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動力系統裡 Silnikov 式同宿軌道

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In order to construct a Silnikov type homoclinic orbit associated with a periodic orbit, I used a numerical method to generate an approximate homoclinic orbit of the system of ODEs and then used shadowing techniques to find computable criteria which can rigorously verify the existence of a true transversal homoclinic orbit near the computed approximate orbit. This technique was applied to the Lorenz equations. This is ongoing work with B.A.Coomes and H.Kocak of the University of Miami and it has not yet been finished.

In order to construct a Silnikov saddle-focus homoclinic orbit, I considered a singularly perturbed system of ordinary differential equations where the slow dynamics on the centre manifold of the unperturbed system had a hyperbolic equilibrium with an associated fast nondegenerate homoclinic orbit. The singularly perturbed system was dependent on two parameters, one of them being the "singular parameter". Beyn and Stiefenhofer considered such a system and used an analytic approach to show under certain conditions that there is a curve in the two-dimensional parameter space along which the perturbed system has a homoclinic orbit. In joint work with Flaviano Battelli, I showed that the non-resonance conditions assumed by Beyn and Stiefenhofer were not necessary. Also I investigated the transversality properties of the homoclinic orbits and constructed systems of this kind which possess Silnikov saddle-focus homoclinic orbits. (In fact, this work was completed earlier than expected, before the research plan took effect, and was published as
Battelli, F. and Palmer, K.J. (2003). Singular perturbations, transversality and Sil'nikov saddle-focus homoclinic orbits, *Journal of Dynamics and Differential Equations* **15** (2003), 357-425.)

The problems considered above all involve hyperbolicity, which is essentially the same as exponential dichotomy. Massera and Schäffer showed that a nonautonomous linear differential system with bounded coefficient matrix has an exponential dichotomy on a half-line if and only if it is *uniformly noncritical*. I extended this result to the whole line and also investigated to what extent the result holds when the coefficient matrix is not bounded. I also introduced the new notion of *exponential expansivity* and examined its relationship to exponential dichotomy. This article is entitled "Exponential dichotomies and expansivity" and will appear in *Annali di Matematica Pura ed Applicada*.

Next I have done some work with Stefan Siegmund of the University of Frankfurt, Germany, also on nonautonomous systems of linear differential equations. For such a system on the whole line, an *attractor-repeller pair* is a pair of complementary solution subspaces, the angle between which is bounded below by a positive number and such that the solutions in the attractor subspace grow at a faster rate than those in the repeller subspace. This is a weaker concept than exponential separation. We showed that there exists at most one attractor-repeller pair for a given dimension of the attractor subspace. Next we showed that if a system has attractor-repeller pairs of all dimensions, then it is diagonalizable and in a unique way. Finally we showed that if the coefficient matrix is bounded and uniformly continuous and all equations in the hull have attractor-repeller pair of all dimensions, then the system is integrally separated. This work has been published in
Palmer, K.J. and Siegmund, S. (2004). Generalized attractor-repeller pairs, diagonalizability and integral separation. *Advanced Nonlinear Studies* **4**, 189-207.