Staircase Approximation for Transients of Multisection Dispersive Transmission Lines with Nonlinear Loads

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Abstract

In this paper, we propose a staircase approximation to solve for the transients of dispersive transmission lines with nonlinear loads. Numerical examples, verified with the FDTD method and the traditional frequency-domain approach, are included. Such an approach is with easy formulation and straightforward numerical solutions, especially when dealing with multisection lines.

I Introduction

As the clock rate of digital circuits goes higher and higher, dispersion of transmission lines takes a more crucial role than before. With dispersion, the pulse shape is distorted, which leads to variations of duration, rise time and fall time. If the pulse shape is distorted, nearby signals are difficult to distinguish if care is not taken. In short, only when signals can be predicted and controlled beforehand do the digital circuits function correctly, especially when the dispersion effect is a major concern.

For microwave circuits, frequency response can be obtained either directly in frequency domain or indirectly in time domain via FFT. Although the harmonic balance method [1] is widely adopted in solving nonlinear loads in frequency domain, transmission lines in that method are usually dealt with in time domain, which inevitably faces the problem with dispersion and nonlinear loads if high frequency effects are taken into account.

To treat the transient response of dispersive transmission lines, the frequency-domain method is adopted traditionally. It suffers, however, not only the difficulty with the presence of nonlinear loads but also the slow numerical integration when taking the

0-7803-7070-8/01/$10.00 ©2001 IEEE

inverse Fourier transform. The FDTD, another candidate, is capable of settling nonlinear loads, but has difficulties when frequency-dependent factors must be considered. In addition, the FDTD becomes involved in dealing with multisection transmission lines, which further limits its applications.

The scale domain method [2] can solve for the transients of dispersive lines with nonlinear loads through a straightforward formulation. The next section will give a sketch of the staircase approximation, a simplification of the scale domain method. Section III provides some numerical examples, and the last section draws some conclusions.

II Formulation

The dispersive telegraphist's equations in time domain are

$$-\frac{\partial}{\partial x}v = Ri + \frac{\partial}{\partial t}(L * i) \qquad (1)$$

$$-\frac{\partial}{\partial x}i = Gv + \frac{\partial}{\partial t}(C * v) \qquad (2)$$

where the star "*" represents the convolution operation to account for dispersion. Approximate the signals by

$$v(x,t) \approx \sum_{j=1}^{n} v_j(x) h_j(t) \qquad (3)$$

$$i(x,t) \approx \sum_{j=1}^{n} i_j(x) h_j(t) \qquad (4)$$

where $h_j(t)$ is the unit rectangular pulse with duration Δt. This is called the staircase approximation since the voltage and current signals at a given x is now approximated by a staircase- like function. By inserting (3) and (4) into (1) and (2) and integrating with respect to t, the original system of continuous time-dependent equations transforms to a system of discrete time-independent matrix equations

$$-\frac{d}{dx}[v] = ([R] + \frac{1}{\Delta t}[L])[i] \qquad (5)$$

$$-\frac{d}{dx}[i] = ([G] + \frac{1}{\Delta t}[C])[v] \qquad (6)$$

where $[v]$ and $[i]$ are column matrices and $[R]$, $[L]$, $[G]$, $[C]$ are square matrices with

$$[R] = R[I] \qquad (7)$$

$$[G] = G[I] \qquad (8)$$

$$[L] = L[M]^{-1} \qquad (9)$$

$$[C] = C[M]^{-1}(\varepsilon_i[I] + \frac{1}{\Delta t}[Dis]) \qquad (10)$$

Here [I] stands for identity matrix and

$$[M] = \begin{bmatrix} 0.5 & 0 & \cdots & \cdots & 0 \\ 1 & 0.5 & \ddots & & \vdots \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ \vdots & & \ddots & 0.5 & 0 \\ 1 & \cdots & \cdots & 1 & 0.5 \end{bmatrix}$$

$$[Dis] = d_{ij} = \begin{cases} \frac{a}{b^2}[e^{b(i-j+1)\Delta t} - 2e^{b(i-j)\Delta t} + \\ \quad e^{b(i-i-1)\Delta t}] & , \quad i > j \\ \frac{a}{b^2}[e^{b(i-j+1)\Delta t} - e^{b)i-j)\Delta t} - \\ \quad b\Delta t] & , \quad i = j \\ 0 & , \quad i < j \end{cases}$$

Assume Debye dispersion [3]

$$C(\omega) = C\varepsilon_r(\omega)$$

$$\varepsilon_r(\omega) = \varepsilon_\infty + \chi(\omega)$$

$$\chi(\omega) = \frac{\varepsilon_s - \varepsilon_\infty}{1 + j\omega t_0}$$

So

$$\chi(t) = \frac{\varepsilon_s - \varepsilon_\infty}{t_0} e^{-t/t_0} U(t)$$

by inverse Fourier transform, and

$$a = \frac{\varepsilon_s - \varepsilon_\infty}{t_0}$$

$$b = -\frac{1}{t_0}$$

correspondingly. Here ε_s means the relative permittivity at dc, ε_∞ is the relative permittivity at $\omega = \infty$, and t_0 stands for the Debye relaxation constant.

Once these matrices are determined, equations can be manipulated in a way similar to those in the time-harmonic case. Problems with linear loads thus can be solved directly, and nonlinear loads can be dealt with by a common iterative scheme [2].

III Numerical Results

To prove the usefulness of the staircase approximation, consider a two-section lossless ($R = G = 0$) transmission lines shown in Fig. 1, and shunt with a capacitor and a nonlinear load at the interconnection and the right terminal. At the same time, generator with an internal resistor is placed at the left end.

As the first example, assume both sections of the transmission lines are with the same parameters, $\ell = 0.5(m)$, $L = 0.5(\mu H/m)$, and $C = 0.2(nF/m)$. In addition, a matched generator excites unit rectangular pulse with duration $w = 1(n\sec)$. The nonlinear loads are described by $i = 0.01 \times v^2$ for $v > 0$ and $i = 0$ for $v \le 0$, and shunt with capacitors $C_s = 50(pF)$. The resultant voltage signal at $x = 1.0(m)$ calculated by the staircase approximation (solid line) and the FDTD (dashed line) are shown in Fig. 2. Both match well. The zero voltage before $10(n\sec)$ is due to the delay of propagation. The rise and fall of the voltage signal between $10(n\sec)$ and $20(n\sec)$ are related to the charge and discharge of capacitors caused by the finite duration pulse. The rise after $20(n\sec)$ is excited by the reflection from the interconnection.

Next, let's remove the nonlinear loads and introduce the Debye dispersion to both transmission lines with parameters $\varepsilon_s = 9$, $\varepsilon_\infty = 4$ and $w_0 = 1/t_0 = 5\pi \times 10^8$. Apply the same pulse excitation and replace the internal resistor by $R_g = 150(\Omega)$. The voltage response calculated at $x = 0.5(m)$ and $x = 1.0(m)$ by the staircase approximation (solid line) and the frequency-domain transform method (dotted line) are illustrated in Fig. 3. The zero voltage are again due to the propagation delay. The smoother shapes and smaller magnitude, compared with the previous example, reveal the effect of dispersion. The agreement of both curves validates the capability of our method in dealing with dispersive transmission lines.

Last, but not least, assume the same parameters used in the second example and apply the nonlinear loads utilized in the first example. The results are exhibited in Fig. 4, in which the solid line is with the parameters $\Delta t = 0.1$ ($n$sec) and 512 bases while the dashed line adopts $\Delta t = 0.2$($n$sec) and 256 bases. The results show that the convergence of our method is pretty good.

IV Conclusion

We have proposed the staircase approximation and shown its usefulness in dealing with transients of multisection dispersive transmission lines with nonlinear loads. Numerical results verified with the FDTD and the conventional frequency-domain method have been exhibited. This method can be easily formulated and applied to problems with frequency-dependent loads, which is important for more realistic applications.

V Reference

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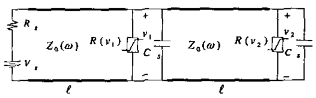

Figure 1

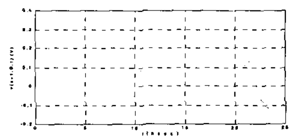

Figure 2

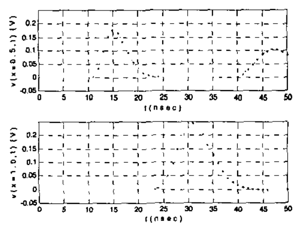

Figure 3

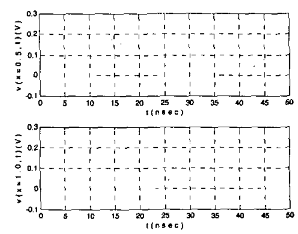

Figure 4