

行政院國家科學委員會專題研究計畫成果報告

在任意異向彈性體內移動之差排的暫態分析

Transient Analysis of A Moving Dislocation in A General Anisotropic Solid

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一、中文摘要

本計畫求得一個具任意柏格向量之差排在任意異向彈性體內從靜止狀態開始以等速移動的彈力顯式解。相關分析是採用一個不需積分轉換的六維形式。本計畫進一步求得差排所輻射出來的能量通量並據此計算作用在立方晶體之差排的拖曳力。

關鍵詞：移動差排、異向彈性體、拖曳力。

Abstract

The elastic fields of a dislocation with general Burgers vector starting from rest and moving with constant velocity in a general anisotropic solid are given in closed form. The analysis is made using a sextic formalism that does not require integral transforms. An explicit expression for the energy flux radiated from the dislocation is derived. The drag force on the dislocation is calculated for a class of cubic materials.

Keywords: Moving Dislocation, Anisotropic Elastic Solid, Drag Force.

2、INTRODUCTION

The problems of moving dislocations that have been studied can be classified into three categories: (i) steady motion of a dislocation

moving with constant velocity for all time; (ii) transient motion of a dislocation starting from rest and moving with constant velocity; (iii) transient motion starting from rest and moving with time-dependent velocity. The solution to the first problem is essentially complete for both the isotropic and anisotropic cases as reviewed in [1] and [2]. The second and the third problems have been studied for screw and edge dislocations in isotropic materials ([3]-[5]) and in anisotropic materials ([6]-[8]). In the latter works the dislocation problem was treated as an equivalent half-space problem with mixed boundary conditions. The treatment is strictly valid only for materials with certain symmetries. In the present paper closed form solution to the second problem is provided for genuine general anisotropic materials and for general types of dislocations.

The customary approach to the analysis of the transient motion is based on integral transform methods and inversion by the Cagniard-de Hoop [9] technique. Recently a sextic formalism for self-similar dynamic problems was proposed by Wu [10]. In the proposed formalism the general solution is directly expressed in terms of the eigenvalues

and eigenvectors of a six-dimensional eigenvalue problem. A major advantage of the proposed formulation is that no integral transforms are required. This fact greatly facilitates derivations of explicit solutions. Here the sextic formalism is employed to treat the moving dislocation problem.

The problem of interest may be considered as the superposition of the following three problems: (i) a static dislocation with the Burgers vector $\hat{\mathbf{a}}$ at the origin, (ii) a stationary dislocation with Burgers vector $-\hat{\mathbf{a}}$ which suddenly appears at $t = 0$ at the origin, and (iii) a moving dislocation with Burgers vector $\hat{\mathbf{a}}$ that suddenly appears at $t = 0$ at the origin and moving thereafter with constant velocity ν . The solution to the first problem is well known (see [11] for example) and is in fact the solution to the second problem as $t \rightarrow \infty$. The second problem, in turn, is a special case of the third problem with $\nu = 0$. Thus one only needs to solve the third problem to construct the desired solution.

An important quantity in dislocation dynamics is the drag force on the moving dislocation. The drag force is the driving force required for the elastic fields associated with the moving dislocation to have neither an energy source nor an energy sink at the dislocation. Eshelby [12] has given estimates on the drag force due to continuum isotropic elasticity for nonuniformly moving screw dislocations at velocities that are small compared with the shear wave speed. Clifton and Markenscoff [13] have derived the drag force for screw and edge dislocations that start from rest and move thereafter at constant subsonic velocity in an isotropic

solid. They also considered supersonic screw dislocations. In this paper an explicit expression for the drag force for general anisotropic solid is derived. It is shown that the expression is closely related to the prelogarithmic energy factor for a steadily moving dislocation.

3 · FORMULATION

For two-dimensional self-similar problems, the velocity and displacement gradients may be expressed as [10]

$$\dot{\mathbf{u}} = 2 \operatorname{Re} \left[\sum_{k=1}^3 f_k(\check{\mathcal{S}}_k) \frac{\partial \check{\mathcal{S}}_k}{\partial t} \mathbf{a}_k(\check{\mathcal{S}}_k) \right], \quad (1)$$

$$\mathbf{u}_{,1} = 2 \operatorname{Re} \left[\sum_{k=1}^3 f_k(\check{\mathcal{S}}_k) \frac{\partial \check{\mathcal{S}}_k}{\partial x_1} \mathbf{a}_k(\check{\mathcal{S}}_k) \right], \quad (2)$$

and the stresses $\mathbf{t}_2 = (t_{12}, t_{22}, t_{32})^T$ as

$$\mathbf{t}_2 = 2 \operatorname{Re} \left[\sum_{k=1}^3 f_k(\check{\mathcal{S}}_k) \frac{\partial \check{\mathcal{S}}_k}{\partial x_1} \mathbf{b}_k(\check{\mathcal{S}}_k) \right], \quad (3)$$

where Re stands for the real part,

$$\check{\mathcal{S}}_k = y_1 + p_k(\check{\mathcal{S}}_k) y_2, \quad y_1 = x_1/t, \quad y_2 = x_2/t.$$

The quantities $p_k(\check{\mathcal{S}})$ and $\mathbf{a}_k(\check{\mathcal{S}})$ are, respectively, the eigenvalue and eigenvector of the following eigenvalue problem:

$$[\mathbf{Q} + \rho(\mathbf{R} + \mathbf{R}^T) + \rho^2 \mathbf{T} - \dots \check{\mathcal{S}}^2 \mathbf{I}] \mathbf{a}(\check{\mathcal{S}}) = 0, \quad (4)$$

where ρ is the density, the matrices \mathbf{Q} , \mathbf{R} and \mathbf{T} are related to elastic constants, and \mathbf{I} is the identity matrix. The vector $\mathbf{b}_k(\check{\mathcal{S}})$ is

given by

$$\mathbf{b}_k(\check{\mathcal{S}}) = (\mathbf{R}^T + p_k(\check{\mathcal{S}}) \mathbf{T}) \mathbf{a}_k(\check{\mathcal{S}}). \quad (5)$$

The eigenvalues $p_k(\check{\mathcal{S}})$ are selected such that the imaginary parts are positive when they are complex.

For $y_2 = 0$, define

$$\sum_{k=1}^3 \frac{1}{\hat{\chi}_k(y_1)} \mathbf{b}_k(y_1) \mathbf{b}_k^T(y_1) = \frac{i}{2} \mathbf{L}(y_1) \quad (6)$$

where $\chi_k(y_1) = 2\mathbf{a}_k^T(y_1) \mathbf{b}_k(y_1)$. The matrix \mathbf{L} is one of the Barnett-Lothe tensors. The matrix \mathbf{L} is real in the subsonic regime, complex in the intersonic regime, and purely imaginary in the supersonic regime.

4. SOLUTION

Consider first the fundamental solution of a dislocation that suddenly appears at $t = 0$ at the origin and moves with a subsonic constant velocity ν is given. The solution of $f_k(\check{\mathcal{S}}_k)$ is obtained as

$$f_k(\check{\mathcal{S}}_k) = \frac{1}{2f(\check{\mathcal{S}}_k - \nu)\chi_k(\check{\mathcal{S}}_k)} \mathbf{b}_k^T(\check{\mathcal{S}}_k) \mathbf{s}. \quad (7)$$

The corresponding $\dot{\mathbf{u}}^*$ and \mathbf{t}_2^* are given by

$$\dot{\mathbf{u}}^*(x_1, x_2, t, \nu) = -\frac{1}{ft} \text{Im} \left[\sum_{k=1}^3 \frac{\check{\mathcal{S}}_k \mathbf{a}_k(\check{\mathcal{S}}_k) \mathbf{b}_k^T(\check{\mathcal{S}}_k)}{(\check{\mathcal{S}}_k - \nu) \hat{\chi}_k(y_1, y_2)} \right] \mathbf{s} = \quad (8)$$

$$\mathbf{t}_2^*(x_1, x_2, t, \nu) = \frac{1}{ft} \text{Im} \left[\sum_{k=1}^3 \frac{\mathbf{b}_k(\check{\mathcal{S}}_k) \mathbf{b}_k^T(\check{\mathcal{S}}_k)}{(\check{\mathcal{S}}_k - \nu) \hat{\chi}_k(y_1, y_2)} \right] \mathbf{s} = \quad (9)$$

where Im stands for the imaginary part.

Consider next a dislocation with the Burgers vector $\hat{\mathbf{a}}$ starting from rest at $t = 0$ and moving thereafter with constant velocity ν . The initial position of the dislocation is assumed at the origin. The solution to the present problem can be obtained by the superposition of those to the following three problems:

Problem I: A static dislocation with the Burgers vector $\hat{\mathbf{a}}$ at the origin.

Problem II: A stationary dislocation with Burgers vector $-\hat{\mathbf{a}}$ that suddenly appears at $t = 0$ at the origin.

Problem III: A moving dislocation with

Burgers vector $\hat{\mathbf{a}}$ that suddenly appears at $t = 0$ at the origin and moving thereafter with constant velocity ν .

The solution to Problem III is the fundamental solution. The solution to Problem I can be deduced from the fundamental solution by taking $\nu = 0$ and $t \rightarrow \infty$ as discussed in the preceding section.

The solution to Problem II can also be deduced from the fundamental solution by taking $\nu = 0$. Thus $\dot{\mathbf{u}}$ can be expressed as:

$$\dot{\mathbf{u}}(x_1, x_2, t) = \dot{\mathbf{u}}^{(s)}(x_1, x_2) + \dot{\mathbf{u}}^{(d)}(x_1, x_2, t, \nu),$$

where $\dot{\mathbf{u}}^{(s)}$ and $\dot{\mathbf{u}}^{(d)}$ are defined as

$$\dot{\mathbf{u}}^{(s)}(x_1, x_2) = \dot{\mathbf{u}}^*(x_1, x_2, \infty; 0),$$

$$\dot{\mathbf{u}}^{(d)}(x_1, x_2, t) = \dot{\mathbf{u}}^*(x_1, x_2, t, \nu) - \dot{\mathbf{u}}^*(x_1, x_2, t, 0)$$

The resulting expression for $\dot{\mathbf{u}}^{(d)}$ is

$$\dot{\mathbf{u}}^{(d)}(x_1, x_2, t) = -\frac{\nu}{ft} \text{Im} \left[\sum_{k=1}^3 \frac{\mathbf{a}_k(\check{\mathcal{S}}_k) \mathbf{b}_k^T(\check{\mathcal{S}}_k)}{(\check{\mathcal{S}}_k - \nu) \hat{\chi}_k(y_1, y_2)} \right] \mathbf{s}?$$

Similarly $\mathbf{t}_2^{(d)}$ is given by

$$\mathbf{t}_2^{(d)}(x_1, x_2, t) = \frac{\nu}{ft} \text{Im} \left[\sum_{k=1}^3 \frac{\mathbf{b}_k(\check{\mathcal{S}}_k) \mathbf{b}_k^T(\check{\mathcal{S}}_k)}{\check{\mathcal{S}}_k (\check{\mathcal{S}}_k - \nu) \hat{\chi}_k(y_1, y_2)} \right] \mathbf{s}?$$

5. DRAG FORCE

The energy flux \dot{E} flowing into the core of a dislocation moving along the x_1 -axis can be calculated by [14]

$$\dot{E} = 2 \lim_{u_1 \rightarrow 0} \left(\lim_{u_2 \rightarrow 0} \int_{-u_1}^{u_1} \mathbf{t}_2^T(\zeta_1, u_2, t) \dot{\mathbf{u}}(\zeta_1, u_2, t) d\zeta_1 \right)$$

The result is

$$\dot{E}(\nu) = \frac{1}{2ft} \mathbf{s}^T (\mathbf{L}'(\nu) \nu - \mathbf{L}(\nu) + \mathbf{L}(0)) \mathbf{s}$$

The drag force required for the elastic fields associated with the dislocation motion to have neither an energy source nor an energy sink at the dislocation is given by [13]

$$F_{drag} = -\frac{\dot{E}}{\nu}.$$

To assess the anisotropy effect on the drag force, consider an edge dislocation moving in a cubic material with the elastic constants characterized by the following two dimensionless parameters A and B:

$$A = \frac{2C_{66}}{C_{11} - C_{12}}, B = \frac{C_{12}}{2C_{66}},$$

where $A > 0$ and $B > -1/(3A)$. The material with $A = 1$ corresponds to the isotropic material. The Burgers vector $\hat{\mathbf{a}}$ is taken as $\hat{\mathbf{a}} = b_1 \mathbf{e}_1$, \mathbf{e}_1 being the unit vector in the x_1 -direction. Figure 1 shows the dependence of the drag force on the dislocation velocity for $A = 1, 2, 4, 6, 8, 10$ and $B = 1$. The curve for $A = 1$ is the same as that obtained by Clifton and Markenscoff [13] for the isotropic material with the Poisson ratio $\epsilon = 1/3$. For small dislocation velocities, the drag force can be approximated as

$$F_{drag} \approx -\frac{L_{11}''(0)b_1^2}{4ft} \nu$$

and is linearly proportional to ν . The drag force becomes infinite as a limiting speed is approached. For $A = 1$, the limiting speed is the shear wave speed c_2 . The limiting speeds vary from $0.90c_2$ for $A = 2$ to $0.44c_2$ for $A = 10$. To see the physical meaning of the limiting speeds, typical wave front curves for the materials with $A > 1.3$ are plotted in Figure 2. The outer curve is the qL (quasi-Longitudinal) wave front and the inner qS (quasi-Shear) wave front. The qS wave front has four cuspidal triangles, two centered on the x_1 -axis and two on the x_2 -axis. The triangular regions are lacunas where there is no disturbance [15]. Thus

along the x_1 -direction, in addition to the fast shear wave traveling with c_2 , there is another slower shear wave. The limiting speed is actually the slower shear wave speed.

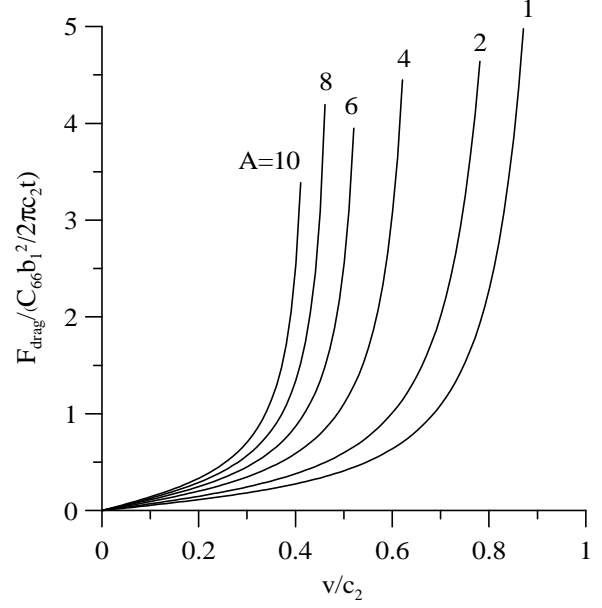


Figure 1. The dependence of the drag force on the dislocation velocity for $A = 1, 2, 4, 6, 8, 10$.

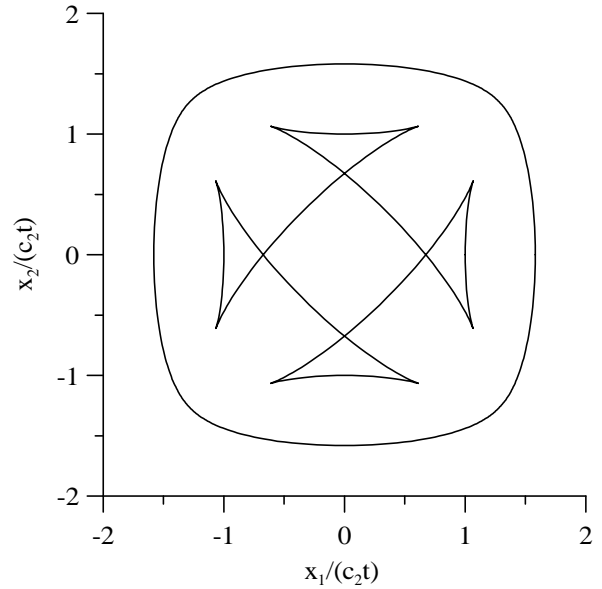


Figure 2. Typical wave front curves for the materials with $A > 1.3$.

6. CONCLUSIONS

A closed form solution to the transient problem of a dislocation starting from rest and moving with constant subsonic velocity has been presented. The materials considered are of general anisotropy and the dislocations with general Burgers vectors. The elastic fields of the moving dislocation are used to derive the energy flux radiated from the dislocation. It is shown that the energy flux may be expressed in terms of the energy factor for a steadily moving dislocation.

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