Multi-Stage MMSE Weighted Decision Feedback Multi-User Detector for DS-CDMA and MC-CDMA

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Abstract – **In this paper, we propose the MMSE weighted decision feedback (WDF) multi-user detector (MUD), which is an enhancement of the MMSE decision-feedback MUD. In contrast to MMSE-DF MUD, in MMSE-WDF past decisions are scaled by weighting factors that reflect their reliability. Furthermore, a multi-stage structure is utilized to improve the decision quality. An algorithm is also presented for jointly optimizing the reliability weighting factors and the coefficients of feedforward and feedback filters for every stage. Simulation results show that some variations of MMSE-WDF perform very closely to the single-user bound for E_b/N_0 lower than 25 dB and the number of active users less than 30.**

I. Introduction

Spread spectrum multi-user communication systems suffer from multiple access interference (MAI). Multi-user detection (MUD) [1,2] is a special receiver signal processing technique that mitigates MAI by treating them as signals and jointly detecting all users' information symbols. MUD achieves performance gain over conventional (single-user) receivers because by treating MAI as signals rather than thermal noise the structure of MAI is exploited, thus realizing the full potentials of spread spectrum communications.

Direct-sequence code division multiple access (DS-CDMA) and multi-carrier CDMA (MC-CDMA) are widely discussed spread spectrum communication techniques for wireless communications. Unified signal models have been established for DS-CDMA and MC-CDMA [1-4]. MUD algorithms based on this unified signal model are in general applicable to both DS-CDMA and MC-CDMA. Some of these MUD algorithms include parallel and serial interference cancellation, and minimum mean square error decision feedback (MMSE-DF) MUD [1,2,5]. In parallel interference cancellation (PIC), MAI is estimated and subtracted from the received signal in parallel, while in successive interference cancellation (SIC) interference cancellation is performed successively. On the other hand, in MMSE-DF, the outputs of matched filter banks are linearly combined with past decisions to suppress MAI, where the combination coefficients are optimized using the MMSE criterion. A common feature of these MUD algorithms is that perfect past decisions are assumed. Unfortunately, the MAI estimate is inevitably noisy, and when the estimate is in error, MAI can potentially be strengthened, instead of cancelled, by the action of PIC, SIC, or MMSE-DF MUD. A significant improvement to the PIC was suggested in [3], where a partial cancellation scheme was proposed. The nonlinear partial PIC (PPIC) appropriately scales the MAI estimates before being subtracted from the received signal, thus reducing the sensitivity to errors in the estimated MAI. In [3], these scaling factors are determined by trial-and-error. Further improvements to PIC and SIC, known as weighted interference cancellation (WIC), are proposed in [5]. In WIC, past decisions are scaled by weighting factors that reflect their reliability and can be computed using a simple algorithm.

In this paper, we extend the ideas presented in [5] to the MMSE-DF MUD, and propose a new multi-stage MUD referred to as the MMSE weighted decision feedback (MMSE-WDF) MUD. MMSE-WDF is less susceptible to errors in past decisions because in MMSE-WDF, past decisions are scaled by reliability weighting factors. Furthermore, a multi-stage structure is utilized to improve the decision quality. An algorithm is also derived for jointly optimizing the reliability weighting factors and the coefficients of feedforward and feedback filters on a stage-by-stage basis using the MMSE criterion.

II. System Model

Consider the (one-shot) discrete-time baseband-equivalent signal model of synchronous DS-CDMA over flat fading channels or quasi-synchronous MC-CDMA over frequency-selective fading channels with $K \geq 1$ active users. The received signal can be modeled as an $L \times 1$ vector $\mathbf{r}$ given by

$$\mathbf{r} = \begin{bmatrix} \mathbf{s}_1 & \mathbf{s}_2 & \cdots & \mathbf{s}_K \end{bmatrix} \begin{bmatrix} A_1 & 0 & \cdots & 0 \\ 0 & A_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & A_K \end{bmatrix} \begin{bmatrix} d_1 \\ d_2 \\ \vdots \\ d_K \end{bmatrix} + \mathbf{z}, \quad (1)$$
$$= \mathbf{SAd} + \mathbf{z} = \mathbf{Pd} + \mathbf{z}$$

where L is the spreading factor, $\mathbf{s}_k$, A_k and d_k are, respectively, the signature vector, channel gain, and information symbol of the k-th user, and $\mathbf{z}$ is a zero-mean, circularly symmetric white Gaussian noise vector with autocorrelation matrix $E[\mathbf{z}\mathbf{z}^H]=N_0\mathbf{I}_K$.

Equation (1) is a well-known baseband-equivalent model for synchronous DS-CDMA systems operating in a flat-fading environment [2]. It can be shown that (1) can also model down-link and quasi-synchronous uplink MC-CDMA signals [4], which combine frequency-division multiplexing (OFDM) and CDMA techniques to combat delay-spread multipath fading. Algorithms proposed in this paper are derived based on (1), and are therefore equally applicable to DS-CDMA and MC-CDMA.

III. Multistage MMSE Weighted Decesion Feedback Multi-User Detection

Fig. 1 shows the block diagram of the s-th stage of MMSE-WDF. The decision of the s-th decision stage (s=1,…,S, where S is the total number of stages) for user k is given by

$$\hat{d}_k^{(s)} = Dec\left(\tilde{d}_k^{(s)}\right), \tag{2}$$

where

$$\tilde{d}_k^{(s)} = \sum_{i=1}^{L} F_{i,k}^{(s)*} r_i + \sum_{i=1}^{k-1} B_{i,k}^{(s)*} \omega_i^{(s^*)} \hat{d}_i^{(s^*)} + \sum_{i=k+1}^{K} B_{i,k}^{(s)*} \omega_i^{(s-1)} \hat{d}_i^{(s-1)}, \tag{3}$$

in which $0 \le \omega_i^{(s)} \le 1$ is the reliability weighting factor for the decision $\hat{d}_i^{(s)}$, $F_{i,k}^{(s)}$ and $B_{i,k}^{(s)}$ are respectively the coefficients of the feedforward and feedback filters of the s-th stage, and Dec(•) is the decision function. The factor $\omega_i^{(s)}$ is derived so that it increases monotonically with the probability of $\hat{d}_i^{(s)} = d_i$. The parameter s^* in (3) can be equal to s-1 or s. When $s^* = s$-1, the feedback filter only uses the decisions from previous stage. This case is referred to as the "parallelizable MMSE-WDF" (P-MMSE-WDF) because the detection of all users in each stage is fully parallelizable. On the other hand, when $s^* = s$, decisions in current stage are used in feedback filter whenever possible. This is referred to as the serial or "non-parallelizable" MMSE-WDF (NP-MMSE-WDF). It can be seen from (3) that the fed-

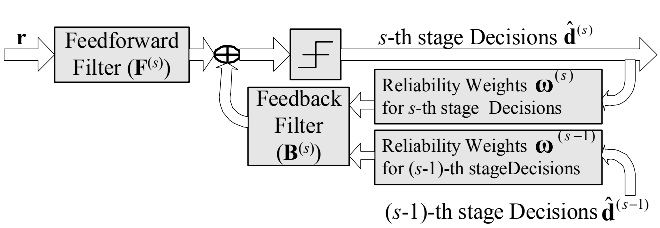

Fig. 1 Block diagram of MMSE-WDF for s-th stage

back decisions in MMSE-WDF are scaled by $\omega_i^{(s)}$ in order to reduce the effect of past decision errors. A special variation of NP-MMSE-WDF, referred to partial decision feedback (PDF), is obtained if only the decisions in the current stage are fed into the feedback filter, i.e., $\omega_i^{(s-1)} = 0$ for $i = k+1,…,K$ in (3). An algorithm has also been derived to jointly optimize the filter coefficients $F_{i,k}^{(s)}$ and $B_{i,k}^{(s)}$ and reliability weighting factors using the MMSE criterion. Details of the algorithm are shown in the appendix.

It should be noted that (2) and (3) include several previously proposed detectors as special cases. For example, setting all reliability weighting factors to 0 degenerates (2) and (3) into the conventional linear MMSE receiver. On the other hand, the conventional MMSE-DF corresponds to the single-stage NP-MMSE-WDF with PDF and unity reliability weighting factors.

IV. Reliability Weighting Factors And Filter Coefficients

The reliability weighting factors for the MMSE-WDF are derived by first recognizing that the detected symbol of the k-th user at the s-th stage can be expressed as

$$\hat{d}_k^{(s)} = m_k^{(s)} d_k, \tag{4}$$

where $m_k^{(s)}$ is an equivalent multiplicative noise introduced to model decision errors. For MMSE-WDF, the reliability weighting factors and filter coefficients are then derived by minimizing the estimated mean square error (MSE) of each user at every stage. Detailed derivations are given in the Appendix. It can be shown that the reliability weights and filter coefficients are functions of **P**, and can be computed as follows. First, we set $\omega_k^{(0)} = 0$ for k = 1, 2, …, K. The reliability weighting factors and filter coefficients for MMSE-WDF are then computed from s = 1 to s = S using the following recursions:

For k = 1 … K:

1. Determine the coefficients of feedforward and feedback filters according to (A.19).
2. Compute $SIR_{k,\min}^{(s)}$ using (A.21), and set the reliability weighting factor to

$$\omega_{o(k,s)}^{(s)} = 1 - 2Q\left(\sqrt{SIR_{k,\min}^{(s)}}\right). \tag{5}$$

In (5), $Q(\bullet)$ is the Gaussian tail function defined as

$$Q(x) = \int_x^{\infty} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}t^2\right) dt. \tag{6}$$

for $x > 0$.

V. Simulation Results

The performance of the proposed schemes is evaluated by computer simulation for synchronous uplink DS-CDMA over wireless communication channels. Information bits from each user are modulated using BPSK and spread using short scrambling sequences with spreading gain 32 as defined in the wideband CDMA specifications [6]. The wireless channels between the users and receivers are modeled as uncorrelated flat Rayleigh fading channels corrupted by AWGN, i.e., the channel gains A_k, $k = 1 \ldots K$, are independent zero-mean circularly symmetric complex Gaussian random variables with unity variance, and are uncorrelated from symbol to symbol. Perfect channel estimation is assumed in these simulations. Furthermore, performance of conventional receivers including the linear MMSE MUD, MMSE-DF, and the single user bound (SUB) are also simulated as baselines for comparison.

Fig. 2 shows the average BER as functions of K at 20dB E_b/N_0, while Fig. 3 shows the average BER as functions of E_b/N_0 when K=30, where E_b is the transmitted energy per user bit. In the legend of these figures, "P" and "NP" respectively refer to P-MMSE-WDF and NP-MMSE-WDF. It can be seen from Fig. 2 that the single-stage NP-MMSE-WDF with PDF outperforms MMSE-DF as the number of active user increases. This is because MAI is more severe when K is large, and the reduction of error propagation due to reliability weighting in NP-MMSE-WDF is more obvious. Furthermore, Figs. 2 and 3 indicate that NP-MMSE-WDF significantly outperforms P-MMSE-WDF. This is because NP-MMSE-WDF always uses the newest decisions at the cost of longer decision delays. Finally, for $K \leq 30$, with just 3 stages NP-MMSE-WDF performs very closely to SUB when $E_b/N_0 < 25$ dB as shown in Figs. 2 and 3.

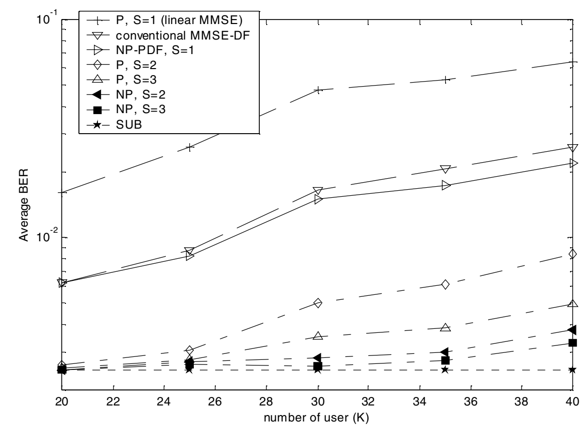

Fig. 2 Capacity figure of MMSE-WDF, linear MMSE, and MMSE-DF under the condition of 20 dB E_b/N_0.

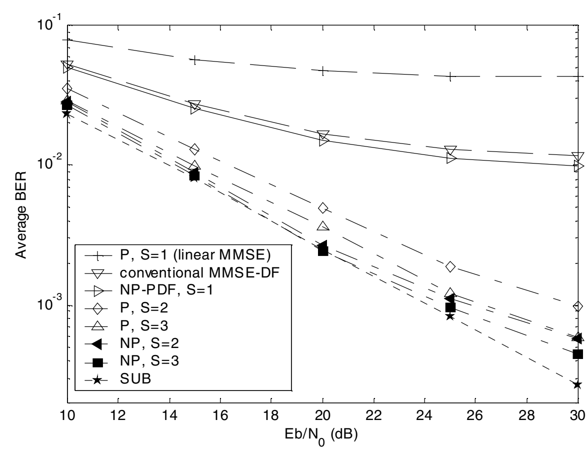

Fig. 3 Comparisons of MMSE-WDF, linear MMSE, and MMSE-DF with K=30.

VI. Conclusion

A generalized multi-stage multi-user detector referred to as MMSE weighted decision feedback (MMSE-WDF) is proposed in this paper. In contrast to conventional MMSE decision-feedback MUD, in MMSE-WDF decisions are weighted by reliability weighting factors before being feedback. Two variations of MMSE-WDF are discussed. The parallelizable MMSE-WDF (P-MMSE-WDF) uses only decisions from the previous stage for MAI cancellation, while the nonparallelizable MMSE-WDF (NP-MMSE-WDF) also uses decisions from the current stage. An algorithm is also proposed to systematically and jointly optimize reliability weighting factors and coefficients of feedforward and feedback filters on a stage by stage basis using MMSE criterion. Simulation results show that 1) with the same number of stages, NP-MMSE-WDF significantly outperforms P-MMSE-WDF, 2) when optimal reliability weighting factors and filter coefficients are used, the NP-MMSE-WDF performs very closely to the single-user bound for E_b/N_0 lower than 25 dB and the number of active users less than 30.

Rerferences

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Appendix

In this Appendix we derive the reliability weighting factors and coefficients of feedforward and feedback filters for MMSE-WDF. As mentioned earlier, the decision of the k-th user at the s-th stage can be expressed as

$$\hat{d}_k^{(s)} = m_k^{(s)} d_k \,, \tag{A.1}$$

where $m_k^{(s)}$ is a discrete random variable with at most M^2 possible values, where M is the cardinality of the signal constellation. Although the derivations to be presented can be extended to the general case, in the following we assume, for simplicity, that $d_k = 1$ or -1 with equal likelihood for all k. In this case, we have

$$m_k^{(s)} = \begin{cases} -1 & \text{with probability } p_k^{(s)} \\ 1 & \text{with probability } 1\text{-}p_k^{(s)} \end{cases}, \tag{A.2}$$

where

$$p_k^{(s)} = \mathrm{Prob}\left(\hat{d}_k^{(s)} \neq d_k\right). \tag{A.3}$$

From (3), we have

$$\tilde{d}_k^{(s)} = \begin{bmatrix} \mathbf{F}_k^{(s)H} & \mathbf{B}_k^{(s)H} \end{bmatrix} \begin{bmatrix} \mathbf{r} \\ \mathbf{\Omega}_k^{(s)} \mathbf{M}_k^{(s)} \mathbf{d}_k^{(s)} \end{bmatrix}, \tag{A.4}$$

where $\mathbf{F}_k^{(s)}$ is an $L\times1$ vector, $\mathbf{B}_k^{(s)}$, and $\mathbf{d}_k^{(s)}$ are $K\times1$ vectors respectively denoted by

$$\mathbf{F}_k^{(s)} \equiv \begin{bmatrix} F_{1,k}^{(s)} & F_{2,k}^{(s)} & \cdots & F_{L,k}^{(s)} \end{bmatrix}^T, \tag{A.5}$$

$$\mathbf{B}_k^{(s)} \equiv \left[B_{1,k}^{(s)}, \cdots, B_{k-1,k}^{(s)}, 0, B_{k+1,k}^{(s)}, \cdots, B_{K,k}^{(s)}\right]^T, \tag{A.6}$$

and

$$\mathbf{d}_k^{(s)} \equiv \left[d_1, \cdots, d_{k-1}, 0, d_{k+1}, \cdots, d_K\right]^T, \tag{A.7}$$

while $\mathbf{\Omega}_k^{(s)}$ and $\mathbf{M}_k^{(s)}$ are $K\times K$ diagonal matrices with j-th diagonal component respectively given by

$$\left(\mathbf{\Omega}_k^{(s)}\right)_{j,j} = \begin{cases} \omega_j^{(s*)} & \text{for } j < k \\ 0 & \text{for } j = k \\ \omega_j^{(s-1)} & \text{for } j > k \end{cases} \tag{A.8}$$

and

$$\left(\mathbf{M}_k^{(s)}\right)_{j,j} = \begin{cases} m_j^{(s*)} & \text{for } j < k \\ 0 & \text{for } j = k \\ m_j^{(s-1)} & \text{for } j > k \end{cases}. \tag{A.9}$$

Assuming that

$$E\left[\left(\mathbf{\Omega}_k^{(s)} \mathbf{M}_k^{(s)} \mathbf{d}_k^{(s)}\right)\left(\mathbf{\Omega}_k^{(s)} \mathbf{M}_k^{(s)} \mathbf{d}_k^{(s)}\right)^H\right] = \left(\mathbf{\Omega}_k^{(s)}\right)^2 \tag{A.10}$$

and $m_k^{(s)}$ is independent of d_k, $p_k^{(s)}$ can be estimated by

$$p_k^{(s)} = Q\left(\sqrt{SIR_k^{(s)}}\right), \tag{A.11}$$

where $Q(\bullet)$ is the Gaussian tail function defined in (6) and $SIR_k^{(s)}$ is the s-th stage relevant signal to interference ratio encountered by k-th user, given by

$$SIR_k^{(s)} = \frac{E\left[\left(\mathrm{Re}\left[\mathbf{F}_k^{(s)H} \mathbf{P} \mathbf{e}_k^K\right]\right)^2\right]}{E\left[\left(\mathrm{Re}\left[\tilde{d}_k^{(s)} - \mathbf{F}_k^{(s)H} \mathbf{P} \mathbf{e}_k^K\right]\right)^2\right]}, \tag{A.12}$$

and $\mathbf{e}_k^K$ is k-th $K\times1$ standard vector. Hence, from (A.2) we have

$$E\left[m_k^{(s)}\right] = 1 - 2Q\left(\sqrt{SIR_k^{(s)}}\right). \tag{A.13}$$

It is desirable to minimize the average probability of error of the S-th stage given by

$$P_{av} \equiv \frac{1}{K}\sum_{k=1}^{K} prob\left(\hat{d}_k^{(S)} \neq d_k\right). \tag{A.14}$$

(A.4) to (A.13) can be used recursively to express P_{av} in terms of all weightings, and in principle all filter coefficients and reliability weighting factors that minimize P_{av} can be jointly found. Unfortunately, this task is mathematically intractable. A more practical approach is to optimize them on a stage-to-stage basis by minimizing the mean-square error (MSE). The MSE in the soft output of the s-th stage is defined as

$$\varepsilon_k^{(s)} \equiv E\left[\left\|\tilde{d}_k^{(s)} - d_k\right\|^2\right] = \begin{bmatrix}\mathbf{F}_k^{(s)H} & \mathbf{B}_k^{(s)H}\end{bmatrix}\Sigma_k^{(s)}\begin{bmatrix}\mathbf{F}_k^{(s)} \\ \mathbf{B}_k^{(s)}\end{bmatrix} - 2\,\mathrm{Re}\left[\mathbf{F}_k^{(s)H}\mathbf{P}\mathbf{e}_k^K\right] + 1, \quad (A.15)$$

where

$$\Sigma_k^{(s)} \equiv \begin{bmatrix}\mathbf{P}\mathbf{P}^H + N_0\mathbf{I}_K & \Theta_k^{(s)H} \\ \Theta_k^{(s)} & \mathbf{\Omega}_k^{(s)2}\end{bmatrix}, \quad (A.16)$$

and

$$\Theta_k^{(s)} \equiv \mathbf{\Omega}_k^{(s)} E\left[\mathbf{M}_k^{(s)}\right]\left(\mathbf{I}_K - \mathbf{e}_k^K\left(\mathbf{e}_k^K\right)^T\right)\mathbf{P}^H. \quad (A.17)$$

Furthermore, by the orthogonality principle the solutions satisfy

$$E\left[\begin{bmatrix}\mathbf{r} \\ \mathbf{\Omega}_k^{(s)}\mathbf{M}_k^{(s)}\mathbf{d}_k^{(s)}\end{bmatrix}\left(\tilde{d}_k^{(s)*} - d_k^*\right)\right] = \mathbf{0}, \quad (A.18)$$

and one of them is given by

$$\begin{cases}\mathbf{\Omega}_{k,opt}^{(s)} = E\left[\mathbf{M}_k^{(s)}\right] \\ \mathbf{F}_{k,opt}^{(s)} = \left(\mathbf{P}\mathbf{\Pi}_k^{(s)}\mathbf{P}^H + N_0\mathbf{I}_K\right)^{-1}\mathbf{P}\mathbf{e}_k^K \\ \mathbf{B}_{k,opt}^{(s)} = -\left(\mathbf{I}_K - \mathbf{e}_k^K\left(\mathbf{e}_k^K\right)^T\right)\mathbf{P}^H\mathbf{F}_{k,opt}^{(s)}\end{cases}, \quad (A.19)$$

where

$$\mathbf{\Pi}_k^{(s)} \equiv \mathbf{I}_K - \left(\mathbf{\Omega}_{k,opt}^{(s)}\right)^2\left(\mathbf{I}_K - \mathbf{e}_k^K\left(\mathbf{e}_k^K\right)^T\right). \quad (A.20)$$

The corresponding $SIR_{k,\min}{}^{(s)}$ in (A.12) is given by

$$SIR_{k,\min}^{(s)} = \frac{\left(\mathbf{\Gamma}_k^{(s)T}\mathbf{e}_k^K\right)^2}{\mathbf{\Gamma}_k^{(s)T}\left\{\mathbf{\Pi}_k^{(s)} - \mathbf{e}_k^K\left(\mathbf{e}_k^K\right)^T\right\}\mathbf{\Gamma}_k^{(s)} + \frac{N_0}{2}\left\|\mathbf{F}_{k,opt}^{(s)}\right\|^2}, \quad (A.21)$$

where

$$\mathbf{\Gamma}_k^{(s)} \equiv \mathrm{Re}\left[\mathbf{P}^H\mathbf{F}_{k,opt}^{(s)}\right]. \quad (A.22)$$

It should also be noted that (A.19) is derived based on the assumptions in (A.10), which do not hold in general. However, using assumptions in (A.10) avoids mathematical intractability and still achieves good performance as shown by computer simulations.