JOURNAL OF THE KOREAN ASTRONOMICAL SOCIETY
32: 1 ∼ , 2001

A SIMPLE VOLUME TRACKING METHOD FOR COMPRESSIBLE TWO-PHASE FLOW

KEH-MING SHYUE
Department of Mathematics, National Taiwan University Taipei 106, Taiwan: shyue@math.ntu.edu.tw
(Received ???. ??, 2001; Accepted ???. ??, 2001)

ABSTRACT

Our goal is to present a simple volume-of-fluid type interface-tracking algorithm to compressible two-phase flow in two space dimensions. The algorithm uses a uniform underlying Cartesian grid with some cells cut by the tracked interfaces into two subcells. A volume-moving procedure that consists of two basic steps: (1) the update of volume fractions in each grid cell at the end of the time step, and (2) the reconstruction of interfaces from discrete set of volume fractions, is employed to follow the dynamical behavior of the interface motion. As in the previous work with a surface-tracking procedure for general front tracking (LeVeque & Shyue 1995, 1996), a high resolution finite volume method is then applied on the resulting slightly nonuniform grid to update all the cell values, while the stability of the method is maintained by using a large time step wave propagation approach even in the presence of small cells and the use of a time step with respect to the uniform grid cells. A sample preliminary numerical result for an underwater explosion problem is shown to demonstrate the feasibility of the algorithm for practical problems.

Key Words : volume tracking method, surface tracking method, stiffened gas equation of state, compressible two-phase flow

I. INTRODUCTION

In previous work by LeVeque and the author (LeVeque & Shyue 1995, 1996), a simple surface-tracking type front-tracking algorithm has been developed for the efficient numerical resolution of problems with discontinuous solutions governed by nonlinear hyperbolic systems of conservation laws in both one and two space dimensions. In this algorithm, we choose a uniform underlying grid with some grid cells subdivided by tracked interfaces made up of moving points in 1D and linear segments in 2D, approximately aligned with discontinuities in the flow field. In each time step, we solve Riemann problems at each cell boundaries, and use the resulting solutions, speeds of strong waves in particular, to determine a new set of interfaces that approximate the expected locations of discontinuities in the solution at the end of the time step. A high resolution finite volume method is then applied on the resulting nonuniform grid to update all the cell values. Here a large time step wave propagation approach is employed to overcome a major difficulty associated with the limit on the time step when small cells are created by the tracked front cutting through the grid, while maintaining conservation of the algorithm, see (Chern *et al.* 1986, Chern & Colella 1987, Mao 1993) for the other possible approaches to deal with this so-called small cell problem appearing in the aforementioned front-tracking literatures. Note that if we have chosen the new interface locations well, the resulting solution will remain sharp and be smooth away from these new interfaces. When the update of the solution is done, the old interfaces can then be eliminated by recombining the adjacent cells.

While our front tracking algorithm has been applied quite successfully to solve many shock wave and interface problems arising in gas dynamics and geophysics, see (Shyue 1993, Shyue 1998) for additional results, it is well-known however that this algorithm with the surface-tracking procedure included does not work efficiently to a class of problems with complicated topologically changes of tracked fronts in multiple space dimensions. To improve the algorithm when such a scenario occurs, it is our goal in this paper to describe a relatively simple volume-tracking procedure for the propagation of interfaces, Combining that front-moving procedure to the large time step wave propagation approach for the solution update would yield potentially a more robust interface-tracking algorithm for an easy handling of complex front motion, This will be described in more details in Section II, see (Bell *et al.* 1991) for a similar approach. A sample numerical result for compressible two-phase flow will be shown in Section III to give a preliminary validation of the proposed algorithm.

II. FRONT TRACKING METHOD

Irrespective of the number of space of dimension, it is quite common that, in each time step, an Eulerian-based front tracking method for hyperbolic systems of partial differential equations (*e.g.*, Chern & Colella 1987, Glimm *et al.* 1998, LeVeque & Shyue 1995, 1996) would consist of the following steps:

1. [*Front propagation step*] Flag tracked interfaces by checking solutions of Riemann problems, determine the size of the next time step, and the

location of the tracked interfaces at the next time step,

2. [*Create new grid step*] Modify the current grid by inserting these new tracked interfaces; some cells will be subdivided and the values in each subcell must be initialized,
3. [*Solution update step*] Take a time step on this nonuniform grid using a conservative finite volume method to update the cell averages,
4. [*Remove old grid step*] Delete the old tracked interfaces from the previous time step; some subcells will be combined and a value in the combined cell must be determined from the subcell values.

Figure 1 shows a typical example from a surface-tracking procedure for front propagation in two space dimensions. In that procedure, a one-dimensional Riemann problem in direction normal to each tracked interface is solved by using the values from the adjacent cells as data. Since it is expected that the solution to this Riemann problem would consist of only one strong wave, corresponding to the shock or interface (contact discontinuity or slip line) being tracked, and other weaker waves, we may then follow the strong wave we want to track to a new location at the end of the time step, see (LeVeque & Shyue 1995) for some discussions on how one might choose a time step in the method. Clearly, in order to have the new tracked interfaces to form a continuous piecewise linear curve as shown in Fig. 1, the solutions of neighboring Riemann problems should be used in some coordinated manner to determine the new interfaces. There are various ways that this can be done *via* some sort of curve fitting through points determined by the strong waves from the Riemann solutions. In (LeVeque & Shyue 1998), we present one simple approach for that in more detail. Note that the final interface and points on the new grid are determined by where this curve intersects the grid line. Nevertheless, if we do not view the interface we introduce as being the definitive location of a tracked front, but rather as a grid interface that is sufficiently well located and aligned that the solution can be well captured on the resulting grid, we have some flexibility on this score.

Once the new grid is constructed, the solution can then be advanced using a fully conservative shock capturing method. This method should be able to deal with the irregular cells near the tracked interface. In particular, it must maintain stability even if some of these cells are very small relative to the underlying mesh size used to determine the time step, and also hopefully to maintain second order accuracy in the smooth flow on either side. To accomplish this, one possible way is to use a high resolution method based on the large time step wave propagation approach, developed by LeVeque, see (LeVeque 1988, LeVeque & Shyue 1996). The main idea is that waves arising from

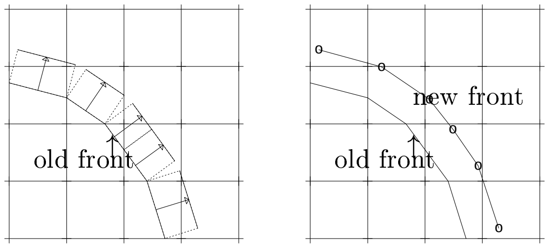

Fig. 1.— A typical surafce-tracking procedure for front propagation. On the left: The original tracked interfaces are updated using the strong wave speeds obtained from the normal Riemann problems during the current time step. On the right: Piecewise linear interpolant through the points are made and the final interface and points on the new grid are determined by where this curve intersects the grid line.

the solution of Riemann problems at the cell boundaries are propagated the appropriate distance determined by the wave speed and time step, and used to update cell averages in any grid cell they encounter. The wave may affect more than one cell if the neighboring cell is very small. In this manner the stencil of the method adjusts automatically so that the CFL (Courant-Friedrich-Lewy) condition is always satisfied regardless of the configuration of the grid.

Note that the algorithm we have just described is not a moving-grid method in that the grid system of the later method would be typically adjusted to fit the location of the tracked discontinuity. It should be mentioned that there are many instances to have the desire of keeping the discontinuity sharp, and consequently the advantage of a front tracking method over a shock capturing method. One example among them is in the simulation of compressible two-phase flow with very different fluid component (say, solid and gas) separated by interfaces, and under extreme flow condition, see (Miller & Puckett 1996). In this case, it would be very difficult in general to devise a shock-capturing type method to deal with the numerical smearing of interface in a proper manner (of course, for some simpler cases, it is still a possible task to do, see (Shyue 1998)). Other famous example is the detonation wave computation in combustions, see (LeVeque & Shyue 1995). Clearly, there are many outstanding difficulties associated with the development of a robust front tracking method in multiple space dimensions. Chern *et al.* pointed out some of them in their paper (Chern *et al.* 1986).

In this paper, we will address one of them, namely, the devise of an efficient method for the handling of complicated topological change of interfaces. Firstly, it is well-known in the scientific community that the surface-tracking type of the procedure as shown in Fig. 1 is not a good candidate to be considered; this is so for its difficulty in both algorithmic and programming issues to realize the problem. Alternatively, in the following we will focus our attention on a popular volume-tracking approach to move front which is much easier to program than the surface-tracking method does. More importantly, it has the capability to handle the splitting and mergering of fronts as well, see (Bell *et al.* 1991) for example.

In a typical volume tracking method, we choose a uniform underlying grid, and have volume fraction function Y for representing specific fluid component in each cell, say in cells when $Y = 1$ we have liquid, in cells when $Y = 0$ we have gas, while in cells when $Y =\in (0, 1)$, we have liquid-gas mixture, see Fig. 2. Note that based on the discrete data for volume fractions, it is easy to reconstruct the approximate location of the interface by using numerical techniques such as the simple line interface calculation (SLIC) of Noh and Woodward, the center of mass of Saltzman, the central difference of Hirt and Nichols, the gradient method of Parker and Youngs , and the least-squares methods of Pilliod and Puckett. These techniques differ from the way in determining the interface that retains the same volume fraction within the cell, see (Rider & Kothe 1998) and reference therein for more details.

0	0	0	0
0.59	0.15	0	0
1	0.91	0.11	0
1	1	0.56	0

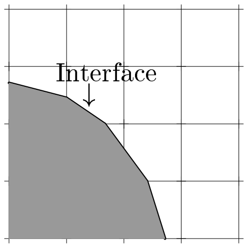

Fig. 2.— A typical setup in a volume-tracking procedure for interface tracking. On the left: discrete data for the volume fractions of fluid component is given, say for liquid $Y = 1$, for gas $Y = 0$, and for liquid-gas mixture $Y \in (0, 1)$. On the right: reconstruction of the interface from volume fractions.

Figure 3b) shows three examples of the results obtained using the center of mass reconstruction technique for the volume fractions given in Fig. 3a). From the figure, it is easy to observe that each of the reconstructed interface does not form a continuous curve, even though the true interface is a continuous one. This kind of interface structure is often seen in a volume tracking method. In fact, because the interface is usually made only using information from the nearby cells and does not need to connect to each other, it is possible to make use of this property and deal with the change of topology in an easy manner when employing a volume tracking method.

In this case, to advance the tracked interface from the current time to the next, in two dimensions, we first solve a linear transport equation of the form for volume fraction Y,

$$\frac{\partial Y}{\partial t} + u\frac{\partial Y}{\partial x} + v\frac{\partial Y}{\partial y} = 0, \qquad (1)$$

using the current discrete data in each grid cell. From the resulting volume fractions, we then apply an interface reconstruction technique for determining the new interface location and grid. Here u and v are the underlying velocity field in the x- and y-directions, respectively.

Having found the new interface location and the new grid at the end of the time step, as in a surface-tracking method for hyperbolic systems of partial differential equations, we may continue the volume tracking method by performing Steps 3 and 4 of the general front tracking procedure. Thus, it should be clear that the only difference between the surface-tracking and volume-tracking methods is on the way how the front is propagated, and the new grid is made. Note that a one-dimensional version of the front tracking code (written in Fortran) is available on web:

http://www.math.ntu.edu.tw/ shyue/code.

For readers who are interested in a two-dimensional version of the code, it is available upon request.

III. NUMERICAL RESULTS

To give an example of how our newly proposed interface-tracking algorithm works for practical problems, we consider a simplified two-phase flow problem with two different fluid components, liquid and gas, separated by immiscible interfaces. In this problem, we use an Eulerian formulation of the equations in which, on the whole part (liquid or gas part) of the domain, the fluid is governed by the full set of the compressible Euler equations of the form,

$$\frac{\partial}{\partial t}\begin{bmatrix} \rho \\ \rho u \\ \rho v \\ E \end{bmatrix} + \frac{\partial}{\partial x}\begin{bmatrix} \rho u \\ \rho u^2 + p \\ \rho u v \\ Eu + pu \end{bmatrix} + \frac{\partial}{\partial y}\begin{bmatrix} \rho v \\ \rho u v \\ \rho v^2 + p \\ Ev + pv \end{bmatrix} = 0.$$

Here ρ is the density, u and v are the velocities in the x- and y-direction respectively, p is the pressure, and E is the total energy . We assume a general compressible material that the specific internal energy, denoted by e, satisfies the stiffened gas equation of state,

$$e = \frac{1}{\gamma - 1}\left(\frac{p + \gamma\mathcal{B}}{\rho}\right),$$

and $E = \rho e + \rho(u^2 + v^2)/2$. Here γ is the usual ratio of specific heats ($\gamma > 1$), and $\mathcal{B}$ is a prescribed pressure-like constant, As for the initial condition, at the depth of 0.3m below a horizontal air-water surface, there is

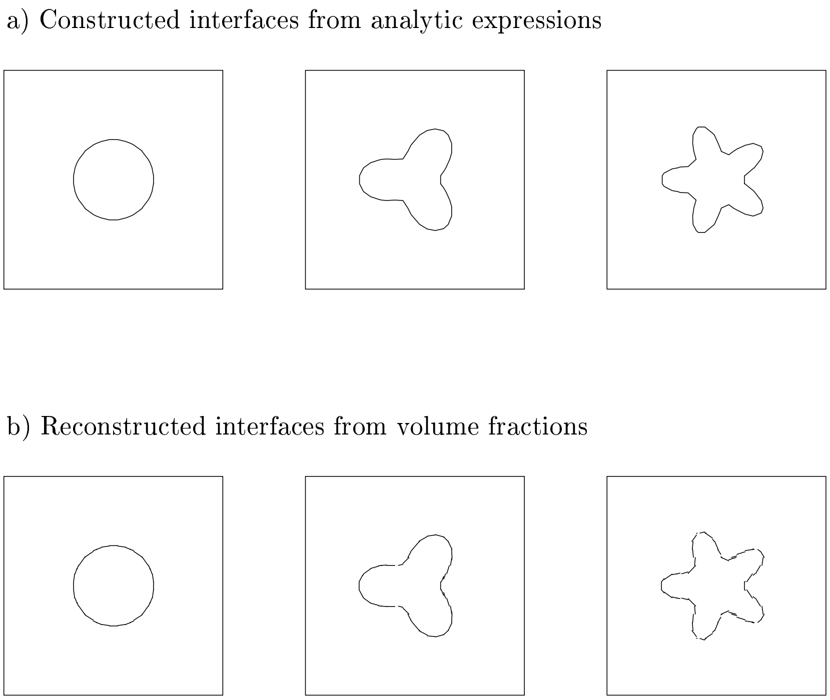

Fig. 3.— Three examples are shown for the approximate interface locations: a) made by using standard marker-and-cell type method from analytic expression of the interface, b) made by using the center of mass interface reconstruction algorithm from voulme fractions.

a circular gas bubble of radius $r_0 = 0.12$m in water. Inside the bubble, the state variables are

$$(\rho,\ p,\ \gamma,\ \mathcal{B}) = \Big(1250\ \mathrm{kg/m}^3,\ 10^9\ \mathrm{Pa},\ 1.4,\ 0\Big),$$

while outside the bubble they are

$$(\rho,\ p,\ \gamma,\ \mathcal{B}) = \Big(10^3\ \mathrm{kg/m}^3,\ 10^5\ \mathrm{Pa},\ 4.4,\ 6\times 10^8\ \mathrm{Pa}\Big),$$

see Fig. 4 for a schematic setup of the problem. Initially both the gas and water are in a stationary position, but due to the pressure difference between the fluids, breaking of the bubble results in a circularly outward-going shock wave in water, an inward-going rarefaction wave in gas, and an interface lying in between that separates the gas and the water. Soon after, this shock wave is diffracted through the nearby air-water surface, causing the subsequent deform of the interface topology from a circle to oval. Note that initially the air-water surafce is at the standard atmospheric condition.

Figure 4 shows numerical results of the interface motion at six different times, $t = 0$, 0.1, 0.2, 0.4, 0.6, and 0.8 ms. Here the bubble interface is tracked by using the proposed interface-tracking method with a 200×200 rectangular grid and Courant number 0.5, whereas the air-water free surface is untracked. It can be shown that the results obtained here agree quite well with the results obtained using the standard surface-tracking type front-tracking method (LeVeque & Shyue 1996); this is the way it should be, see (Grove & Menikoff 1990) also for a similar computation where both the bubble and air-water interfaces are tracked by using a surface-tracking method.

IV. CONCLUSION

We have presented a simple volume-of-fluid type approach to interface tracking. The algorithm uses a volume-tracking procedure to advance tracked interfaces from the current time to the next in a uniform underlying Cartesian grid. The cell values in the resulting slightly nonuniform grid is updated by using a high resolution finite volume method based on a wave propagation formulation. Numerical results shown in the paper indicate that for a distinguished interface without complicated topological change this method is at least as robust as the standard surface-tracking method. Ongoing work is to further validate the method, and use the method as a tool to study chaotic fluid mixing problem which is important in many astrophysical flows and other field of applications as well.

ACKNOWLEDGEMENTS

This work was supported in part by National Science Council of Republic of China Grant NSC-86-2115-M-002-005. The author thanks the organizers and staffs

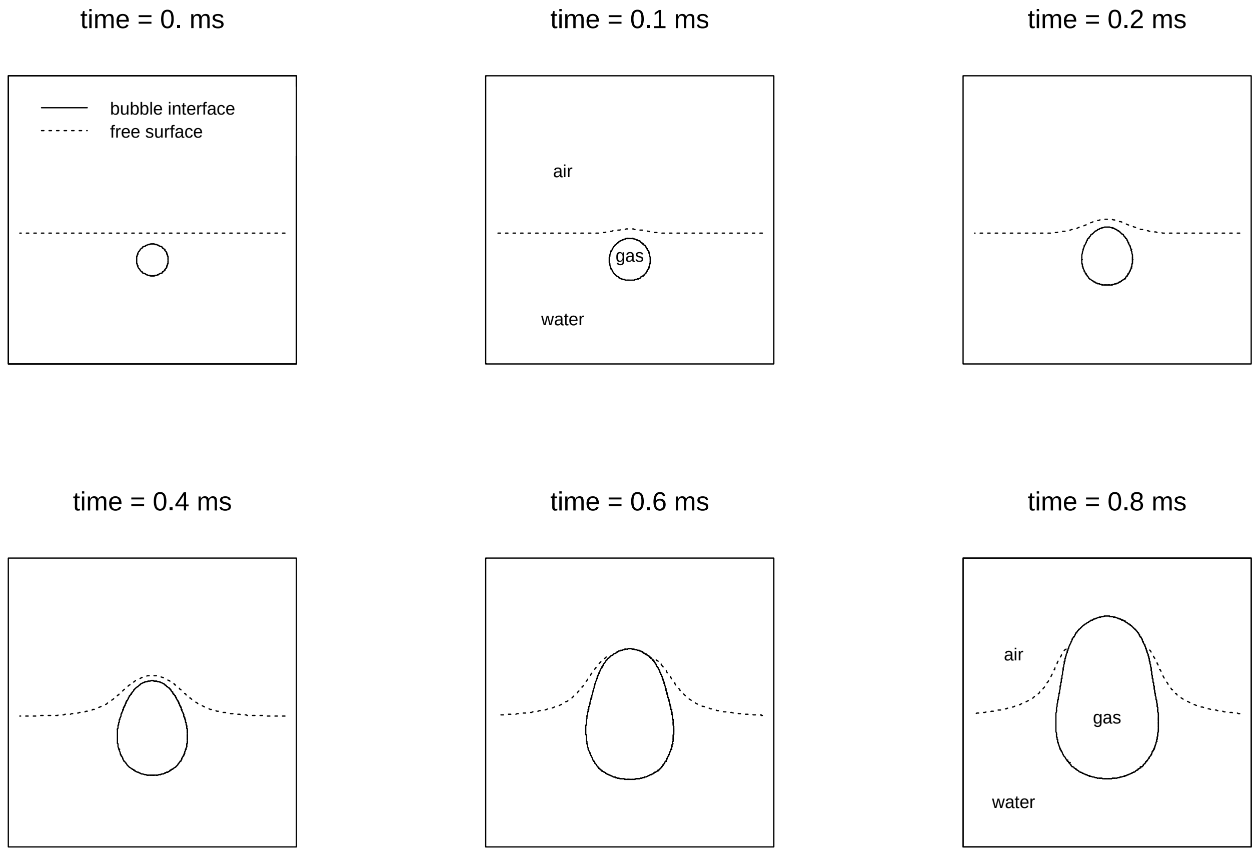

Fig. 4.— Time evolution of the bubble interface for an underwater explosion problem with free surface. In each subplot, the solid line is the approximate location of the tracked bubble interface obtained using by our front tracking algorithm based on the volume-of-fluid representation of , while the dashed line is the approximate location of the air-water free surface which is captured in the current computation. The grid is 200×200.

of the workshop for their hospitality and support.

REFERENCES

Bell, J. B., Colella, P., & Welcome, M. L. 1991, UCRL-JC-105251, preprint

Chern, I-L. & Colella, P. 1987, UCRL-97200, preprint

Chern, I-L., Glimm, J., McBryan O. A., Plohr, B., & Yaniv, S. 1986, J. Comput. Phys., 63,83

Courant, R. & Friedrichs, K. O. 1948, Supersonic Flow and Shock waves, Wiley-Interscience, New York

Glimm, J., Grove, J., Li, X. L., Shyue, K.-M., Zeng, Y., & Zhang, Q. 1998, SIAM J. Sci. Comput., 19, 703

Grove, J. & Menikoff, R. 1990, J. Fluid Mech., 219, 313

Hyman, J. M. 1984, Physica D, 12, 396

LeVeque, R. J. 1988, J. Comput. Phys., 78, 36

LeVeque, R. J. & Shyue, K.-M. 1995, SIAM J. Sci. Comput., 16, 348

LeVeque, R. J. & Shyue, K.-M. 1996, J. Comput. Phys., 123, 354

Mao, D.-K. 1993, J. Comput. Phys., 104, 377

Miller, G. H. & Puckett, E. G. 1996, J. Comput. Phys., 128, 134

Roe, P. L. 1981, J. Comput. Phys., 43, 357

Rider, W. J. & Kothe, D. B. 1998, J. Comput. Phys., 141, 112

Shyue, K.-M. 1993, Front Tracking Methods based on Wave Propagation, Ph.D. thesis, University of Washington

Shyue, K.-M. 1998, J. Comput. Phys., 142, 208