

Equilibrium and Stability at the Relativistic Plasma-Magnetic Field Interface

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We have discussed that the instability at relativistic plasma-magnetic field interface may lead to enhanced mixing as the nonequilibrium at the non-relativistic plasma-magnetic field interface does. Both effects result from the plasma motion relative to the magnetic field.

EQUILIBRIUM and stability are the two major problems about the boundary behavior at the plasma-magnetic field interface. They play a great role in both laboratory and astrophysical plasmas. We consider first a simple situation: The plasma is nonrelativistic and confined in a half space by a uni-directed magnetic field. In magnetohydrodynamic (MHD) approximation a steady state between the plasma and magnetic field exists when the pressure P in the fluid equals the magnetic pressure $B^2/8\pi$ in the magnetic field. The stationary solution is uniquely determined by the condition of pressure balance, regardless of the fluid motion relative to the field. The sharpness of the interface is limited only by the resistivity of the fluid, which may be small. However when we are interested in the boundary phenomena, the MHD approximation is always inadequate. This can be easily visualized by the fact that the structure of the interface must be related with the plasma distribution. In examination of the equations of a non-relativistic collisionless plasma, Ferraro⁽¹⁾, Dungey⁽²⁾, and Grad⁽³⁾ have shown that when the plasma has no motion relative to the magnetic field the plasma behaves essentially like the classical conducting fluid. The interface between the plasma and field has a sharpness of the order of the characteristic cyclotron radius R of the plasma ions.

The instability is known to be ineffective in this situation. On the one hand, the Kruskal-Schwarzschild instability does not emerge in the absence of the gravitational field. On the other hand, flute instabilities also do not appear since there is no bending of the lines of force. Furthermore, the effect of the finite

(1) V. C. A. Ferraro, *J. Geophys. Res.* 57, 15 (1952).

(2) J. W. Dungey, *Cosmic Electrodynamics* (Cambridge: Cambridge University Press), pp. 141-143 (1958).

(3) H. Grad, *Phys. Fluids*, 4, 1366 (1961).

cyclotron radius can only enhance the stability⁽⁴⁾.

The situation is changed when the collisionless plasma is moving relative to the field as shown by Parker⁽⁵⁾, Lerche⁽⁶⁾, and Lerche and Parker⁽⁷⁾. They found that equilibrium state does not exist. The essential reason of this is that if the collisionless plasma flows along the field B with a bulk velocity v and if the number density is N , then the total current is $NevR/\text{cm}$ parallel to B , leading to a magnetic field in the boundary layer R of magnitude

$$A(cB) \simeq 4\pi NevR, \quad (1)$$

which is perpendicular to the main field B . Writing $R = Mwc/eB$ where M is the mass, e the charge, and w the thermal velocity of the plasma ions, and remembering that pressure balance requires that

$$Nmw^2 \simeq B^2/8\pi,$$

it follows that

$$\Delta B/B = 0(v/w). \quad (2)$$

Since in the nonrelativistic case, v may be as large as w , or even larger, it is evident that the new field ΔB generated by the parallel motion of the plasma may be of the same as or even larger than the original B confined by the pressure of the plasma. There is no evident way to confine ΔB which, further, has the effect to deflect the plasma into the field. It is pointed out by Parker and Lerche that this non-equilibrium leads to enhanced mixing of the plasma and field on a small scale of the order of R .

Some astrophysical phenomena, such as the extended geomagnetic tail, surges in the solar atmosphere, the interface between novae and supernova ejecta and the interstellar field, the 'jet' in M87, interstellar gas clouds containing magnetic fields moving through the interstellar medium, possible cosmic-ray streaming between galaxies, etc., may involve the non-equilibrium effect.

The situation is again changed when the confined plasma is relativistic but moves relative to the field nonrelativistically. Such a situation is more likely to occur in some astrophysical circumstances, e. g., at the interface between novae and supernova ejecta and the interstellar field, at the interface between the pulsar emission and the magnetic field of supernova remnants, etc.. The previous conclusions about the nonrelativistic plasma can not apply to this case now. It is easily seen from equation (2) that the non-equilibrium effect is not signifi-

(4) G. Schmidt, *Physics of High Temperature Plasmas* (N. Y. Academic Press), Chap. 8 (1966).

(5) E. N. Parker, J. Geophys. Res. 72, 2315 (1967).

(6) I. Lerche, J. Geophys. Res. 72, 5295 (1967).

(7) I. Lerche and E. N. Parker, Ap. J. 150, 731 (1967).

cant. This is because of the fact that $\Delta B/B \simeq (v/c) \ll 1$, when $v \ll c$. Then any mixing of the plasma and field can result only from the instability existing at the interface layer.

As to the problem of stability, we also expect that the situation would be different from the nonrelativistic one. This is based on the fact that a 'hot' plasma is highly possible to yield their energy to the electromagnetic disturbance and induce instabilities. The hot plasma instabilities have a great variety; e.g., the fire-hose instability, the two-beam instability, which are of hydrodynamic type, and the streaming instability⁽⁸⁾, the magnetosonic⁽⁹⁻¹¹⁾ instability, which are of resonant type, etc.. Although these kind of instabilities are mainly studied for an infinitely homogeneous plasma, conceivably they can also exist in a finitely confined plasma. The question is whether the boundary effect will enhance or reduce the instability.

As we have mentioned above, the MHD description about the boundary phenomena becomes inadequate as one goes into the structure of the finite transition layer R. This indicates the diminishing of the hydrodynamic effect at the boundary region. Physically, this can be understood by the fact that since the plasma density at that region is rapidly decreasing toward the magnetic field, the collective behavior is hence drastically reduced. The plasma at the transition layer loses its fluid characteristics. On this basis, we assert that the existence of the finite transition layer can retain the plasma stability 'hydrodynamically', i. e., less instability of hydrodynamic type exists.

The story might be different for the resonant instabilities. Across the transition layer, one can easily see that only longitudinal plasma oscillations along the magnetic field will survive. The modes of electromagnetic oscillations across the magnetic field will be 'Quantized'. The latter occurs because the electromagnetic wave must become standing in order that the oscillatory fields vanish asymptotically into the vacuum region. In fact, any perturbation to the system must be the combination of 'unquantized' longitudinal and 'quantized' transverse oscillations—the so-called hybrid modes. We also know that the wave-particle resonance occurs only when the particle's velocity is close to the velocity of the wave traveling. So in the boundary region we expect only those particles moving along the field can have effective resonance with the perturbed wave. The plasma streaming relative to the magnetic field hence becomes the only possibility of inducing instability. The detailed analysis of the instabilities which we have

(8) See, e.g., ref. 4, Chap. 7.

(9) I. Lerche, Ap. J. 147, 689 (1967).

(10) D.G. Wentzel, Ap. J. 152, 987 (1968).

(11) E. Tademaru, Ap. J. 158, 959 (1969).

just discussed in a qualitative manner is complicated and has not yet led to entirely certain results. Nevertheless we may summarize the discussion by inferring that an effective instability at the interface can result from the plasma streaming along the field. With this in mind we claim that while being the cause of nonequilibrium in nonrelativistic plasmas, the motion relative to the field results in the instability in relativistic plasmas. In both cases, enhanced mixing between the plasma and magnetic field occurs although they have different origins.

In conclusion we remark that nonequilibrium and instability constitute the two major processes in the turbulent interface between the plasma and magnetic field. In astrophysical phenomena as we have mentioned in which both relativistic and nonrelativistic components participate, the two physical mechanisms probably play an equal role and combine together to enhance the mixing at the interface.