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A New Point-of-View at Plastic Materials of the Prandtl-Reuss Type

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ABSTRACT

A very efficient integration strategy for constitutive relations of a class of plastic materials is proposed, and therefrom a new point-of-view is developed that a rate-type constitutive law may be superseded by an integral constitutive law.

INTRODUCTION

For plastic materials obeying von Mises' yield condition, the normality flow rule and isotropic hardening rule, the Prandtl-Reuss equation is used for their constitutive description. Basically, a rate-type point-of-view is taken therein and rate quantities are employed in the formulation. The usual analysis approaches require numerical integration of the rate constitutive equations over a discrete sequence of time steps. Marcal(1965) proposed the tangent stiffness-radial return method; Rice and Tracey(1973) modified it and presented the secant stiffness method, applicable only to elasto-perfectly plasticity. Krieg and Krieg(1977) proposed the radial return method and Nagtegaal(1982) extended it to account for material hardening behavior. Schreyer et al.(1979) proposed the elastic predictor-radial corrector method, which was initially given by Mendelson(1968). A review of the methods will not be included here, however. The aim of this paper is to show that for the Prandtl-Reuss material the stress can be expressed as an integral over some function of strain rate and that a single material parameter suffices to describe the whole deformation history. The presented strategy applies to not only isotropic but also kinematic or mixed hardening rules. Numerically the proposed integral equation makes it possible a very efficient integration strategy involving no iterations and suitable for large as well as small deformation. In contrast to the rate constitutive law the proposed integral relation may represent a class of materials slightly larger than the rate-type materials and imply a new theoretical and experimental point-of-view.

FORMULATION

Our model problems include associative J_2 plasticity and non-linear mixed hardening rules which are specified by a hardening parameter $\kappa(\bar{\epsilon}^p)$ and a plastic tangent modulus $h'(\bar{\epsilon}^p)(\equiv dh/d\bar{\epsilon}^p)$ for the back stress, both of which depend on the history of plastic deformation as expressed through the effective plastic strain scalar $\bar{e}^p$ given by the growth law

$$\dot{\bar{e}}^p = \sqrt{\frac{2}{3}\mathbf{d}^p : \mathbf{d}^p} \tag{1}$$

where $\mathbf{d}^p$ is the plastic strain rate and ( ) : ( ) denotes the trace of the martix product. The yield criterion is given by

$$\begin{aligned} f &\equiv \frac{3}{2}\xi : \xi - \kappa^2(\bar{e}^p) = 0 \\ \xi &= S - \alpha \end{aligned} \tag{2}$$

where S and α denote the deviator of the Cauchy stress σ and the back stress (center of yield surface) respectively. The evolution equations are given as:

$$(i)\ \textit{associative flow rule} : \mathbf{d}^p = \dot{\lambda}\,\xi\,; \tag{3}$$

$$(ii)\ \textit{hypoelasticity} : \dot{S} = 2G(\mathbf{d}' - \mathbf{d}^p)\,; \tag{4}$$

$$(iii)\ \textit{kinematic hardening rule} : \dot{\alpha} = \frac{2}{3}h'(\bar{e}^p)\mathbf{d}^p\,; \tag{5}$$

$$(iv)\ \textit{pressure}\ P : \dot{P} \equiv -\frac{1}{3}\mathrm{tr}(\dot{\sigma}) = -K\,\mathrm{tr}(\mathbf{d})\,, \tag{6}$$

where $\dot{\lambda}$ is a scalar parameter, G and K are shear and bulk moduli, $\mathbf{d}'$ is the deviator of the total strain rate $\mathbf{d}$, and $\mathrm{tr}(\cdot)$ denotes the trace of a tensor. The effective stress $\bar{\sigma}$ is defined as

$$\bar{\sigma} = \sqrt{\frac{3}{2}\xi : \xi} \tag{7}$$

and is equal to $\kappa(\bar{e}^p)$ in the present model when material is on yielding.

By the use of eqs.(1),(2),(3) and (7),

$$\dot{\lambda} = \frac{3\dot{\bar{e}}^p}{2\kappa(\bar{e}^p)} \tag{8}$$

and

$$\mathbf{d}^p = \frac{3\dot{\bar{e}}^p}{2\kappa(\bar{e}^p)}\xi\,. \tag{9}$$

Substracting eq.(5) from eq.(4) and using eq.(3) give

$$\dot{\xi} + (2G + \frac{2}{3}h')\dot{\lambda}\xi = 2G\mathbf{d}' \tag{10}$$

The solution is

$$\xi = \xi_0 + 2Ge^{-\Phi(t)}\int_0^t e^{\Phi(\tau)}\,\mathbf{d}'(\tau)\,d\tau \tag{11}$$

in which

$$\Phi = \int_0^t (2G + \frac{2}{3}h')\dot{\lambda}\, d\tau = \int_0^{\bar{e}^p(t)} \frac{3G + h'(\bar{e}^p)}{\kappa(\bar{e}^p)}\, d\bar{e}^p \tag{12}$$

where we have used eq.(8) and assumed $\bar{e}^p(0) = 0$. The back stress α and pressure P are obtained by time integration of eq.(5) and eq.(6):

$$\alpha = \alpha_0 + \int_0^{\bar{e}^p(t)} \frac{h'(\bar{e}^p)}{\kappa(\bar{e}^p)} \xi(\bar{e}^p)\, d\,\bar{e}^p \tag{13}$$

and

$$P = P_0 - K \int_0^t \mathrm{tr}(\mathbf{d}(\tau))d\tau \tag{14}$$

in which ξ obtained from eq.(11) can be expressed as a function of $\bar{e}^p$ since $\bar{e}^p$ is a monotonic function of time t during the process of plastic deformation. In the above, ξ_0, α_0, and P_0 are initial constants. The Cauchy stress σ can then be formulated as

$$\sigma = \sigma_0 + K \int_0^t \mathrm{tr}(\mathbf{d}(\tau))d\tau\mathbf{1} + 2Ge^{-\Phi(t)} \int_0^t e^{\Phi(\tau)}\mathbf{d}'(\tau)d\tau + \int_0^{\bar{e}^p(t)} \frac{h'(\bar{e}^p)}{\kappa(\bar{e}^p)}\xi(\bar{e}^p)\, d\,\bar{e}^p \tag{15}$$

where $\sigma_0 = -P_0\mathbf{1} + S_0$ and $\mathbf{1}$ indicates the second order unit tensor. Furthermore, if no plastic deformation has occurred, Φ is reduced to zero and eq.(15) just describes an elastic response.

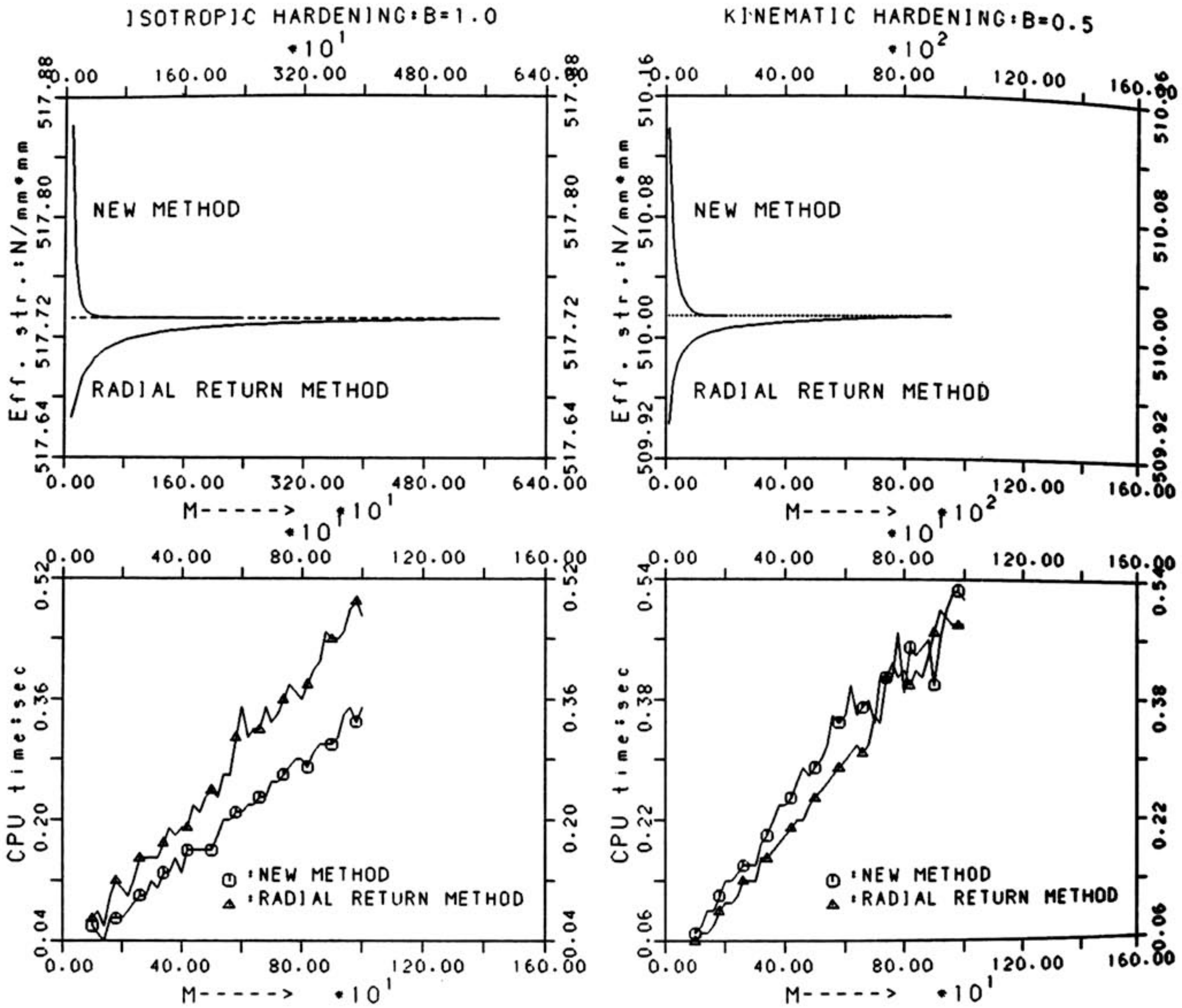
At this stage the key point to continue this problem becomes how to determine the history of effective plastic stain $\bar{e}^p$ from a known total strain rate within a time increment. The answer is apparent and all we have to do here is to enforce the consistency condition at the end of each time increment. Mathematically, by substituting eq.(11) into eq.(2), we have the relation

$$\kappa^2(\bar{e}^p(t)) = \frac{3}{2}\left(\xi_0 + 2Ge^{-\Phi(t)}\int_0^t e^{\Phi(\tau)}\,\mathbf{d}'(\tau)\,d\tau\right) : \left(\xi_0 + 2Ge^{-\Phi(t)}\int_0^t e^{\Phi(\tau)}\,\mathbf{d}'(\tau)\,d\tau\right) \tag{16}$$

by which is established the relation between $\bar{e}^p(t)$ and $\mathbf{d}(t)$ over the time interval [0,t].

EXAMPLE

Consider a plane strain problem where at $t = t_n$ the deviatoric stress is $(S_{11}, S_{22}, S_{12})_n$ $=(210,-90,20)MPa$ for isotropic hardening ($\beta = 1$) and $(S_{11}, S_{22}, S_{12})_n$ $=(230,-100,40)MPa$, $(\alpha_{11}, \alpha_{22}, \alpha_{12})_n = (20,-10,20)MPa$ for kinematic hardening ($\beta = 0.5$). The strain rate deviator is $(d'_{11}, d'_{22}, d'_{12}) = (1/3, 1/3, 1/2)\tau, \tau = t - t_n$. The hardening behavior is characteried by $\kappa(\bar{e}^p) = 1000\,(0.000146 + \bar{e}^p)^{1/3}\, MPa$ and $h(\bar{e}^p) = (1 - \beta)\,\kappa(\bar{e}^p)$. G is taken as $14070MPa$. The final result at $\tau = 0.5$ sec is shown in the figures. The number M represents the number of subincrements for the radial return method presented by Simo and Taylor(1985) or that of integration points for the new method. It is seen that the present method requires fewer points and much less CPU time in a VAX8650 system. The CPU time consumed for increasing one integration point is lower than that for incresing one subincrement for the isotropic hardening case and a little bit higher for the kinematic hardening case. For this example, the total computational time is reduced about 90 percent to the same accuracy.



DISCUSSION AND CONCLUSION

We have proposed an integration strategy for the associative J_2 plasticity with nonlinear mixed hardening rules. The preposition indeed changed the rate equation into an integral one. The integral constitutive law can be at the most worst situation integrated numerically to any accuracy as desired. Illustration and comparison with a well-known traditional approach were made. The integral equation represents a class of materials slightly larger than the rate equation and may be viewed as a guideline for further theoretical and experimental investigations.

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