = \nabla^2(\phi_I)_2 = 0 \qquad (25)$$

$$\frac{\partial(\phi_I)_1}{\partial z} = 0 \qquad (26)$$

$$\frac{\partial(\phi_I)_2}{\partial z} = 0 \qquad (27)$$

$$(\phi_I)_i - \frac{g}{w^2}\frac{\partial(\phi_p)_i}{\partial z} = 0 \qquad (28)$$

$$\frac{\partial(\phi_I)_i}{\partial x} = -n_0 R \frac{\partial(\phi_p)_i}{\partial x} \qquad (29)$$

$$(\phi_I)_i(x \to -\infty) \Rightarrow outgoing \qquad (30)$$

(T)區

$$\nabla^2(\phi_T)_1 = \nabla^2(\phi_T)_2 = 0 \qquad (31)$$

$$\frac{\partial(\phi_T)_1}{\partial z} = 0 \qquad (32)$$

$$\frac{\partial(\phi_T)_2}{\partial z} = 0 \qquad (33)$$

$$(\phi_T)_i - \frac{g}{w^2}\frac{\partial(\phi_T)_i}{\partial z} = 0 \qquad (34)$$

$$\frac{\partial(\phi_T)_i}{\partial x} = -n_0 R \frac{\partial(\phi_p)_i}{\partial x} \qquad (35)$$

$$(\phi_T)_i(x \to -\infty) \Rightarrow outgoing \qquad (36)$$

(P)區

$$\nabla^2(\phi_p)_1 = \nabla^2(\phi_p)_2 = 0 \qquad (37)$$

$$\frac{\partial(\phi_p)_1}{\partial z} = 0 \qquad (38)$$

$$\frac{\partial(\phi_p)_2}{\partial z} = 0 \qquad (39)$$

$$(\phi_p)_i - \frac{g}{w^2}\frac{\partial(\phi_p)_i}{\partial z} = 0 \qquad (40)$$

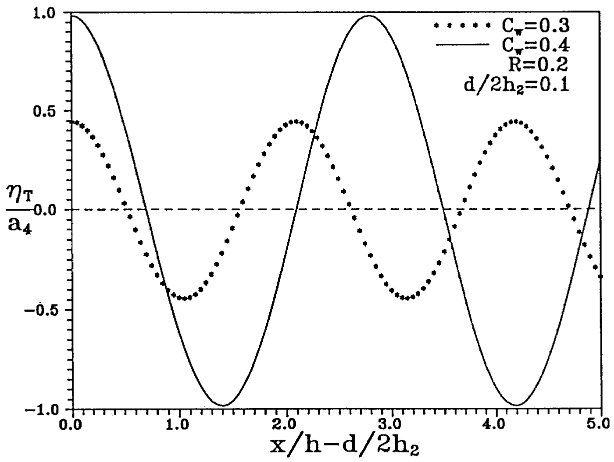
$$(\phi_p)_i + i\left[Rw(\phi_p)_i + \frac{w}{g}(\phi_I)_i\right] = 0 \qquad (41)$$

$$(\phi_p)_i + i\left[Rw(\phi_p)_i + \frac{w}{g}(\phi_T)_i\right] = 0 \qquad (42)$$

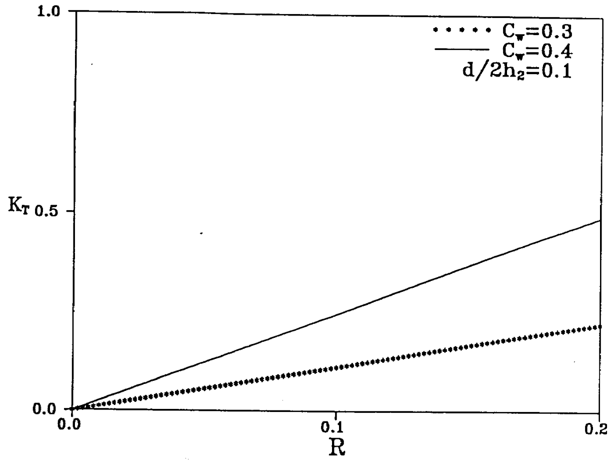
皆下來吾人便可以用微擾分析法來分析勢函數及波高的情況，其結果將在下一節中討論。

四、結果與討論

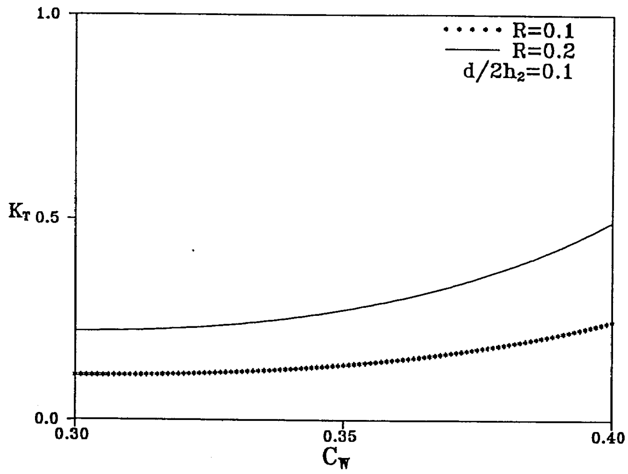
在冗長的分析之後，各項參數間的關係可由以下數個圖中得知



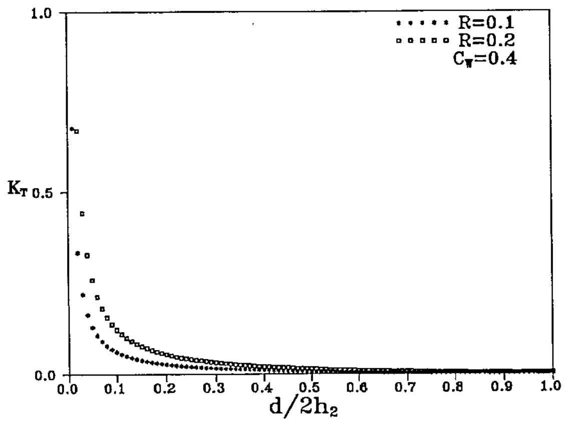
圖二 穿透波波高圖



圖三 K_T 與透水雷諾數 R 關係圖



圖四 K_T 與 C_w 關係圖



圖五　K_T 與 $d/2h_2$ 關係圖

現將以上各圖所得結論簡述如下

1. 內重力波流經多孔介質透水牆時，其入射波及透射波的振幅都受到雙層流密度及深度的影響，而透射波位移受多孔雷諾數、水波參數及透水牆厚度的影響。
2. 以多孔雷諾數 R 作為微擾逼近法的微小參數，由於 R 為慣性力與黏滯力的比值，在 R 很小的情況下，慣性項的作用就可被忽略，而僅存黏滯項的作用，於是多孔介質透水牆的滲透能力及厚度，嚴重的影響了滲透波及反射波的成因，在透水牆厚度較小時，滲透係數幾乎成正比關係，但是滲透係數卻與透水牆厚度成反比關係。
3. 由於多孔介質透水牆內的速度分佈及入射區、滲透區的速度勢和水波位移，可以發現在牆的左右兩側的流況有很大的不同，但是固液兩態的交互作用普遍存在於自然界，藉由本計畫模式化的探討，期望能對物理現象有另一方面的體會

五、參考文獻

1. Biot, M. A.(1962). “Mechanics of Deformation and Acoustic Propagationin Porous Media.”, *J. .of Applied Phys*., Vol. 33, No.4, pp.1482-1498.
2. C.S.Yih(1979) *Fluid Mechamics*, West River Press, Ann-Arbor Michigan.