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斜向波對多孔式港池消波牆影響之研究（1/2）

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ABSTRACT

The analysis of wave field in the harbor has been studied by many investigations. But in all earlier researches, the boundary condition applied on surfaces of porous structures is the strict slipping condition, inducing the paradoxical inference that the wave always permeates through the porous wall in normal direction. By this inference, when the incoming wave is parallel to the porous wall, the unreasonable phenomenon that the wave permeates through the wall completely is inevitable.

To remedy the above paradoxical phenomenon, a new partial-slipping boundary condition on surfaces of the thin porous wall is proposed in the principal investigator’s earlier researches. Moreover, in order to apply this boundary condition in analysis of wave field in the harbor, an analytical solution of oblique impact waves upon a thin porous wall with a vertical bank behind is obtained in the present study.

A simplified reflection coefficient and the boundary condition usable for porous breakwaters surrounding inside the harbor are obtained. And the boundary element method (BEM) is adopted to simulate the wave field along with initial development and test.

The following step of this study is to continuously develop the BEM model for investigating more reasonable physical mechanism of harbor wave fields.

Keywords: *harbor wave fields, thin porous wall, partial-slipping boundary condition.*

1. INTRODUCTION

In earlier researches for analyzing wave fields in the harbor using numerical approaches, the boundary condition applied allows only the normal penetration into the porous medium. Hence, as long as the incoming wave is parallel to the porous wall, an unreasonable phenomenon that waves permeating through completely is inevitable.

Based on the concept that the wave direction remains unchanged as it permeates into the thin porous medium, the principal investigator’s earlier research “Study of paradox of wave, current parallel to a porous wall” (NSC 90-2611-E002-015) (accepted by Journal of Engineering Mechanics, ASCE) proposed a new partial-slipping boundary condition at interfaces of the thin porous wall to eliminate the paradoxical phenomenon that when the incoming wave direction is parallel to the porous wall, the wave permeates completely no matter how large the porosity of the porous breakwater is.

Furthermore, in the principal investigator’s research “Boundary layer analysis of oblique water waves impact on porous walls” (NSC 92-2611-E002-029), viscous effect is considered in analysis of laminar boundary layer on thin porous walls. And it is found that the wave direction changes due to the viscous effect, but it only appears inside the boundary layer similar to the Stokes’ second problem type of flow. Therefore, it is confirmed that the analytical solution applied in our proposed boundary condition on thin porous walls indeed meets the physical expectation.

For further application of our partial-slipping boundary condition on porous structures to analyze wave fields in the harbor, the present study derives the analytical solution of oblique incoming waves impacting on the thin porous wall with an impermeable bank behind. According to the result, a simplified reflection coefficient and the boundary condition for porous structures inside the harbor are thus found.

The boundary element method (BEM) is adopted in the present study for its better simulating accuracy. And our proposed partial-slipping boundary condition is introduced into the BEM model in place of the original slipping condition. The usability of this model has been tentatively proved by preliminary verification.

2. FORMULATION OF OBLIQUE WAVE FIELD

2.1 Governing Equations

The physical domain including 3 flow regions is shown in Figure 1.

The fluid is divided into regions (1) and (2) by a thin porous wall (region (3)) while the water wave propagates from the unbounded far field of region (1), acting on the porous wall with an impermeable vertical bank behind it. For a linear water wave problem investigated in the present study, the fluid is assumed to be inviscid, incompressible, and irrotational.

For region (1) and region (2), we have

$$\underline{V}_j = \nabla \Phi_j, \qquad j = 1,2, \tag{1}$$

where $\underline{V}_j$ and Φ_j are velocity vectors and velocity potentials of fluid, respectively; and the subscript j denotes the flow regions (see Figure 1). And the continuity equation and linear momentum equation are

$$\nabla \cdot \underline{V}_j = 0, \qquad j = 1,2, \tag{2}$$

$$\rho \frac{\partial \underline{V}_j}{\partial t} = -\nabla p_j + \rho \underline{g}, \qquad j = 1,2. \tag{3}$$

By substituting (1) into (2), (2) becomes

$$\nabla^2 \Phi_j = 0, \qquad j = 1,2. \tag{4}$$

Then by substituting (1) into (3) and integrating (3) with respect to y, (3) becomes

$$p_j = -\rho \frac{\partial \Phi_j}{\partial t} - \rho g (y - h), \qquad j = 1,2, \tag{5a}$$

when it is not on the surface of seepage (e.g. see Muskat (1946)); and

$$p_j = 0, \qquad j = 1,2, \tag{5b}$$

when it is on the surface of seepage, where p_j is the pressure of the fluid and ρ is the density of the fluid.

For region (3) that is within the porous wall, we have

$$\underline{V}_3 = \nabla \Phi_3, \tag{6}$$

where $\underline{V}_3$ and Φ_3 are velocity vector and velocity potential of fluid inside the porous wall, respectively (see Figure 1). And, referring to Huang (1991), the continuity equation and linear momentum equation are

$$\nabla \cdot \underline{V}_3 = 0, \tag{7}$$

$$\rho \frac{\partial \underline{V}_3}{\partial t} = -\nabla p_3 + \rho \underline{g} - \frac{\mu n_0}{k} \underline{V}_3, \tag{8}$$

where μ is dynamic viscosity of fluid; n_0 and k are porosity and specific permeability of the porous wall, respectively. By substituting (6) into (7), (7) becomes

$$\nabla^2 \Phi_3 = 0. \tag{9}$$

Then by substituting (6) into (8) and integrating (8) with respect to y, (8) becomes

$$p_3 = -\rho \frac{\partial \Phi_3}{\partial t} - \rho g (y - h) - \frac{\mu n_0}{k} \Phi_3, \tag{10a}$$

when it is not on the surface of seepage; and

$$p_3 = 0, \tag{10b}$$

when it is on the surface of seepage.

2.2 Boundary Conditions

On the bottom (i.e. $y = 0$), the normal velocity equals to zero, i.e., we get

$$\frac{\partial \Phi_j}{\partial y} = 0, \qquad j = 1,2,3. \tag{11}$$

On the free surface of fluid, $y = h$ and $-\infty < x < -d/2$ for region (1) and $d/2 < x < d/2 + L$ for region (2), we have the conventional kinematic and dynamic boundary conditions as

$$\frac{\partial \Phi_j}{\partial y} = \frac{\partial \eta_j}{\partial t}, \qquad j = 1,2, \tag{12}$$

$$\frac{\partial \Phi_j}{\partial t} + g \eta_j = 0, \qquad j = 1,2, \tag{13}$$

where η_j is the vertical deviation of water surface from the mean free surface. By combining (12) with (13), we get

$$\frac{\partial^2 \Phi_j}{\partial t^2} + g \frac{\partial \Phi_j}{\partial y} = 0, \qquad j = 1,2. \tag{14}$$

On the free surface of fluid inside the porous wall, $y = h$ and $-d/2 < x < d/2$, the kinematic and dynamic boundary conditions are

$$\frac{\partial \Phi_3}{\partial y} = n_0 \frac{\partial \eta_3}{\partial t}, \tag{15}$$

$$\frac{\partial \Phi_3}{\partial t} + \frac{\mu n_0}{\rho k} \Phi_3 + g \eta_3 = 0. \tag{16}$$

By combining (15) with (16), we get

$$\frac{\partial^2 \Phi_3}{\partial t^2} + \frac{\mu n_0}{\rho k}\frac{\partial \Phi_3}{\partial t} + \frac{g}{n_0}\frac{\partial \Phi_3}{\partial y} = 0. \tag{17}$$

At the far field for region (1), $0 < y < h$ and $x \to -\infty$, the radiation boundary condition is

$$\lim_{l_0 x \to -\infty} \Phi_1 \to \text{outgoing.} \tag{18}$$

On the bank for region (2), $0 < y < h$ and $x = d/2 + L$, the normal velocity vanishes, i.e.

$$\frac{\partial \Phi_2}{\partial x} = 0. \tag{19}$$

At the interfaces of the porous wall and the fluid, $0 < y < h$ and $x = \pm d/2$, the continuities of mass flux and pressure are considered. From (1) and (6), we have

$$\frac{\partial \Phi_j}{\partial x} = n_0 \frac{\partial \Phi_3}{\partial x}, \qquad j = 1,2, \tag{20}$$

and according to (5) and (10) with surface of seepage effect being ignored for convenience, it is obtained that

$$\frac{\partial \Phi_j}{\partial t} = \frac{\partial \Phi_3}{\partial t} + \frac{\mu n_0}{\rho k}\Phi_3, \qquad j = 1,2. \tag{21}$$

Since the present study focuses on the problem of periodic linear water wave, we may introduce a time factor $e^{-i\omega t}$ into the time-dependent variables, where ω is the angular frequency of water wave. And according to Snell's law, we can write

$$\Phi_j(x,y,z,t) = \mathrm{Re}\left[\phi_j(x,y)e^{i(k_0 \sin\theta z - \omega t)}\right], \qquad j = 1,2,3, \tag{22}$$

In addition, by referring to (8), a flow-efficiency parameter is defined as

$$R = \frac{reac\tan ce \;\; force}{resis\tan ce \;\; force} = \frac{\rho \omega k}{\mu n_0}, \tag{23}$$

which represents the capability to flow through the porous medium (see Huang (1991)). After getting rid of $e^{-i\omega t}$, the time-independent boundary value problem can thus be constructed.

2.3 Partial-Slipping Boundary Condition on Thin Porous Wall

When the porous wall is extremely thin, only the flow fields of region (1) and region (2) are considered, i.e. we don't want to solve for $j = 3$.

Based on the concept similar to that in Beavers and Joseph (1967), obliquely incoming wave does not always permeate through the porous wall in normal direction, but propagates with a certain degree of angle from the normal line. Thus, the partial-slipping condition, instead of slipping condition, is necessary to be considered for oblique water waves impacting on the thin porous wall. Moreover, since the porous wall is extremely thin, the in and out refractive angles may cancel each other. Therefore, we don't need to be concerned about the refractive angle and may regard the wave direction as unchanged when it permeating into the thin porous wall.

For the oblique incoming wave as shown in Figure 2, the flow is in the direction of x' axis and passes through $\underline{x}_1\left(-\frac{l}{2}, y, z'\right)$ and $\underline{x}_2\left(\frac{l}{2}, y, z'\right)$ on the wall. Therefore, it is required to take velocity vectors at $\underline{x} = \underline{x}_1$ and $\underline{x} = \underline{x}_2$ as well. Then, referring to Huang et al. (1993), we get

$$\left.\frac{\partial \phi_3}{\partial x'}\right|_{\left(-\frac{l}{2}, y, z'\right)} = \left.\frac{\partial \phi_3}{\partial x'}\right|_{\left(\frac{l}{2}, y, z'\right)} = \frac{1}{l}\left(\phi_3\Big|_{\left(\frac{l}{2}, y, z'\right)} - \phi_3\Big|_{\left(-\frac{l}{2}, y, z'\right)}\right). \tag{24}$$

During coordinate transformation by an angle and substituting $\frac{\partial \phi_3}{\partial x'} = \frac{1}{\cos\theta}\frac{\partial \phi_3}{\partial x}$ and $l = \frac{d}{\cos\theta}$ into (22), it is obtained that

$$\left.\frac{\partial \phi_3}{\partial x}\right|_{\left(-\frac{d}{2}, y, z_1\right)} = \left.\frac{\partial \phi_3}{\partial x}\right|_{\left(\frac{d}{2}, y, z_2\right)} = \frac{1}{d}\left(\phi_3\Big|_{\left(\frac{d}{2}, y, z_2\right)} - \phi_3\Big|_{\left(-\frac{d}{2}, y, z_1\right)}\right)\cos^2\theta. \tag{25}$$

On the other hand, by the continuous mass flux condition and continuous pressure condition at interfaces, (20) can be rewritten as

$$\left.\frac{\partial \phi_1}{\partial x}\right|_{\left(-\frac{d}{2}, y, z_1\right)} = n_0 \left.\frac{\partial \phi_3}{\partial x}\right|_{\left(-\frac{d}{2}, y, z_1\right)}, \tag{26}$$

$$\left.\frac{\partial \phi_2}{\partial x}\right|_{\left(\frac{d}{2}, y, z_2\right)} = n_0 \left.\frac{\partial \phi_3}{\partial x}\right|_{\left(\frac{d}{2}, y, z_2\right)}, \tag{27}$$

and (21) can be expressed as

$$iR\phi_1\Big|_{\left(-\frac{d}{2}, y, z_1\right)} = (iR-1)\phi_3\Big|_{\left(-\frac{d}{2}, y, z_1\right)}, \tag{28}$$

$$iR\phi_2\Big|_{\left(\frac{d}{2}, y, z_2\right)} = (iR-1)\phi_3\Big|_{\left(\frac{d}{2}, y, z_2\right)}. \tag{29}$$

(29) minus (28) gives

$$iR\left(\phi_2\Big|_{\left(\frac{d}{2}, y, z_2\right)} - \phi_1\Big|_{\left(-\frac{d}{2}, y, z_1\right)}\right) = (iR-1)\left(\phi_3\Big|_{\left(\frac{d}{2}, y, z_2\right)} - \phi_3\Big|_{\left(-\frac{d}{2}, y, z_1\right)}\right). \tag{30}$$

Now, substituting (26), (27), and (30) into (25) to get rid of ϕ_3, the final form of boundary conditions at interfaces are obtained as

$$\left.\frac{\partial \phi_1}{\partial x}\right|_{\left(-\frac{d}{2},y,z_1\right)} = \left.\frac{\partial \phi_2}{\partial x}\right|_{\left(\frac{d}{2},y,z_2\right)} = \frac{n_0}{d}\frac{iR}{1-iR}\left(\phi_1\Big|_{\left(-\frac{d}{2},y,z_1\right)} - \phi_2\Big|_{\left(\frac{d}{2},y,z_2\right)}\right)\cos^2\theta. \tag{31}$$

From Figure 2, it is found that $z = z_1 - \frac{d}{2}\tan\theta = z_2 + \frac{d}{2}\tan\theta$, hence (46) can be rewritten as

$$\left.\frac{\partial \phi_1}{\partial x}\right|_{\left(-\frac{d}{2},y,z+\frac{d}{2}\tan\theta\right)} = \left.\frac{\partial \phi_2}{\partial x}\right|_{\left(\frac{d}{2},y,z-\frac{d}{2}\tan\theta\right)} \tag{31a}$$

$$= \frac{n_0}{d}\frac{iR}{1-iR}\left(\phi_1\Big|_{\left(-\frac{d}{2},y,z+\frac{d}{2}\tan\theta\right)} - \phi_2\Big|_{\left(\frac{d}{2},y,z-\frac{d}{2}\tan\theta\right)}\right)\cos^2\theta.$$

3. ANALYTICAL SOLUTIONS AND NUMERICAL APPLICATIONS

3.1 Analytical Solution of Oblique Wave Field

By applying governing equation (4) and boundary conditions (11), (14), (18), (19), and (31a), the time-independent analytical solution of the irrotational part of flow can be readily obtained as

$$\phi_1(x,y,z) = \phi_0\left[e^{i\xi\left(x+\frac{d}{2}\right)} + A_0 e^{-i\xi\left(x+\frac{d}{2}\right)}\right]\cosh(k_0 y)e^{i\zeta\left(z-\frac{d}{2}\tan\theta\right)}, \tag{32}$$

$$\phi_2(x,y,z) = i\phi_0\frac{1-A_0}{\sin\xi L}\cos\left(\xi\left(x-\frac{d}{2}-L\right)\right)\cosh(k_0 y)e^{i\zeta\left(z+\frac{d}{2}\tan\theta\right)}, \tag{33}$$

with

$$\phi_0 = -\frac{iag}{\omega\cosh(k_0 h)}, \tag{34}$$

$$A_0 = \frac{\xi d(1-iR) - n_0 R\cos^2\theta(1-i\cot(\xi L))}{\xi d(1-iR) + n_0 R\cos^2\theta(1+i\cot(\xi L))}, \tag{35}$$

where $\xi = k_0\cos\theta$, $\zeta = k_0\sin\theta$ and k_0 satisfies the dispersion relation

$$\omega^2 - gk_0\tanh(k_0 h) = 0. \tag{36}$$

By (13), the profile of potential wave field which takes off the incoming wave η_0 in region (1) and wave profile in region (2) are

$$\left.\frac{\eta_1}{a}\right|_{y=h} = A_0 e^{-i\xi\left(x+\frac{d}{2}\right)}e^{i\zeta\left(z-\frac{d}{2}\tan\theta\right)}, \tag{37}$$

$$\left.\frac{\eta_2}{a}\right|_{y=h} = i\frac{1-A_0}{\sin\xi L}\cos\left(\xi\left(x-\frac{d}{2}-L\right)\right)e^{i\zeta\left(z+\frac{d}{2}\tan\theta\right)}. \tag{38}$$

The reflection coefficient C_r and transmission coefficient C_t, most important factors in the analysis of the wave field, can be expressed as

$$C_r = \left|\frac{\eta_1}{a}\right|^2 = \mathrm{Re}(A_0)^2 + \mathrm{Im}(A_0)^2, \tag{39}$$

$$C_t = \left|\frac{\eta_2}{a}\right|^2 = \frac{\mathrm{Im}(A_0)^2}{\sin^2\xi L}\cos^2\left(\xi\left(x-\frac{d}{2}-L\right)\right). \tag{40}$$

For harbor design, if inner sides of the harbor are designated as porous breakwaters, the boundary condition at their interfaces can be simplified as

$$\left.\frac{\partial \phi_1}{\partial x}\right|_{x=-\frac{d}{2}} = -i\alpha\phi_1\Big|_{x=-\frac{d}{2}}, \tag{41}$$

where α is the impedance coefficient. From (32), it can steadily be shown that

$$\alpha = -\xi\frac{1-A_0}{1+A_0}. \tag{42}$$

3.2 Numerical Approach for Harbor Analysis – Boundary Element Method

The analysis of wave field in the harbor has been studied by many investigations, such as Hannoura and McCorquodale (1985), Sulisz (1985), Kobayashi and Jarcobs (1985a, 1985b), Kobayashi et al. (1987), and Lee (1987), etc. Nevertheless, in all earlier researches, the boundary condition applied on surfaces of porous structures is the strict slipping condition of potential theory, which induces an unreasonable inference that the wave will always permeate through the porous wall in normal direction regardless what the incident angle is. By this inference, even though the wave field seems reasonable in most of the physical domain in the harbor, the paradoxical phenomenon that the wave permeates through the porous wall completely must appear when the incoming wave is parallel to the porous wall.

Therefore, in the present study, the partial-slipping boundary condition is applied on the surfaces of porous walls inside the harbor, investigating the wave field and physical properties by using proper numerical approaches for simulation.

From (39), the simplified reflection coefficient of the thin porous wall and a vertical bank impacted by oblique waves is obtained. Furthermore, from (41) and (42), the boundary condition applicable for surfaces of porous walls inside the harbor is found.

Due to the better simulating accuracy (see Figure 3), the boundary element method (BEM) is chosen here for analysis. And the original BEM program with its source codes is provided by Dr. Ching-Ho Su.

At the present stage, the preliminary development of the BEM model has been accomplished. The partial-slipping boundary condition is introduced into the above BEM model and takes the place of the original slipping condition. After further test of this model, the computing result is fairly accepted so far.

4. CONCLUSIONS

By using the partial-slipping boundary condition derived from the concept similar to that in Beavers and Joseph (1967), an analytical solution of oblique water waves impacting on a thin porous wall with a vertical bank behind is obtained in the present study.

Moreover, according to the result of this designed problem, the present study finds out the simplified reflection coefficient and boundary condition applied at interfaces of porous walls surrounding inside the harbor.

The boundary element method (BEM) is designated for analysis of wave fields in the harbor owing to the comparatively better accuracy. And the fundamental construction of the BEM model has been achieved under verification.

The next goal of this study is to use our newly developed partial-slipping boundary condition to investigate wave fields of oblique impact waves on a harbor with thin porous walls inside by applying the BEM model. The result is also expected to be the theoretical basis for optimization of harbor oscillation and elimination of wave energy.

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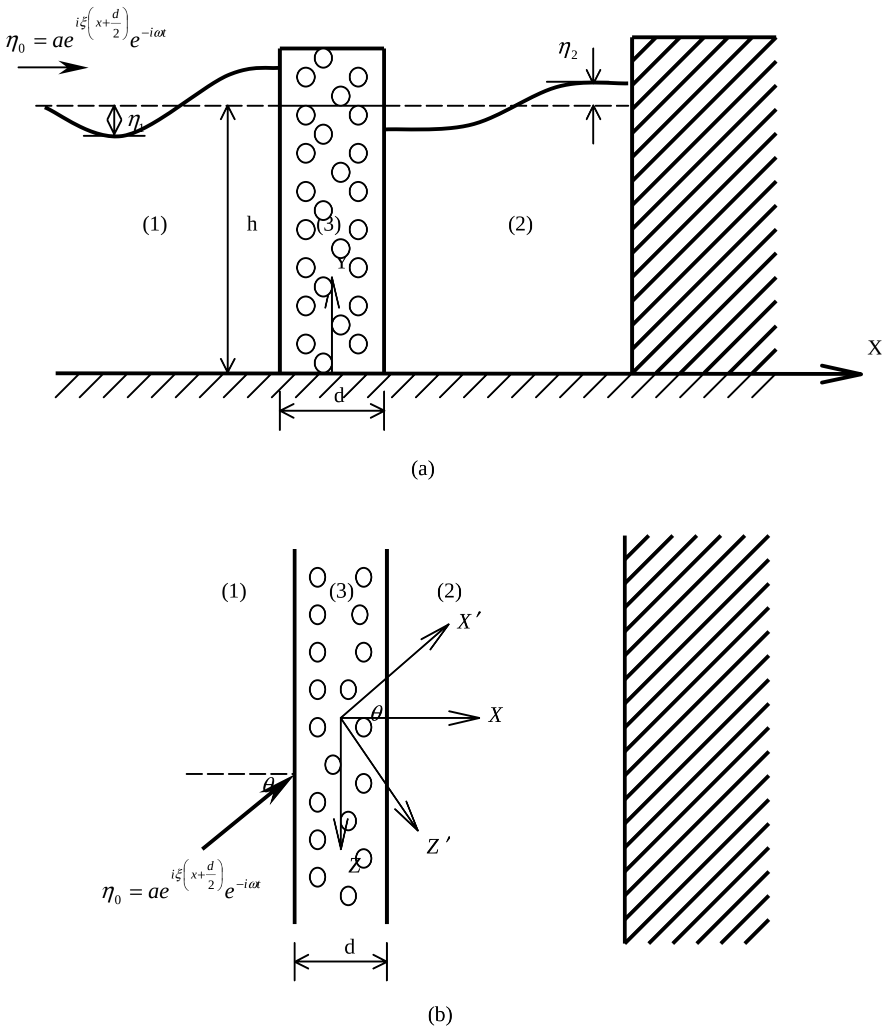

Figure 1 Schematic diagram of oblique water wave impacting on a porous wall and a bank: (a) side view (b) top view

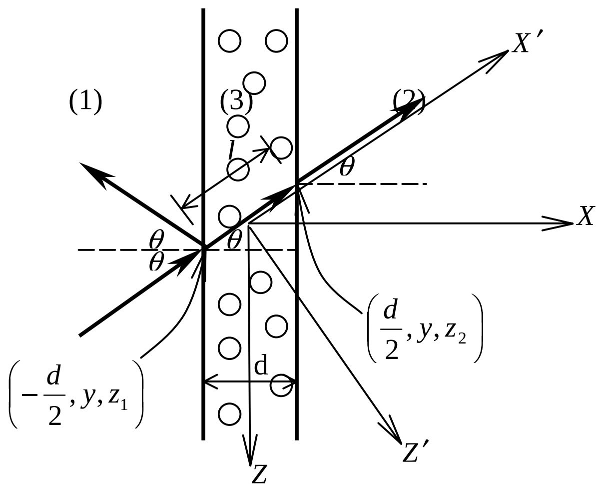

Figure 2 Schematic diagram of oblique water wave impacting on a thin porous wall in the present study

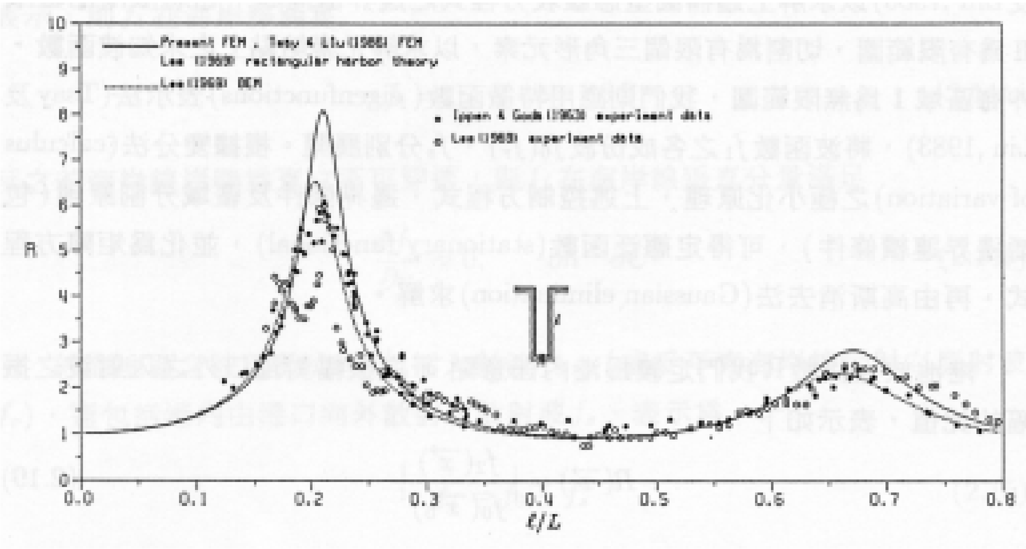

Figure 3 Response curve for a fully open rectangular harbor