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Monitoring Structural Changes with Generalized Fluctuation Test

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中文摘要

本文根據一般化波動檢定的原則來分析結構性變化的監測檢定，並且推導出此檢定的極限特性。本文也提出以移動估計值來建立兩種不同的監測檢定，模擬結果顯示監測效果比使用遞迴性的監測檢定效果為佳。

關鍵詞：結構性變化、監測、一般化波動檢定。

Abstract

In this paper we introduce the generalized fluctuation test for monitoring structural changes and establish a result characterizing the limiting behavior of this class of tests. As applications of the generalized fluctuation test, tests based on the maximum and range of the fluctuation of moving estimates are proposed. We also derive the boundary functions for the proposed tests and tabulate simulated critical values. Our simulations indicate that these tests compare favorably with the recursive-estimates-based test by Chu, Stinchcombe, and White~(1996) when a change occurs late.

Keywords: Generalized fluctuation test, Monitoring, Structural Changes.

Introduction

The structural change problem has long been an important research topic in the statistics and econometrics literature. Much research effort has been devoted to tests for structural changes and estimation of change points in a given sample; recent results include, e.g.,Andrews~(1993), Andrews and Ploberger~(1994), Bai~(1994, 1995, 1996),Kuan and Hornik~(1995), and Bai and Perron~(1998). These methods are all ``retrospective" because they are designed to examine what happened in historical data sets. In practice, one may want to monitor new observations to see if a change occurs. Such forward-looking methods are closely related to the sequential test in the statistics literature but receive little attention in econometrics; Chu, Stinchcombe, and White~(1996), henceforth CSW, is an exception.

CSW proposed two tests for monitoring potential structural changes. In their fluctuation test, when new observations arrive, estimates are computed sequentially from all available data (historical sample together with newly arriving data) and compared to the estimate based only on the historical sample. The hypothesis of no change is then rejected if the difference between these two estimates gets too large. This testing procedure aims at detecting a change when new data arrive, whereas the (retrospective) fluctuation test of Ploberger, Krämer, and Kontrus~(1989) checks whether a change exists in historical data. Moreover, the constant critical values for the latter cannot be used for monitoring. The law of iterated logarithm implies that, with probability one, the monitoring statistics would eventually exceed constant boundaries and hence signal a change even when there is none; see e.g., Robbins~(1970). Thus, a challenging task is to find suitable boundary functions such that monitoring tests can maintain proper sizes.

In this paper we introduce a class of tests for monitoring structural changes. This class of tests is referred to as the generalized fluctuation test and includes the fluctuation test of CSW as a special case. We extend Kuan and Hornik~(1995) to establish a general result characterizing the limiting behavior of this test. As applications of the generalized fluctuation test, we propose to monitor changes using the maximum and range of the fluctuation of moving estimates. It is shown that the distributions of these tests are determined by the increments of the generalized Brownian bridge and that the corresponding boundary functions for the proposed tests grow approximately at the rate about square of log(t). By contrast, the boundary functions for the CSW tests grow much faster (approximately at the rate t). As such, the CSW tests are less sensitive to a change occurring late in the monitoring period, but the proposed tests are not. Our simulations confirm that the proposed test indeed detect a late change much more quickly.

Results

Consider the structural change test against β i in the multiple regression model

$y_i = x_i \times \beta_i + \varepsilon_i \quad i = 1,\ldots,T,T+1,\ldots,$

The first result of Theorem 2.1 in

this article indicates that under local alternatives of order T-0.5, a positive homogeneous functional, $\boldsymbol{\lambda}(L_T Y_T)$, has non-trivial local power, where L_T is a linear operator and Y_T is a process calculated from y_i and x_i; the second conclusion says that for non-local alternatives, this statistic diverges and hence is consistent against this class of alternatives. Under the null hypothesis that no structural change occurs,

$\boldsymbol{\lambda}(L_T Y_T) \Rightarrow \boldsymbol{\lambda}(LW)$.

W is a n-dimensional standard Weiner process. The limiting distribution of $\boldsymbol{\lambda}(L_T Y_T)$ is therefore determined by the behavior of (LW).

To illustrate the generalized fluctuation test $\boldsymbol{\lambda}(L_T Y_T)$, we now consider the tests based on recursive estimates and can show that the CSW fluctuation test is a special case of this class of tests.

By Theorem 2.1 in this article and under some technical conditions, the Test Statistics in CSW is,

$$\max RE_T\left({}^{2}\right) = \max_{k=T+1,\ldots,[T{}^{2}]} \frac{k}{)\pm_T\sqrt{T}} \left\| Q_T^{0.5}\left( )_k -)_T \right) \right\|,$$

Under the null hypothesis that no structural changes occur,

Where $\mathbf{W}^0$ is the generalized Brownian bridge on $[0,\infty)$, as shown by CSW. For a suitable boundary function of $\mathbf{W}^0$, the

$$\max RE_T\left({}^{2}\right) \Rightarrow \max\left(L\,W;\left[1,{}^{2}\right]\right) = \max\left(W^0;\left[1,{}^{2}\right]\right)$$

limiting distribution of $\max RE_T(\tau)$ is thus determined by the boundary-crossing probability of $\mathbf{W}^0$ on [1, τ]. From the discussion above we can see that numerous monitoring tests can be constructed by choosing suitable operators and/or functionals. The limiting behaviors of the tests so constructed can be easily characterized by Theorem 2.1.

One important defect of the fluctuation test in CSW is that it requires longer time to detect a change occurring late in the monitoring period. A major reason is that its boundary function grows too fast during the monitoring period. We find that this disadvantage could be greatly improved if we use moving estimates to calculate the test statistics. Intuitively, statistics derived from the moving estimates will react to the change more quickly than the one from CSW since we abandon some "out of date" data.

For example, the fluctuation test of CSW now could be re-written as

$$\max ME_{T,h}\left({}^{2}\right) = \max_{k=T+1,\ldots,[T{}^{2}]} \frac{[Th]}{)\pm_T\sqrt{T}} \left\| Q_T^{0.5}\left(\tilde{}_T\left(k,[Th]\right) -)_T \right) \right\|,$$

The first term in the norm is a moving OLS estimates computed from a

$$\max ME_{T,h}\left({}^{2}\right) \Rightarrow \max\left(L_h W;\left[1,{}^{2}\right]\right)$$

constant window size [Th] in contrast to the one in CSW computed from all historical samples. Under the null hypothesis,

Where $L_h\mathbf{W}(t) = \mathbf{W}_0(t) - \mathbf{W}_0(t-h)$, which is the increments of the Brownian bridge. Also, $L_h\mathbf{W}(t)$ is a stationary Gaussian process. This is in sharp contrast with $\mathbf{W}_0$ which has a growing variance.

From Révész ~(1982) we get that the

asymptotic growth rate of the supremum of the increments of the Wiener process is square of log(t) almost surely. Hence proper boundary functions with constant hitting probability over time for the increments of the Brownian bridge should grow approximately at this rate (at least for large t). We show that one can indeed construct boundary functions using the asymptotic growth rate.

Concluding Remarks

Although we observed from our simulations that the moving-estimates-based tests with a smaller window size can detect a change more quickly, it remains unknown how small the size should be. In fact, it would be of great interest if one can find a window size that is optimal in the sense of shortest detection delay. Another open question is monitoring schemes for nonlinear models. The current approach cannot be used directly because there is no closed form solution for the parameter estimates. These topics are currently under investigation.

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