Jpn. J. Appl. Phys. Vol. 38 (1999) pp. 1725–1729
Part 1, No. 3B, March 1999

Signal Processing Algorithms for Doppler Effect Based Nanometer Positioning Systems

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(Received October 14, 1998; accepted for publication December 1, 1998)

Interferometers based on laser Doppler effect are used as the vehicle to examine the cross relationship and performance impact among sub-systems such as optical layout, mechanical layout, signal processing algorithm, and signal processing circuitry. The classification of interferometers into the three main categories (*e.g.* homodyne, heterodyne and quadrature) and their influence on signal processing algorithms and circuitry which impact interferometer performance is examined. A quadrature configuration, found to have some unique advantages, is detailed. Also presented is an experimental laser encoder set-up equipped with a real-time signal processing circuitry based on a high performance field programmable gate array (FPGA) circuitry with a PC interface for quadrature signal decoding that was constructed to verify the quadrature configuration presented. The experimental data obtained that verifies how this new system architecture can fulfill the stringent demands of today's high performance, high accuracy metrology systems is also examined.

KEYWORDS: quadrature, homodyne, heterodyne, Doppler interferometers, phase decoding, frequency decoding, FPGA

1. Introduction

Laser doppler anemometers (LDA), Laser doppler velocimeters (LDV), Laser doppler vibrometers/interferometers, and diffractive optical laser encoders are all optical metrology systems that use the laser Doppler effect to encode information generated by the displacement and velocity of the measured surface. Using a photo detector to retrieve the intensity of the encoded returning light beam yields the desired encoded or modulated signal. All of these metrology systems are capable of generating nanometer or better positioning signals and thus are extremely valuable to be used as position sensors for the mastering of advanced optical storage systems.

As shown in Fig. 1, one laser beam is incident along the k_1 direction, and the reflection or scattering beam returns along the k_2 direction. When the measured surface moves with velocity V due to the Doppler effect, the returning beam will have a Doppler frequency shift ω_d. We can obtain a relationship between the Doppler frequency f_d and the velocity of the measured surface along the z-direction V_z such that

$$f_\mathrm{d} = \omega_\mathrm{d}/2\pi = [(2/\lambda)\sin(\theta/2)] \cdot V_\mathrm{z}. \tag{1}$$

Integrating the above equation, we can get a relationship between the phase angle and the displacement of the measured surface where

$$\phi_\mathrm{d} = \int f_\mathrm{d} dt = [(2/\lambda)\sin(\theta/2)] \cdot d_\mathrm{z}. \tag{2}$$

Considering a special case when $\theta = 180°$, where both the incident beam and the returning beam are normal to the measured surface, the two above equations can be further simplified to

$$f_\mathrm{d} = [2/\lambda] \cdot V_\mathrm{z}, \text{ and } \phi_\mathrm{d} = [2/\lambda] \cdot d_\mathrm{z}. \tag{3}$$

From the above equations, velocity was found to encode (modulate) to a frequency change signal and displacement was encoded (modulated) to a phase change. With some signal processing techniques, the signal can be decoded (demodulated) to yield the velocity and displacement information needed.

Applications of differential laser Doppler interferometers range from advanced optical storage systems, aircraft vibration testing to acoustic speaker mode shape measurements. Its characteristics such as non-contact, high sensitivity, and high adaptability to different measurement requirements make it one of the most important metrology equipment which exists today.

2. Interferometer Classification

Typical interferometers, according to their optical layout and signal processing algorithms, can be classified into three main categories: homodyne, heterodyne and quadrature.1) Classical interferometry layouts such as Michaelson interferometers and Mach-Zehnder inteferometers2) are examples of typical homodyne interferometers. Homodyne interferometers measure velocity of the vibrating surface by detecting the Doppler frequency change. However, both positive and negative velocities (different directional motion) cause the same Doppler frequency change since there is no negative frequency in the physical world. The homodyne interferometers can thus only measure the absolute value of velocity and cannot identify the direction of motion.

As was mentioned above, since a homodyne interferometer measures velocity of a vibrating surface without indicating the motion direction, its application is somewhat limited. On the other hand, both heterodyne and quadrature inteferometers can measure velocity and motion direction. One beam in the heterodyne interferometer is typically modulated to a different frequency by using an AOM, EOM, or rotating-grating before the interference. This frequency modulation causes the origin shift of the final interference in a frequency domain as shown in Fig. 2. It is with this mechanism the negative velocity is mapped onto the positive frequency range so as to discern the motion direction.

Figure 3 shows a typical heterodyne interferometer setup as was done at IBM.3) The third type of interferometer, the quadrature interferometer, uses an optical or electronic method to generate the final signal in an I/Q (in-phase/quadrature) form that is characterized by two sinusoidal signals with a 90 degrees phase difference. The two signals then form a circular trace on a Lissajous plane as is shown in Fig. 4. Direction of the circular trace corresponds to

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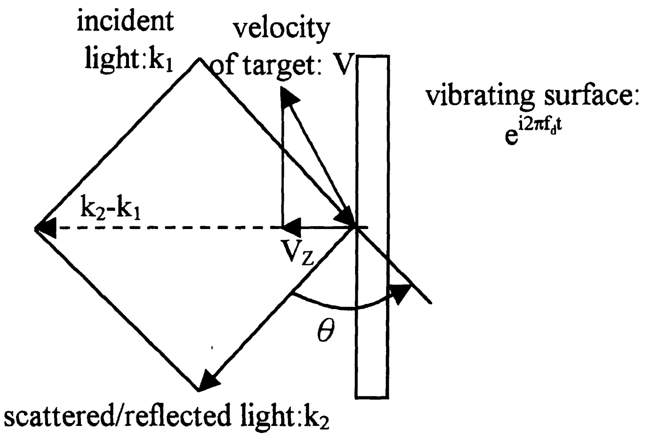

Fig. 1. Laser Doppler effect.

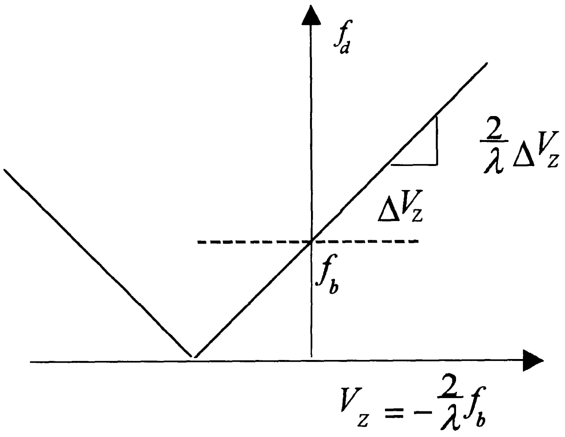

Fig. 2. Heterodyne interferometer mapping negative velocity into positive Doppler frequency.

either a positive or a negative frequency respectively, which maps to either a positive or a negative velocity motion. Figure 5 shows the optical design layout of the quadrature interferometer[4] called *AVID*. Two returning linearly polarized laser beams that are orthogonal to each other in terms of the polarization state are sent through a quarter wave attached next to the NPBS (non-polarized beamsplitter) to become two opposite circularly polarized laser beams. It can be shown that these two circularly polarized light beams can be viewed as a linearly polarized light beam whose azimuth angle corresponds to the relative movement of the two specimens. Using two polarizers attached to two sides of the NBPS, 45 degrees apart in the polarized space, it was possible to convert the above mentioned azimuth angle into a set of intensity signals. Photo detectors were then used to convert this set of intensity signals into electric signals. Since photo-detectors detect the intensity of a laser beam and are thus square law devices, the 45 degrees difference in the polarized space causes the signals to have a 90 degrees difference. The 90 degrees difference between the two electric signal forms the desired quadrature signal.

It is clear that heterodyne and quadrature interferometers can both identify the value and direction of the object motion and thus are the typical choices of modern interferometry systems. Instead of using a laser wavelength as the underlying metrology reference, diffractive optical laser encoders use grating pitch as a reference. The major advantage of a laser encoder versus that of a traditional laser interferometer is that laser encoders can have a far less stringent coherence length requirement. This observation indicates that the measurement range of diffractive optical laser encoders hinges on the length of the grating. As laser encoders can also be configured into a rotary form to measure angular displacement with nanometer resolution, mechanical systems such as high performance spindles and motors that are typically found in today's advanced optical storage systems can thus be characterized in detail.[5–7] In regards to the configuration, the optical layout of laser encoders is similar to quadrature interferometers except that the two returning beams are ± 1 order diffractive beams

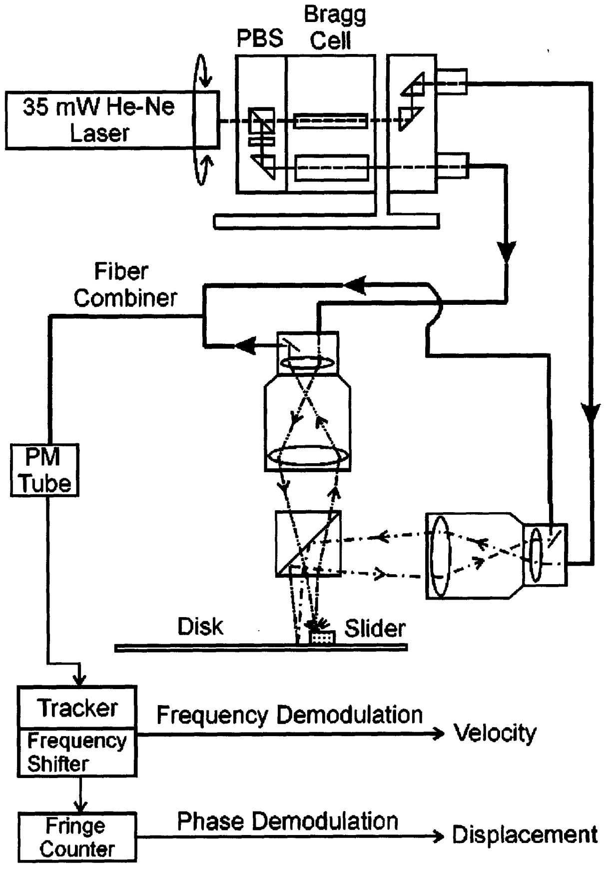

Fig. 3. Typical heterodyne interferometer optical layout.[3]

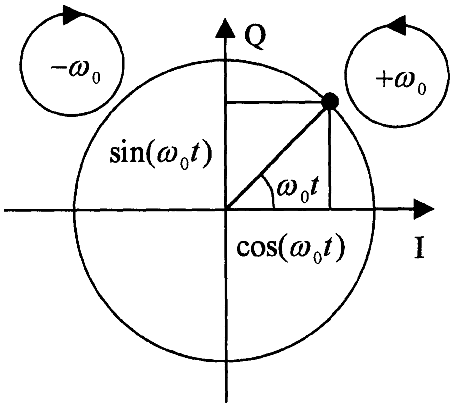

Fig. 4. Quadrature signal Lissajous plane circular trace.

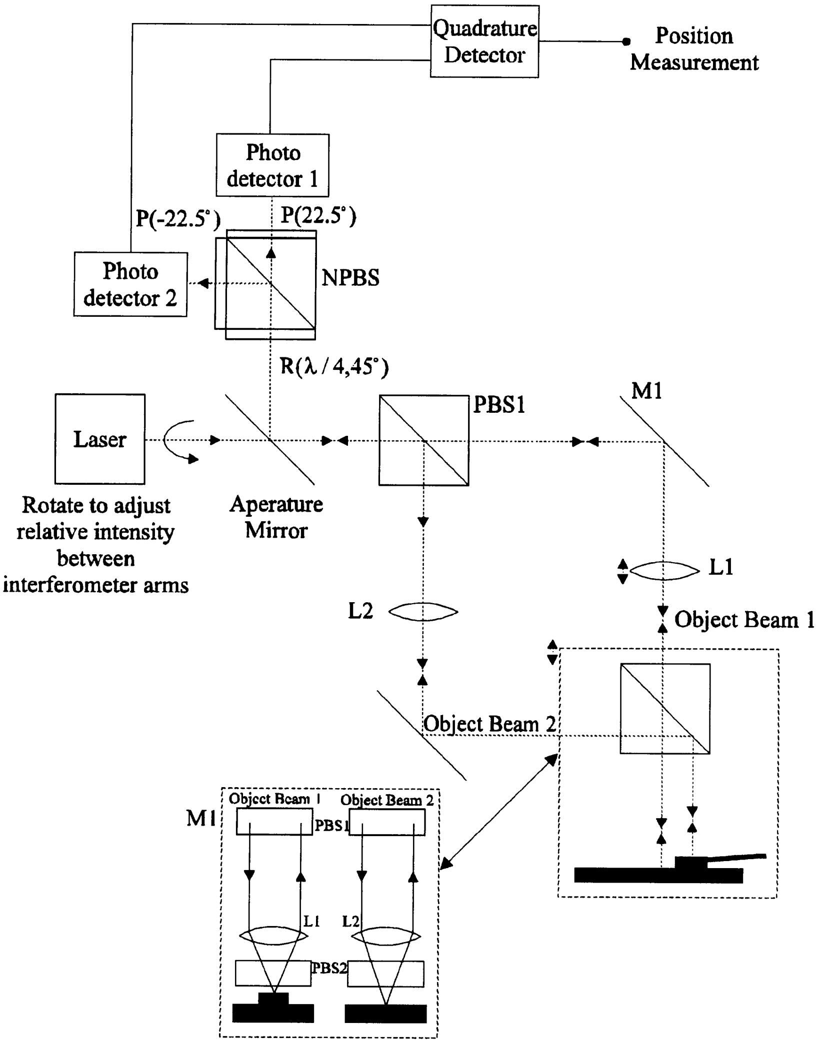

Fig. 5. AVID[4] quadrature interferometer optical layout.

from the grating, although the signals are also in the form of a quadrature signal. As such, the SNR (signal to noise ratio) of the signal from a typical interferometer is highly dependent on the roughness of the measured surface. On the other hand, the SNR of the signal from the laser encoder will always be excellent since the returning beam of the laser encoder always diffract from the grating.

3. Signal Processing Algorithm

Depending on the optical layout, different signal processing algorithms should be used to decode the signal. Differences in optical layouts, signal processing algorithms and circuitry are what determine the performance of the various interferometers. Since signals from heterodyne interferometers are self-modulated away from the base band, the usual decoding scheme adopted in heterodyne interferometers decodes velocity information from a frequency change. Out of the many frequency change detection methods, the simplest approach is to adopt a FVC (frequency voltage converter) chip that converts the frequency change to a voltage signal.[8] A more precise frequency detection method, called a TIA (time-interval analysis), uses a very high and stable base frequency as the reference to count the time-interval between two zero crossings of the signal. The time-interval measured can then be converted to a frequency. Typically, TIA instruments can have up to a pico-second time resolution and can thus detect phase and frequency change very precisely. The dynamic range and resolution of these measurements all depend on the time resolution of the TIA circuit. The major drawback of the TIA method is that the data updating rate depends on the frequency of the measured signal and will change as the target signal frequency changes.

There are commonly many signal-processing methods that are used in quadrature type interferometers.[9] A quadrature signal can also be further synthesized into the form of a heterodyne interferometer signal. This synthesis method involves synthesizing another higher frequency quadrature signal, mixing these synthesized signals with respect to the IQ parts generated by the measurement signal, and then adding the resultant two signals to the final form (Fig. 6). More specifically, the above mentioned synthesis can be mathematically shown to be

$$
\begin{aligned}
&A\sin(\omega t+\phi)\cos(\omega_0 t)+A\cos(\omega t+\phi)\sin(\omega_0 t)\\
&=\frac{A}{2}\{\sin[(\omega+\omega_0)t+\phi]+\sin[(\omega-\omega_0)t+\phi]\\
&\quad+\sin[(\omega+\omega_0)t+\phi]-\sin[(\omega-\omega_0)t+\phi]\}
\end{aligned}
$$

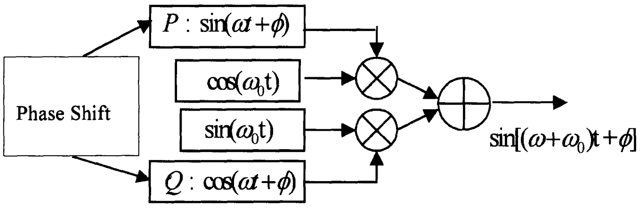

Fig. 6. Quadrature signal synthesis flow.

$$=A\sin[(\omega+\omega_0)t+\phi], \tag{4}$$

where the quadrature signal with frequency ω_0 is the synthesized high frequency signal and ω is the frequency of interest.

If the amplitude of the quadrature signal is stable, an analog level comparator circuitry can be used to detect the phase change of the signal directly. A comparator output result can then be further processed by using some very simple logic to convert the result into a standard A/B quad digital encoder signal form.(10) The major drawback of this method is that the signal form constrains the maximum displacement change during the sampling interval such that only a minimum resolution can be achieved. In comparison, an accumulator can have large changes during a sampling interval.(10)

Another quadrature decoding algorithm is to directly calculate the phase angle on the Lissajous plane using an arctangent function. This method involves using an ADC (analog-digital converter) to sample the signal and then uses the arctangent function to obtain the phase change. To achieve a high throughput, a DSP (digital signal processor) is typically used to speed up the computation. However, the slow DSP processing speed is usually the major constraint on the performance of this method. With the vast improvements to DSP technology, real-time up to hundred mega-Hertz can now be realized. Another advantage lies on the fact that an arctangent operation can decode the phase correctly even if the amplitude changes. With some improvement, quadrature signals from interferometers can be decoded correctly even if the signal SNR is bad. For example, no matter whether the signal is less than ideal due to it is not having equal amplitude, nor not exactly being 90 degrees difference in phase and having some DC offset, the quadrature signal processing approach can all be adopted to achieve accurate results. This modified arctangent method uses a least square model and a linear transformation mathematical technique to fit the imperfect trace and then uses the fitting parameter in real-time to correct the errors. Fundamentally, we can say that this method improves measurement accuracy by using much computational power.(13) As a result of this high computational power requirement, the processing speed becomes somewhat slow.

Table I shows the differences between heterodyne and quadrature signal processing techniques. It is clear from this table that signal processing algorithms and the circuitry adopted are important for system performance improvement. We can see clearly from this table that quadrature signal-based systems have many advantages in terms of implementing high performance metrology systems based on real-time DSP techniques.

4. Experimental Set-up and Results

Figure 7 shows the layout of an experimental linear laser encoder. A 1.6 μm grating was mounted on a micro-stepping (50,000 steps/rev) linear stepper stage that was moving relative to the stationary optics setup. The light source was a He-Ne laser with a 632.8 nm wavelength. The incident laser beam was normal to the grating surface. Both the +1 and −1 order diffractive beams were diffracted and returned by the two sets of symmetric optical arms. The two returning beams, orthogonally polarized, were combined by the last polarizing beamsplitter to complete the circular polarization interferometer configuration. The two beams, split by the NPBS, were linearly polarized and possessed a 45 degrees phase difference in the polarization direction. When the photo-detector measured the intensity, the signal obtained became 90 degrees apart, which is the quadrature signal needed. More specifically, the signals generated from these two photo-detectors are two sinusoidal signals with a 90 degrees phase difference. Typically, this quadrature signal is called a P/Q signal. Displacement can be obtained by decoding the phase while the velocity can be obtained by decoding the frequency signal. In addition the direction of motion can be obtained by exam-

Table I. Comparison of heterodyne and quadrature signal processing technique.

Optical layout & algorithm	Circuitry	Dynamic range		Resolution		Data updating rate
		d	*V*	*D*	*V*	
heterodyne frequency mixing, TIA	more analog less digital	digital accumulator	$2f_c\Delta t_{\min}$	$f_c\Delta t_{\min}$	$\Delta t_{\min}$	$f_c \pm f_d^*$
quadrature P/Q frequency mixing, TIA	more analog less digital	digital accumulator	$2f_c\Delta t_{\min}$	$f_c\Delta t_{\min}$	$\Delta t_{\min}$	$f_c \pm f_d^*$
quadrature P/Q arctangent	less analog more digital complex logic	digital accumulator	$f_s > f_d/2$	$1/2^{\mathrm{EB+2}}$	1/26EB-2	f_s of ADC**
quadrature P/Q level comparator	less analog more digital simple logic	digital counter	$f_s > f_d\mathrm{N}/2$	$1/\mathrm{N}2^{\mathrm{EB}}$	$1/\mathrm{N}2^{\mathrm{EB}}$	f_s 0f ADC**

*f_c denotes carrier frequency, $\Delta t_{\min}$ denotes minimum time resolution of TIA, and f_d denotes Doppler frequency shift.
**N denotes number of interpolation, EB denotes effective number of bits of ADC, and f_s denotes sampling rate of ADC.

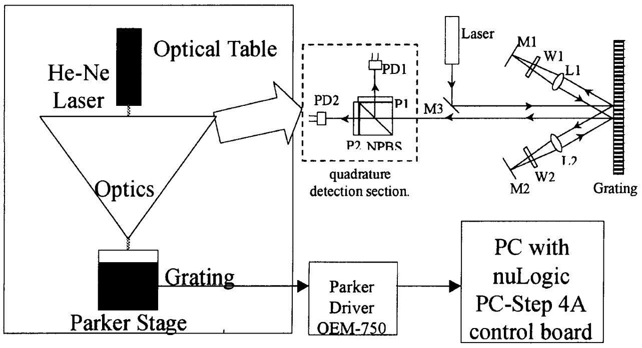

Fig. 7. Linear encoder experimental layout.

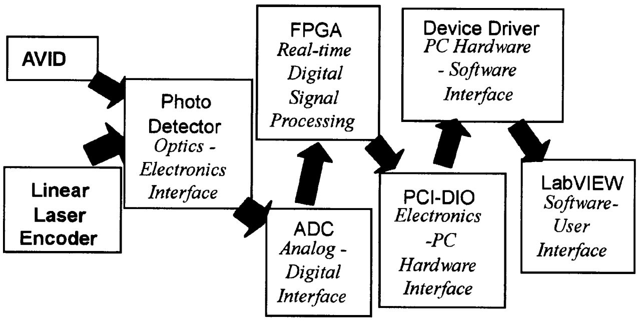

Fig. 8. Linear encoder experimental signal processing structure.

ining the relative lead/lag between the P/Q signals. Due to its high resolution, which is in the nanometer range or better, the output signal frequency from laser encoders can become extremely high when the testing specimen is running at high speed. Decoding this kind of extremely high frequency quadrature signal can be a challenge to signal processing techniques. Figure 8 shows the signal-processing platform adopted in this paper that was designed based on the comparison presented in Table I. A high speed ADC (20 MHz, Burr-Brown ADS801) was used to digitize the signal obtained from the photo-detectors. To decode this signal, a high performance FPGA[11,12] (field programmable gate array) technology was also developed to real-time implement the arctangent phase decoding method to 20 MHz. The real-time decoded displacement information was sent back using a high speed PCI-DIO card installed on a PC computer. A LabVIEW-based program running under a standard PC-Windows platform was then used to further process the information needed for integration in other systems.[13]

Figure 9 shows the experimental results of a 25 μm step response for a micro-stepping stepper stage that was measured from the above mentioned linear laser encoder system. The position versus time plot shows the 25 μm displacement was precisely measured. Both the overshooting and the final settling on arrival were also measured. The experimental data attained verifies that the new system architecture may fulfill the stringent demands of today's high performance, high accuracy metrology systems.

5. Conclusions

The theoretical analysis of the three main categories of interferometric systems (*e.g.* homodyne, heterodyne and quadrature) were classified according to their optical layout and signal processing algorithms. The influence of signal processing algorithms and its implemented circuitry are also examined with an intention to improve system performance. It was found that a quadrature signal-based interferometer is particularly suitable for real-time high frequency metrology system applications. It is also with this understanding that an experimental laser encoder was set up to perform real-time signal processing by using a high performance FPGA circuitry. The real-time quadrature signal decoding was also equipped with a PC interface. Results obtained verify that the newly developed system architecture can fulfill the stringent demands of today's high performance, high accuracy metrology systems.

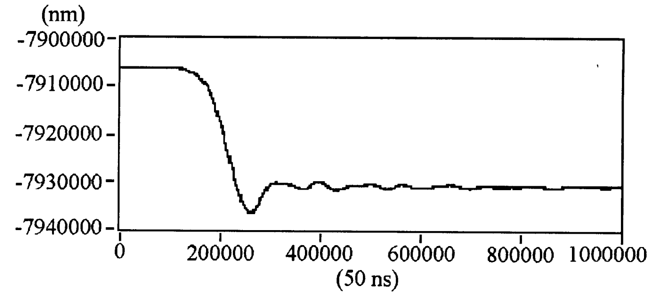

Fig. 9. Step response of a micro-stepping stepper stage as measured from a laser encoder.

Acknowledgement

This research was partially supported by National Science Council, Taiwan, R.O.C. under the contract number NSC84-2622-E-002-007, NSC85-2622-E-002-017R, NSC86-2622-E-002-023R, and NSC88-2622-E-002-001.

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