Stoch Environ Res Risk Assess (2006) 20: 291–295
DOI 10.1007/s00477-005-0028-z

ORIGINAL PAPER

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Uncertainty and importance assessment using differential analysis: an illustration of corrosion depth of spent nuclear fuel canister

Published online: 17 March 2006

Abstract Uncertainty of NRC proposed performance assessment for the corrosion depth of storage canister of nuclear spent fuel in time history is preformed using the differential analysis. The method is demonstrated to evaluate uncertainty propagation of a system with incomplete and insufficient data in the preliminary phase of assessment. The result shows that mean corrosion depth presents a non-linear regression while variance demonstrates a first-order linear increase through 1,000-year time history. By incorporating correlation into pitting factor, uniform corrosion factor, oxygen concentration and chlorine concentration, it is found that the covariance of pitting factor and chlorine concentration has the most significant contribution in the variance of total loss. Incorporating assumed covariance into the system, the analytical form of uncertainty and importance for the corrosion depth evaluation can be well demonstrated from the input parameters.

Keywords Uncertainty analysis · Importance assessment · Nuclear waste · Canister · Groundwater

1 Introduction

Preliminary study of problems of waste package degradation is initiated for nuclear waste final disposal in Taiwan. NRC's criteria (1983) for waste package lifetime of 300–1,000 and 1,000-year prewaste emplacement groundwater travel time are proposed at current status. Uncertainty analysis plays an important role in establishing reliability and reasonable assurance of waste package performance in the licensing process in the future. For the unexpected time scale of waste disposal, uncertainty evaluation must be conducted to measure performance and safety of waste package. Waste package performance has been studied by first-order reliability method (Sutcliffe 1984; Song and Lee 1989). A first-order second moment reliability method applied to the stochastic analysis of radioactive waste package performance is useful for cases where statistical information is incomplete and insufficient. The method can be used to solve subsurface environmental problems where only first- and second-order statistical moments and marginal probability distributions are available (Song and Lee 1989). A method of uncertainty analysis is illustrated by analyzing the corrosion of canisters in a nuclear waste package (Sutcliffe 1984). In his research, uncertainty is demonstrated by the probability distribution in the form of histogram. It results that the method is with less calculation than a Monte Carlo approach. For the preliminary phase of nuclear waste disposal, field data are incomplete and insufficient in many cases. Mean and variance of the field data are substantially well measured than a precise probability distribution. The mean value, or expected value, is a center value for a specified parameter. Variance is a measure of deviation from its mean value. Variance can be taken to identify uncertainty of models or parameters (Shih 2001). This paper promote the study of Shih et al. (2002) to analytically measure uncertainty and importance of parameters in calculation for the corrosion depth of nuclear spent fuel canister by using the differential analysis. Only mean value and variance of the system output are considered to evaluate parameter uncertainty and importance of the problem. Differential analysis has been also discussed and used in the other fields (NRC 1989; Shih 2001).

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2 Differential analysis

It assumes that the mean value and variance constitute the statistical model of the system. The mean, variance and covariance can be respectively expressed as

$$E(p_j) = m^{-1}\sum_{i=1}^{m} p_{ji} \tag{1}$$

$$\mathrm{Var}(p_j) = (m-1)^{-1}\sum_{i=1}^{m}\left(p_{ji} - E(p_j)\right)^2 \tag{2}$$

and

$$\mathrm{Cov}(p_j,p_k) = (m-1)^{-1}\sum_{i=1}^{m}\left(p_j - E(p_j)\right)\left(p_k - E(p_k)\right) \tag{3}$$

where $j \neq k$, m is the sample size.

The differential analysis uses the Taylor series to approximate the system model under consideration (NRC 1989). It assumes that the model under consideration can be represented by a function of the form

$$y = f(p_1, p_2, \ldots, p_n) = y(\mathbf{p}). \tag{4}$$

The base values and ranges are selected for the input variables p_j, $j=1, 2,\ldots, n$. The base values can be presented by the vector

$$\mathbf{p_0} = [p_{10}, p_{20}, \ldots, p_{n0}] \tag{5}$$

Using the multidimensional Taylor series expansion, the first-order approximation to y is

$$y(\mathbf{p}) \cong y(\mathbf{p_0}) + \sum_{j=1}^{n}\frac{\partial f(\mathbf{p_0})}{\partial p_j}(p_j - p_{j0}). \tag{6}$$

In essence, uncertainty analysis is to evaluate variance propagation of input parameters to system response. Respectively, mean and variance of the corrosion depth y can be estimated by

$$E(y) \cong y(\mathbf{p_0}) + \sum_{j=1}^{n}\frac{\partial f(\mathbf{p_0})}{\partial p_j}E(p_j - p_{j0}) = y(\mathbf{p_0}) \tag{7}$$

and

$$\begin{aligned}\mathrm{Var}(y) \quad &\cong \sum_{j=1}^{n}\left(\tfrac{\partial f(\mathbf{p_0})}{\partial p_j}\right)^2 \mathrm{Var}(p_j) \\ &+2\sum_{j=1}^{n}\sum_{k=j+1}^{n}\left[\tfrac{\partial f(\mathbf{p_0})}{\partial p_j}\right] \\ &\times\left[\tfrac{\partial f(\mathbf{p_0})}{\partial p_k}\right]\mathrm{Cov}(p_j,p_k).\end{aligned} \tag{8}$$

Mean is considered as a center value for a group of parameters while variance is a measure of deviation from its center. In the nature, parameter variance can propagate to system output according to the relationship of Eq. 8. The resultant variance of output response is dependent on the variance and covariance of the system inputs.

2.1 Analysis of uncertainty

For spent nuclear fuel storage canister, the loss of containment time is equal to the breach time of canister plus the time for leaching and transport of a specified amount of activity past the waste package boundaries. For simplicity in illustration, we use the following conservative assumptions: (1) the release initiation time is equal to the failure or breach time of canisters; (2) the corrosion in water is occurring from the time of emplacement; (3) the emplacement is above the water table so that the maximum temperature is 100°C; and (4) the water flow rate is so slow that it has no influence on the corrosion rate. Furthermore, it is assumed that only uniform corrosion and pitting take place as degradation mechanisms (Sutcliffe 1984).

As an illustration, we use a formula for corrosion depth proposed by NRC (1983) in the sample calculation,

$$y = k_p k_h \mathrm{e}^{a/T}[\mathrm{O}]^b[\mathrm{Cl}]^c t^m \tag{9}$$

where y is the corrosion depth (mm); k_p is the pitting factor; k_h is the uniform corrosion factor (mm/year^m); T is the absolute temperature (K); [O] is the oxygen concentration in groundwater (µg/g); [Cl] is the chlorine concentration in groundwater (µg/g); t is the exposure time (year); m, a, b, and c are dimensionless empirical parameters; and e is the exponential function.

Corrosion depth (y) is the system output with independent variable t by incorporating uncertainty into k_h, k_p, [O], [Cl], and m. T, a, b, and c are constants in calculation.

From Eqs. 7 and 9, the mean of corrosion depth is derived as

$$E(y) = E(k_p)E(k_h)\mathrm{e}^{a/T}E([\mathrm{O}])^bE([\mathrm{Cl}])^c t^m \tag{10}$$

Using Eq. 8, the variance of corrosion depth can be derived as (Shih et al. 2002)

$$\begin{aligned}\mathrm{Var}(y) = {} & \alpha^2\mathrm{Var}(k_p) + \beta^2\mathrm{Var}(k_h) + \gamma^2\mathrm{Var}([\mathrm{O}]) + \eta^2\mathrm{Var}([\mathrm{Cl}]) + \xi^2\mathrm{Var}(m) \\ & + 2\alpha\beta\mathrm{Cov}(k_p,k_h) + 2\alpha\gamma\mathrm{Cov}(k_p,[\mathrm{O}]) + 2\alpha\eta\mathrm{Cov}(k_p,[\mathrm{Cl}]) \\ & + 2\alpha\xi\mathrm{Cov}(k_p,m) + 2\beta\gamma\mathrm{Cov}(k_h,[\mathrm{O}]) + 2\beta\eta\mathrm{Cov}(k_h,[\mathrm{Cl}]) + 2\beta\xi\mathrm{Cov}(k_h,m) \\ & + 2\gamma\eta\mathrm{Cov}([\mathrm{O}],[\mathrm{Cl}]) + 2\gamma\xi\mathrm{Cov}([\mathrm{O}],m) + 2\eta\xi\mathrm{Cov}([\mathrm{Cl}],m)\end{aligned} \tag{11}$$

where

$$\alpha = E(k_h)\mathrm{e}^{a/T}E([\mathrm{O}])^bE([\mathrm{Cl}])^c t^{E(m)} \tag{12}$$

$$\beta = E(k_p)\mathrm{e}^{a/T}E([\mathrm{O}])^b E([\mathrm{Cl}])^c t^{E(m)} \tag{13}$$

$$\gamma = bE(k_p)E(k_h)\mathrm{e}^{a/T}E([\mathrm{O}])^{b-1}E([\mathrm{Cl}])^c t^{E(m)} \tag{14}$$

$$\eta = cE(k_p)E(k_h)\mathrm{e}^{a/T}E([\mathrm{O}])^b E([\mathrm{Cl}])^{c-1} t^{E(m)} \tag{15}$$

$$\xi = E(k_p)E(k_h)\mathrm{e}^{a/T}E([\mathrm{O}])^b E([\mathrm{Cl}])^c t^{E(m)} \ln(t) \tag{16}$$

in which ln refers to the natural logarithm.

For the uncorrelated case, i.e. k_p, k_h, [O], [Cl], and m are not correlated, the variance of corrosion depth becomes

$$\begin{aligned} \mathrm{Var}(y) = {} & \alpha^2\mathrm{Var}(k_p) + \beta^2\mathrm{Var}(k_h) + \gamma^2\mathrm{Var}([\mathrm{O}]) \\ & + \eta^2\mathrm{Var}([\mathrm{Cl}]) + \xi^2\mathrm{Var}(m). \end{aligned} \tag{17}$$

System uncertainty can be analytically evaluated using Eqs. 11 and 17 for correlated and uncorrelated cases, respectively.

2.2 Analysis of parameter importance

It defines importance as the contribution of normalized variance (φ) in total uncertainty output for each parameter in input. Normalized contribution of uncertainty φ for the correlated case related to Eq. 11 can be derived as

$$\begin{aligned}
\varphi(k_p) &= \varepsilon\alpha^2\mathrm{Var}(k_p)\\
\varphi(k_h) &= \varepsilon\beta^2\mathrm{Var}(k_h)\\
\varphi([\mathrm{O}]) &= \varepsilon\gamma^2\mathrm{Var}([\mathrm{O}])\\
\varphi([\mathrm{Cl}]) &= \varepsilon\eta^2\mathrm{Var}([\mathrm{Cl}])\\
\varphi(m) &= \varepsilon\xi^2\mathrm{Var}(m)\\
\varphi(k_p,k_h) &= 2\varepsilon\alpha\beta\mathrm{Cov}(k_p,k_h)\\
\varphi(k_p,[\mathrm{O}]) &= 2\varepsilon\alpha\gamma\mathrm{Cov}(k_p,[\mathrm{O}])\\
\varphi(k_p,[\mathrm{Cl}]) &= 2\varepsilon\alpha\eta\mathrm{Cov}(k_p,[\mathrm{Cl}])\\
\varphi(k_p,m) &= 2\varepsilon\alpha\xi\mathrm{Cov}(k_p,m)\\
\varphi(k_h,[\mathrm{O}]) &= 2\varepsilon\beta\gamma\mathrm{Cov}(k_h,[\mathrm{O}])\\
\varphi(k_h,[\mathrm{O}]) &= 2\varepsilon\beta\eta\mathrm{Cov}(k_h,[\mathrm{Cl}])\\
\varphi(k_h,m) &= 2\varepsilon\beta\xi\mathrm{Cov}(k_h,m)\\
\varphi([\mathrm{O}],[\mathrm{Cl}]) &= 2\varepsilon\gamma\eta\mathrm{Cov}([\mathrm{O}],[\mathrm{Cl}])\\
\varphi([\mathrm{O}],m) &= 2\varepsilon\gamma\xi\mathrm{Cov}([\mathrm{O}],m)\\
\varphi([\mathrm{Cl}],m) &= 2\varepsilon\eta\xi\mathrm{Cov}([\mathrm{Cl}],m)
\end{aligned} \tag{18}$$

For the uncorrelated case, normalized contributions of uncertainty in Eq. 17 are presented as

$$\begin{aligned}
\varphi(k_p) &= \varepsilon\alpha^2\mathrm{Var}(k_p)\\
\varphi(k_h) &= \varepsilon\beta^2\mathrm{Var}(k_h)\\
\varphi([\mathrm{O}]) &= \varepsilon\gamma^2 Var([\mathrm{O}])\\
\varphi([\mathrm{Cl}]) &= \varepsilon\eta^2\mathrm{Var}([\mathrm{Cl}])\\
\varphi(m) &= \varepsilon\xi^2\mathrm{Var}(m)
\end{aligned} \tag{19}$$

where $\varepsilon = 1/\mathrm{Var}(y)$.

Having calculations for Eqs. 18 and 19, the contributions of parameters in total uncertainty for y can be substantially evaluated and then the importance for parameters can be quantitatively determined.

Table 1 Parameters and constants

Parameter	Value	
k_p[a]	$E = 4.700\times10^0$	$\mathrm{Var} = 2.756\times 10^0$
k_h[a]	$E = 1.706\times 10^{-1}$	$\mathrm{Var} = 1.000\times 10^{-4}$
[O][a]	$E = 7.000\times 10^0$	$\mathrm{Var} = 1.000\times 10^{-2}$
[Cl][a]	$E = 6.600\times 10^0$	$\mathrm{Var} = 6.400\times 10^{-1}$
m[a]	$E = 4.677\times 10^{-1}$	$\mathrm{Var} = 9.800\times 10^{-4}$
a[a]		-1.402×10^3
b[a]		0.200×10^0
c[a]		5.430×10^{-1}
T[a]		3.730×10^2
T[a]		$0–1.000\times 10^3$
Cov(k_p,k_h)		0.0
Cov(k_p,[O])		5.249×10^{-2}
Cov(k_p,[Cl])		4.200×10^{-1}
Cov(k_p,m)		0.0
Cov(k_h,[O])		3.160×10^{-4}
Cov(k_h,[Cl])		2.530×10^{-3}
Cov(k_h, m)		0.0
Cov([O],[Cl])		0.0
Cov([O], m)		0.0
Cov([Cl],m)		0.0

[a]Song and Lee (1989)

3 Result and discussion

Given values of parameters and constants are listed as Table 1. Parts of parameters and constants used in Eq. 9 are cited from Soong and Lee 1989. We assume that k_p and k_h are correlated with [O] and [Cl], respectively. Exposure time is taken as 1,000 years. For the purpose

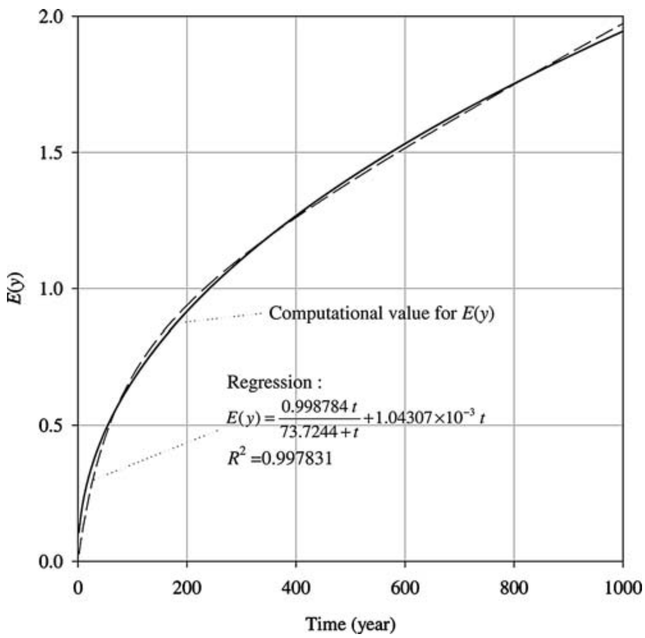

Fig. 1 Mean of the corrosion depth

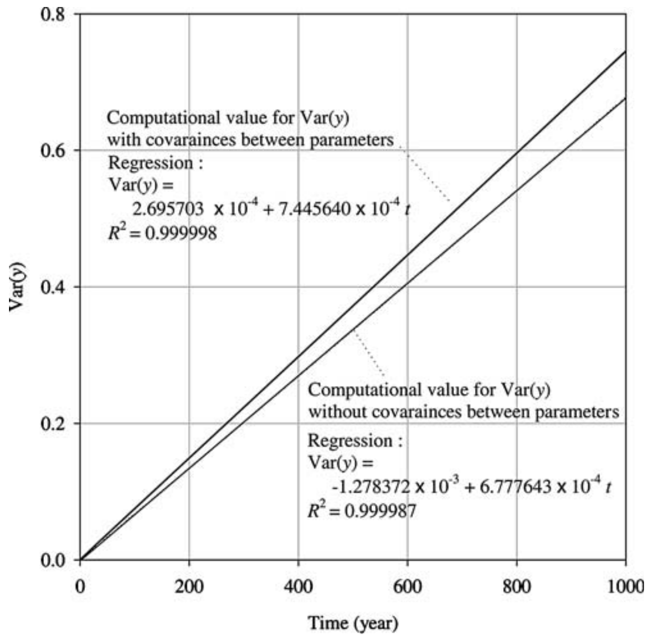

Fig. 2 Variance of the corrosion depth

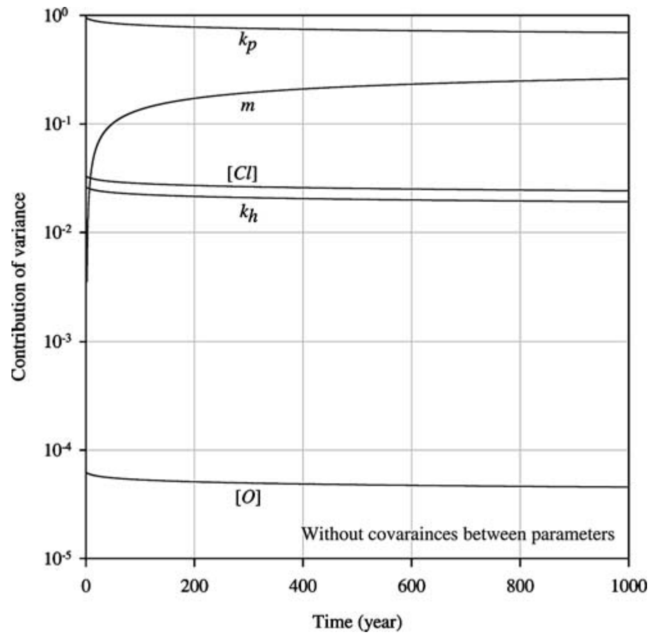

Fig. 4 Importance of parameter for the uncorrelated case

of demonstration, covariance is calculated by the multiplication of variance of parameters in a factor of 10%. It shows that the mean corrosion depth presents a nonlinear regression type through the time of 1,000 years (Fig. 1). The variances of corrosion depth demonstrate a first-order linear increase for both correlated and uncorrelated cases (Fig. 2). The variance of corrosion depth reaches 0.75 for the correlated case while 0.68 for the uncorrelated case. Propagation of uncertainty of corrosion depth for the correlated case is more significant than the uncorrelated case in the time history. It is suggested that the correlated case has a larger uncertainty than the uncorrelated case. The most significant contribution in total uncertainty is Var(k_p) (Fig. 3). Cov(k_p,[O]) is undistinguished for the uncertainty of the system output. It should be noted that Var(k_p) has the highest importance in the uncorrelated case (Fig. 4). Var(m) is sharply increased to the 100 year and keep up

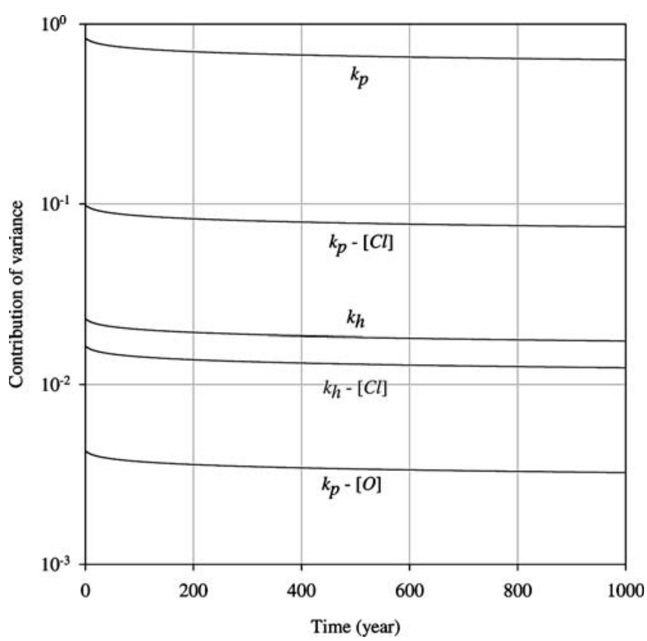

Fig. 3 Importance of parameter for the correlated case

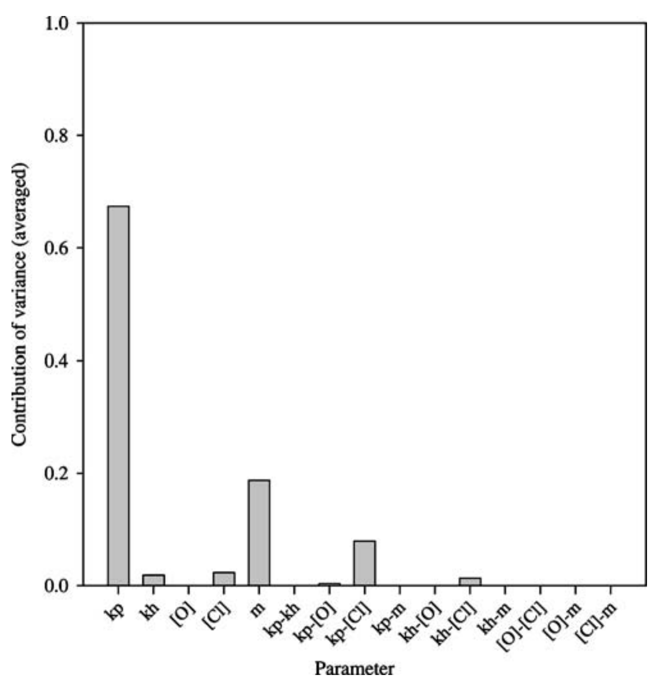

Fig. 5 Importance of parameter through 1,000 years for the correlated case

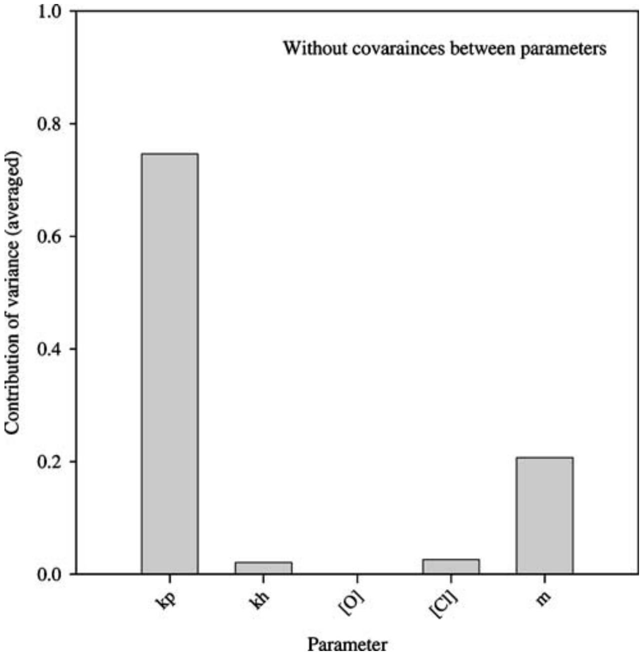

Fig. 6 Importance of parameter through 1,000 years for the uncorrelated case

a fixed value from 100 to 1,000 years for the uncorrelated case (Fig. 4). From the calculation for averaged importance through 1,000-year time history, Cov(k_h,[Cl]) and Var(m) are the considerable parameters for the correlated case (Fig. 5) while Var(k_p) is for uncorrelated case (Fig. 6). In order to analytically demonstrate the uncertainty of performance assessment for the corrosion depth of nuclear spent fuel canister in the current status, covariance values are assumed. It can be expected that the analytical form of uncertainty and importance of parameters for the corrosion depth evaluation can be adequately conducted using the differential analysis.

4 Conclusion

The differential analysis has been used analytically to evaluate the uncertainty of NRC proposed performance assessment for the corrosion depth of storage canisters of nuclear spent fuel in time history. The result shows that through the time of 1,000 years the mean and variance of corrosion depth demonstrate a non-linear regression and first-order linear relationship, respectively. The uncertainty of the corrosion depth demonstrate a linear increase in 1,000-year time history. By incorporating correlation into pitting factor, uniform corrosion factor, oxygen concentration and chlorine oncentration, it is found that the covariance of pitting factor and chlorine concentration has the highest contribution in the variance of total loss than the other parameter pairs. The differential analysis is suggested to evaluate the uncertainty propagation of a system with incomplete and insufficient data in the preliminary phase of assessment. Incorporating assumed covariance data into system, the analytical form of uncertainty and importance of parameters for the corrosion depth evaluation can be well demonstrated using the differential analysis. In the future, uncertainty analysis can be well performed using the proposed method and the sample collected from the real world.

Acknowledgements This research is supported by the Institute of Nuclear Energy Research (INER), Atomic Energy Council, Taiwan, under a fund from the Executive Yuan of the Republic of China.

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