

On the transition process of a swirling vortex generated in a rotating tank

Shih-Lin Huang · Hung-Cheng Chen ·
Chin-Chou Chu · Chien-Cheng Chang

Received: 16 July 2007 / Revised: 5 February 2008 / Accepted: 7 February 2008 / Published online: 7 March 2008
© Springer-Verlag 2008

Abstract In this study, we investigate the transition of a swirling vortex from a one-celled to a two-celled vortex structure in a rotating tank. The main idea is to initiate the flow by siphoning fluid out of the tank and then to lift the siphoning mechanism out of the water within a short period of time. Before it reaches a state of quasi-two-dimensionality, the core region of the vortex can be roughly divided into three stages. (1) A siphoning stage induces the formation of the one-celled vortex. (2) A downward jet impingement stage triggers the transition of the vortex into the two-celled one. (3) A detachment stage of the inner cell leads to a cup-like recirculation zone, which is pushed upward by an axial flow from the boundary layer. This eventually develops into a stable quasi-two-dimensional barotropic vortex. The core region is enclosed by an outer region, which is in cyclostrophic balance. In the siphoning stage, the flow pattern can be well fitted by Burgers' vortex

model. However, in the post-siphoning stage, the present data show a flow pattern different from the existing two-celled models of Sullivan and Bellamy-Knights. Flow details, including flow patterns, velocity profiles, and surface depressions were measured and visualized by particle tracking velocimetry and the dye-injection method with various colors. The one-celled and two-celled flow structures are also similar to the conceptual images of the one- and two-celled tornadoes proposed in the literature.

1 Introduction

It has been of great interest to study a columnar, concentrated vorticity structure spinning up from an environment with background vorticity (see for example Lugt 1995). The major significance of such flows is the complex nature between the interaction of the vortex and its surroundings. One of the well-known examples is the so-called bathtub vortex, which emerges when fluid freely drains out of a rotating tank from its bottom. Anderson et al. (2003) presented a very interesting study of the bathtub vortex with increased tank rotation rates. A rapidly twisting bubble jet penetrates from a highly deformed free surface downward into the drain hole. This strongly concentrated vorticity structure thus spawns spirally inward the flow in the thin Ekman layer at the bottom toward the drain.

Another way to generate barotropic vortices in a rotating tank is to withdraw a certain amount of fluid through the free surface. In the review article by Hopfinger and van Heijst (1993) and Nezlin and Snezhkin (1993), several studies of sink-induced quasi two-dimensional vortices have been presented with a focus on their dynamics, instabilities and mutual interactions. These vortices, which

Electronic supplementary material The online version of this article (doi:10.1007/s00348-008-0477-5) contains supplementary material, which is available to authorized users.

S.-L. Huang · H.-C. Chen · C.-C. Chu (✉)
Institute of Applied Mechanics,
National Taiwan University, Taipei 106, Taiwan, ROC
e-mail: chuucc@iam.ntu.edu.tw

C.-C. Chang
Division of Mechanics,
Research Center for Applied Sciences,
Academia Sinica, Taipei 115, Taiwan, ROC
e-mail: mechchang@gate.sinica.edu.tw

C.-C. Chang
Institute of Applied Mechanics and Taida Institute
of Mathematical Sciences, National Taiwan University,
Taipei 106, Taiwan, ROC
e-mail: changcc@iam.ntu.edu.tw

are intrinsically geostrophic or quasi-geostrophic, have been considered as miniatures with which to simulate large-scale geophysical coherent vortices such as oceanic eddies or atmospheric vortices. In addition, several recent articles are concerned with barotropic vortices generated by withdrawing fluid for a short period of time. For example, Trieling et al. (1998) studied the evolution characteristics of monopolar vortices in an irrotational annular shear flow. In order to extend the study of vortex motion to the strongly nonlinear regime, Stegner and Zeitlin (1998) investigated by experiments the behaviors of monopolar vortices in a thin layer of fluid on a rotating parabolic dish. Flór and Eames (2002) investigated the dynamics of a cyclonic monopolar vortex on a topographic beta-plane with both laboratory experiments and theoretical analysis. However, the complicated flow phenomena associated with the transition process of swirling vortices with relatively high Rossby numbers have not been reported yet, especially the transition of a one-celled to a two-celled vortex structure.

In this study, we explore a sequence of flow phenomena in generating a swirling vortex by siphoning a certain amount of fluid in a rotating tank where siphoning plays a pivotal role. The siphoning mechanism initially sucked fluid out of the tank at the volume flow rate, Q , and then was lifted off water within a short period of time. The strength of the generated vortex measured by its maximum azimuthal velocity is linearly proportional to $\sqrt{Qf/h_0}$, where f is the Coriolis parameter and h_0 is the water depth at the central tank. In particular, we investigate the transition from a one-celled to a two-celled vortex structure, which resembles different aspects of tornadoes in nature (e.g., Davies-Jones et al. 2001). The transition can be divided into three stages: (1) A siphoning stage induces the formation of the one-celled vortex. (2) A downward jet impingement stage triggers the transition of the vortex into a two-celled one. (3) A detachment stage of the inner cell leads to a cup-like recirculation zone, which is pushed upward by an up-drafting flow from the boundary layer. The one-celled structure describes the concentrated process with the upward motion of the vortex core in the siphoning stage; however, the two-celled structure denotes the dissipative process, with a downward motion of the vortex core in the post siphoning. In the literature, there are some one-celled and two-celled vortex models without background vorticity. For example, we have a one-celled vortex model in Burgers' vortex (1948), and two-celled vortex models in the steady Sullivan's vortex (1959) and unsteady Bellamy-Knights' vortex (1970). The present vortex-generation mechanism provides a situation for detailed investigation of the transition of a vortex from a one-celled to a two-celled flow pattern. In the siphoning stage, the flow pattern can be well fitted by Burgers' vortex model; however, in

the post-siphoning stage, the present data shows a flow pattern different from the existing two-celled models of Sullivan and Bellamy-Knights due to the different initial and boundary conditions. Flow details related to the transition, including flow patterns, velocity profiles, and surface depressions were measured and visualized by particle tracking velocimetry (PTV) and the dye-injection method with various colors. Two typical cases of vortex evolution with the Rossby numbers $Ro \approx 18$ and 8, respectively, will be presented.

2 Theoretical background and experimental details

2.1 Theoretical background

In order to compare our vortex structures with theoretical models at various flow stages, we review some well-known analytical solutions in the literature. Figure 1 is a schematic of the physical problem in which the cylindrical coordinates (r, θ, z) are used, and the corresponding velocities are denoted by (v_r, v_θ, v_z) . Let $f = 2\Omega$ be the Coriolis parameter, $Ro = U/fL$ be the Rossby number, where U is the characteristic velocity and L is the characteristic length. Let D be the diameter of the siphoning orifice, Q be the volume flow rate, and h_0 be the water depth at the center of the tank under rigid body rotation.

2.1.1 One-celled vortex structure model

(1) Burgers' vortex. The simplest is the steady Burgers' vortex (1948); the solution is given by

$$v_r(r) = -ar, \quad (1)$$

$$v_\theta(r) = \frac{\Gamma}{2\pi r} (1 - e^{-\frac{ar^2}{2\nu}}), \quad (2)$$

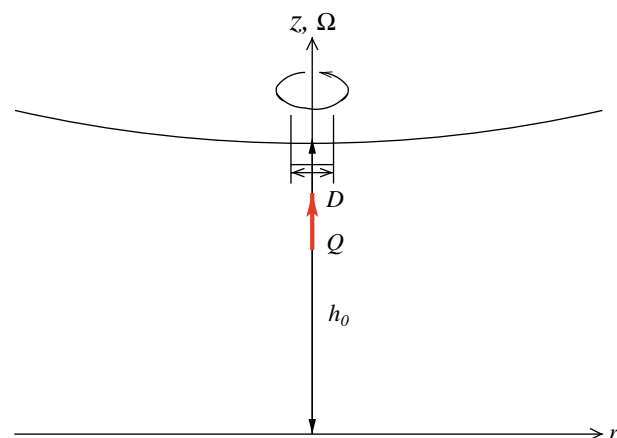


Fig. 1 This figure shows a schematic figure of the experimental model where h_0 is the water depth the center of the rotating tank

$$v_z(z) = 2az \quad (3)$$

where ν is the fluid kinematics viscosity and a is a function of time t that depicts the axially deformed motion. If the axially deformed motion is uniform, $a(t)$ is constant. If normalized by $\bar{v}_\theta = v_\theta/\nu_{\theta_{\max}}$, $\bar{r} = r/r_{\max}$, $\bar{\Gamma} = \Gamma/\Gamma_0$ and $\bar{a} = ar_{\max}^2/2\nu$, we have the azimuthal velocity, v_θ , in the nondimensional form, where $\Gamma_0 = 2\pi\nu_{\theta_{\max}}r_{\max}$ is the reference circulation.

$$\bar{v}_\theta = \frac{\bar{\Gamma}}{\bar{r}}(1 - e^{-\bar{a}\bar{r}^2}). \quad (4)$$

2.1.2 Two-celled vortex structure model

(1) Sullivan's vortex. The steady Sullivan's vortex model (1959) is given by

$$v_r(r) = -ar + 2b\frac{\nu}{r}(1 - e^{-\frac{ar^2}{2\nu}}), \quad (5)$$

$$v_\theta(r) = \frac{\Gamma}{2\pi r}H\left(\frac{ar^2}{2\nu}\right)/H(\infty), \quad (6)$$

$$v_z(r, z) = 2az(1 - be^{-\frac{az^2}{2\nu}}), \quad (7)$$

$$H(x) = \int_0^x e^{-y+b} \int_0^y \frac{1-e^{-q}}{q} dq dy, \quad (8)$$

where $H(x)$ is an integrating function and a, b are constant. Moreover, if $b > 1$, it represents a two-celled structure. Also, the nondimensional form is obtained from the above definition.

$$\bar{v}_\theta = \frac{1}{\bar{r}}H(\bar{a}\bar{r}^2)/H(\infty). \quad (9)$$

(2) Bellamy-Knights' vortex. Additionally, the unsteady Bellamy-Knights' exact solution (1970) is given by

$$v_r = -\frac{3}{b} \frac{r}{\lambda t + \mu} + b(1 - e^{-\frac{r^2}{\lambda t + \mu}}), \quad (10)$$

$$\Gamma = \Gamma_c \frac{H_{b/2\nu}(\eta)}{H_{b/2\nu}(\infty)}, \quad (11)$$

$$v_z = z \left(\frac{2b}{3} \left(\frac{1}{\lambda t + \mu} \right) - \frac{2b}{\lambda t + \mu} e^{-\frac{r^2}{\lambda t + \mu}} \right), \quad (12)$$

where $H_{b/2\nu}(\eta) = \int_0^\eta e^{-y+\frac{b}{2\nu}} \int_0^y \frac{1-e^{-q}}{q} dq dy$, Γ is the circulation and $\eta = r^2/(\lambda t + \mu)$. If normalized, we have the azimuthal velocity, v_θ , in the nondimensional form.

$$\bar{v}_\theta = \frac{1}{\bar{r}}H_{\bar{b}}\left(\frac{-\bar{r}^2}{\bar{t}(1-\bar{b}/3)}\right)/H_{\bar{b}}(\infty). \quad (13)$$

2.2 Experimental details

All the experiments were performed in a rectangular glass water tank with horizontal dimensions of 1.35 m $\times$ 1.35 m

and a height of 0.3 m mounted on a turntable rotating in the positive z -direction at constant angular speed, Ω , from 0.523 to 1.046 s⁻¹, providing an environment with background vorticity. The corresponding Coriolis parameter, f , ranges from 1.046 to 2.092 s⁻¹.

Figure 2 shows a sketch of the experimental apparatus. The facilities, including the vortex generating system, dye injection system, illumination system, and the image recording system, are mounted on the turntable. The depth of the working fluid is 15 cm before the operation of the turntable. However, the free surface is adjusted to a parabolic shape when the fluid reaches the state of rigid-body-rotation. The difference in fluid heights is approximately 2 mm within a 0.3 m $\times$ 0.3 m area of interest in the central portion of the rotating tank. The vortex generating system consists of a siphoning orifice, with an inner diameter 0.75 cm; a water pump; a water reservoir; and the lifting system used to remove the siphoning orifice. As the fluid reaches the state of rigid-body rotation in the rotating tank, the siphoning orifice, which has been inserted into the fluid through the free surface, begins to suck the fluid with flow rates between 40 (without pump) and 160 cm³ s⁻¹ (with pump) for 25 s to generate swirling vortices. When a preset volume of siphoning has been reached, the siphoning orifice is removed automatically by a vertical lifting system. These different flow rates generate vortices with different Rossby numbers, which measures the relative importance of the Coriolis force to the inertia force, under the assumption of a Rankine vortex in the present study.

Flow visualization techniques employed include a dye-injection method, laser-induced fluorescence, and particle seeding illuminated by a sheet of laser light. As shown in Fig. 2, the top-view and side-view images are recorded by high-resolution video camera or by a digital camera. A Particle Tracking Velocimetry (PTV) technique (see Capart et al. 2002) is used to determine the flow velocity by measuring the positions of fluid particles in a sequence of images. The suspended particles, with a diameter of about 50 μ m, are illuminated by a light sheet emitted from a 4-Watt Argon-ion laser, and the images of particles in the flow are recorded by a 1 K $\times$ 1 K high-resolution video camera running at the rate of 30 or 100 frames per second with the exposure time of 3–10 ms. Horizontal laser light sheets are generated at different depths to explore the velocity distribution of the vortex. A vertical light sheet emitted from a 0.2-Watt diode laser mounted on the turntable is also positioned approximately at the vortex core to visualize the flow structures in the meridional plane as well as to estimate the velocity distribution of the up-drafting and down-drafting motions. It should be noted that since the cameras are mounted on the turntable, the recorded images actually show the relative motions of fluid with respect to the rotating reference frames.

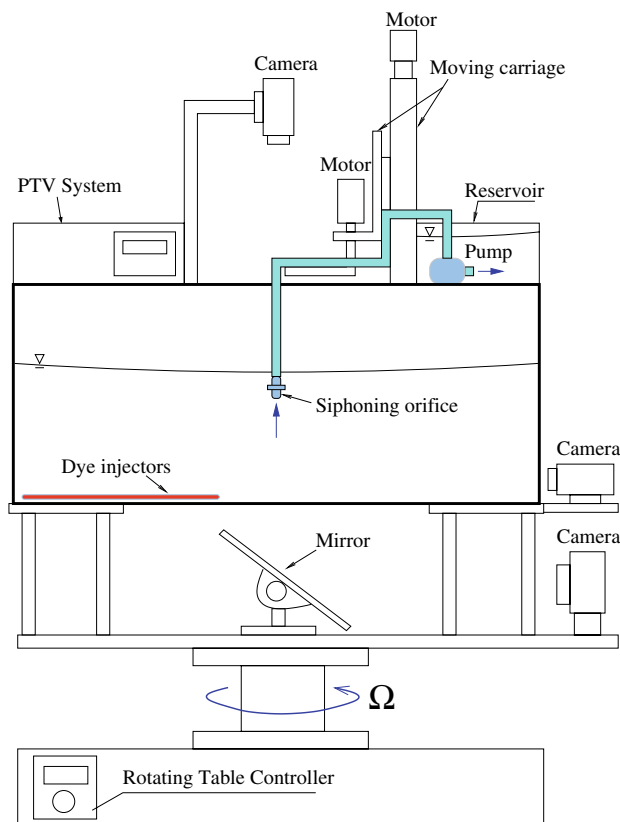


Fig. 2 This figure shows a sketch of the side-view of the experimental apparatus including the rotating tank, the siphoning vortex generator, the dye injectors, and the PTV system with CCD cameras

3 Results and discussion

3.1 PTV measurement and flow visualization

Note that the initial time is counted from the beginning of siphoning. Let $U = v_{\theta_{\max}}$ and $L = r_{\max}$; the Rossby number is now defined as $Ro = U/fL$, and the dimensionless time, $\bar{t}$, is defined as $\bar{t} = t/(L/U)$, where $v_{\theta_{\max}}$ is the maximum azimuthal velocity measured at 1 s prior to the removal of the siphoning mechanism. In the present study, U and L are measured at $\bar{t} = 658$ and 294, and the siphon was removed at $\bar{t} = 685$ and 306 for the cases of $Ro \approx 18$ and $Ro \approx 8$, respectively. To further characterize the flow conditions at all times, we define the time-dependent Rossby number, $Ro_t = v_{\theta_{\max}}(t)/fr_{\max}(t)$, where $v_{\theta_{\max}}(t)$ and its corresponding $r_{\max}(t)$ are measured at time instant t . In the present study, two typical cases of vortex evolution with $Ro \approx 18$ (see Sect. 3.1.1) and 8 (see Sect. 3.1.2) will be described.

3.1.1 Case I with $Ro \approx 18$

Figure 3c shows the nondimensional time evolution of Ro_t and nondimensional core radius, $\bar{r}_{\max} = r_{\max}/L$, in a dimensionless horizontal plane $\bar{z} = 0.19$ for the case where

$Ro \approx 18$, while the distributions of the azimuthal velocity at different times are shown in Fig. 3a and b. The associated vorticity distributions, measured by PTV on the same plane at three different time instants, are also shown in Fig. 3d–f. It is observed that the angular velocities near the core centre increase gradually and attain a maximum value at the end of the siphoning stage; furthermore, they exhibit the concentrations of angular momentum and vorticity. Other than $\bar{z} = 0.19$, PTV measurements were also made in axial planes $\bar{z} = 0.56$ and 0.84; however, similar velocity distributions were observed, as shown in Fig. 4a–c.

It is interesting to understand how the vortex strength in the post-siphoning stages is determined by the initial and background physical conditions. For this, we consider the maximum azimuthal velocity, $v_{\theta_{\max}}$, as a function of the dimensional group of the volume flow rate, Q (by suction), Coriolis parameter, f and depth of water, h_0 . Figure 5 shows the linear relationship of $v_{\theta_{\max}}$ versus $\sqrt{Qf/h_0}$, which also has the dimensions of velocity. This result is quite reasonable, since the swirling vortex would not exist without the background vorticity, and the sucking of more fluid and shallower water should produce a stronger vortex. The agreement was found to be quite good when one of the cases was measured. The larger value of the dimensional group, $\sqrt{Qf/h_0}$, is given, implying that a stronger vortex will be generated. Moreover, the approximated line tends to be steep, which implies a fast decay with the evolutionary time. On the contrary, the line tends to be gentle and approaches the x -axis. Indeed, the relationship is no longer linear as the flow evolves into the detachment stage of the inner cell. In the present study, the linear relationship shown in Fig. 5 can roughly be maintained up to $t = 40$ s. Nevertheless, this linear relationship is still reliable in predicating the initial maximum velocity of the generated vortices.

Another characteristic of the post-siphoning stage is that the surface depressions are also measured by digitizing the free surface at different times. Figure 6 also shows the case for $Ro \approx 18$, where the depression decreases very rapidly in the early stage of post-siphoning roughly from $\bar{t} = 822$ to $\bar{t} = 1,096$, and significantly less rapidly from that moment to a later stage at $\bar{t} = 1,370$. However, in the siphoning stage the siphoning orifice significantly affects the free surface, causing the measurements to be inconsequential.

Figure 7 presents a sequence of the side-view flow visualization of the swirling vortex with $Ro \approx 18$. In the experiment shown in Fig. 7, two dyes were injected at different time stages from the bottom boundary plate. A red dye was injected first to visualize the one-celled structure, followed by a green dye to visualize the two-celled structure. Both the dyes were controlled precisely by a timer; therefore, fascinating phenomena were observed in

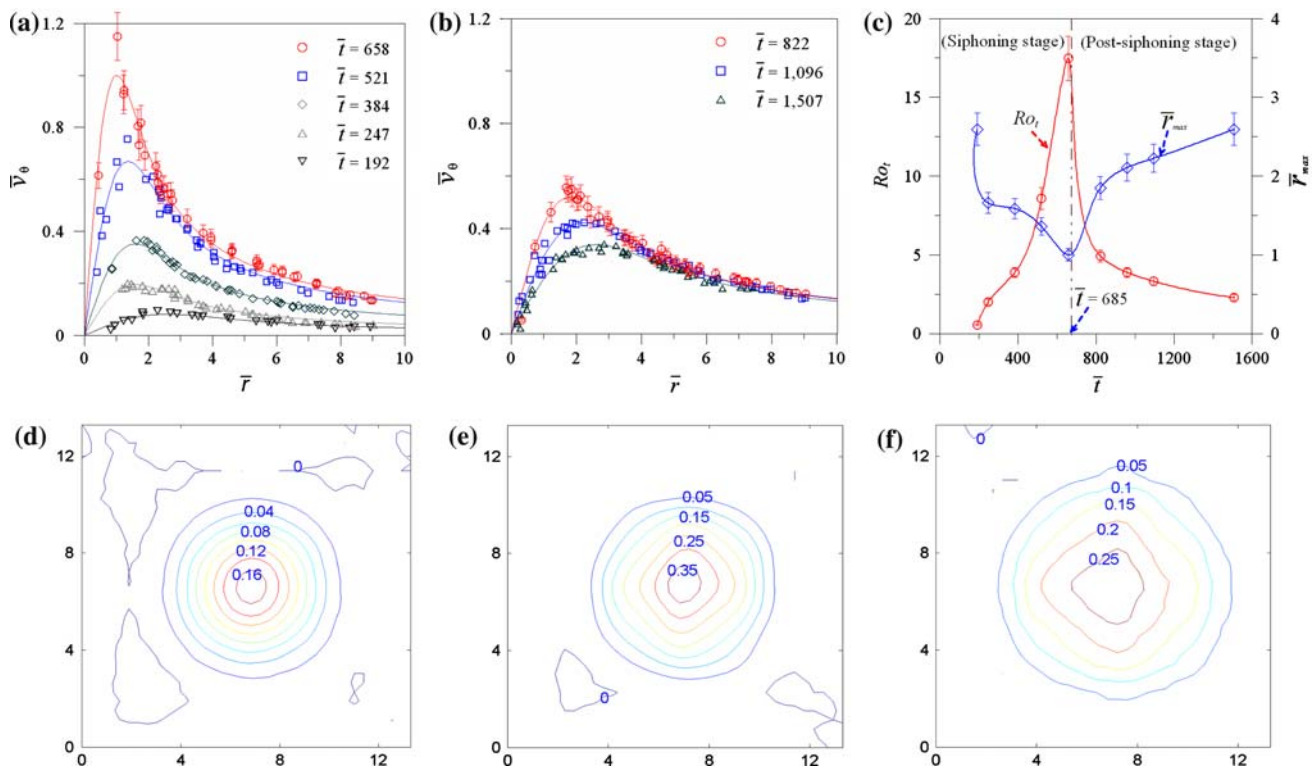


Fig. 3 The results of PTV measurement of velocity distributions in a dimensionless horizontal plane $\bar{z} = 0.19$ ($\bar{z} = z/h_0$) from the tank bottom plate of the vortex at different time instants: **a** before the siphoning orifice is removed; and **b** after the siphoning orifice is removed. The flow conditions are the same as those of Fig. 6. **c** shows the evolution of time correspondent Rossby number, Ro_t , while the

corresponding dimensionless core radii, $\bar{r}_{max}$ ($\bar{r}_{max} = r_{max}/L$, $L = 1.11$ cm and $U = 30.44$ cm/s), are also shown. Error bars shown in **a**, **b** and **c** are plotted according to the root mean square of each measured data. **d–f** express the nondimensional vorticity distributions at $\bar{t} = 247, 521$ and 822

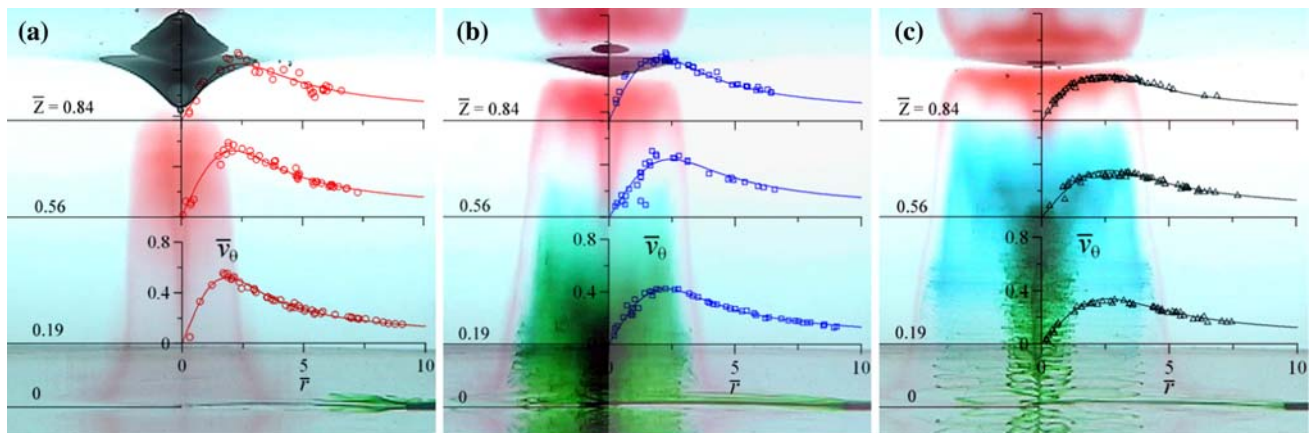


Fig. 4 The results of PTV measurement of velocity distributions in the different horizontal plane of $\bar{z} = 0.19, 0.56$ and 0.84 . **a–c** express, for $Ro \approx 18$, the nondimensional velocity distributions of three

different heights at $\bar{t} = 822, 1,096$ and $1,507$ respectively, overlaying the photo pictures of the flow visualization at the specific times

the swirling vortex over the course of the entire process, from the beginning to the end. Based upon the results of the PTV measurements and flow visualization, as shown in Figs. 3 and 7, three main stages are observed during the evolution of the swirling vortex in the core region before it

reaches a quasi-two-dimensional vortex: (1) a siphoning stage, as shown in Fig. 7a and b; (2) a downward jet impingement stage, as shown in Fig. 7c–g; and (3) a detachment stage of the inner cell, as shown in Fig. 7h–j. Figure 8 shows the detailed three-dimensional sketches

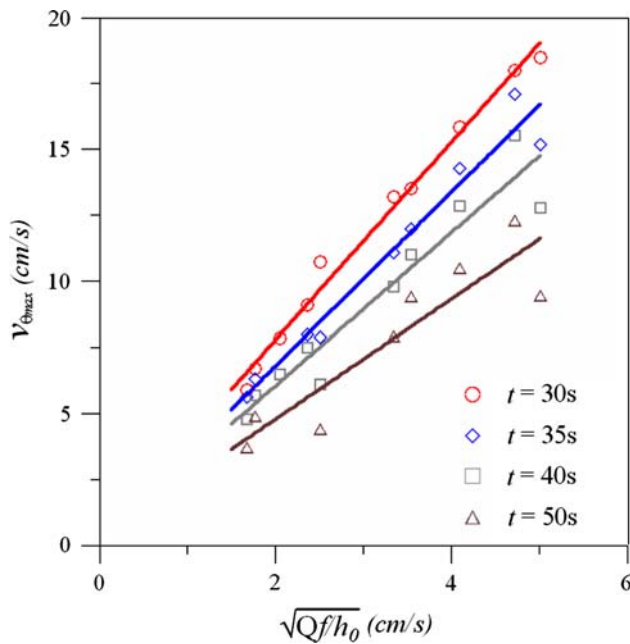


Fig. 5 The relation between $v_{\theta_{max}}$ and $\sqrt{Qf/h_0}$ at different times, where t is counted from the beginning of siphoning, and the siphoning mechanism is lifted off the water at $t = 25$ s

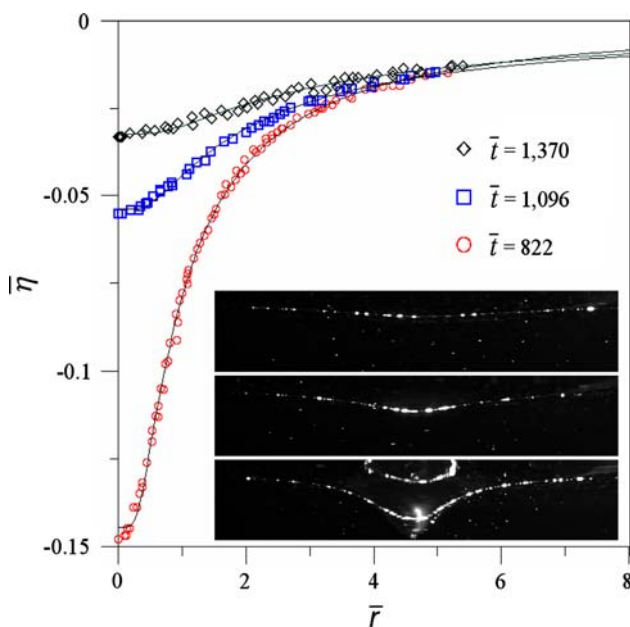


Fig. 6 This figure shows the surface depressions at different times. The flow visualization is obtained by illuminating a vertical laser light sheet on the meridional plane. The dimensionless parameter $\bar{\eta} = \eta/h_0$, where η is the maximum surface depression and $\bar{\eta} = 0$ means the free surface profile before suction under the condition of rigid body rotation

that qualitatively depict the flow structures corresponding to the different stages shown in Fig. 7. Each stage will be described in greater detail in Sect. 3.2.

3.1.2 Case II with $Ro \approx 8$

In this study, a relatively weaker swirling vortex was also investigated to compare the flow behaviours with those associated with the stronger one, $Ro \approx 18$. Figure 9 shows a sequence of photos of a swirling vortex with $Ro \approx 8$ obtained using two-colour flow visualizations. The red dye is injected in the same manner as that shown in Fig. 7, while the yellow dye is released from a stainless-steel tube mounted on the siphoning orifice, where the outlet of the dye-injector is approximately $\bar{z} = 0.49$ below the orifice. In particular, this dye-injector was purposely inserted into the core region to identify (1) the two-celled vortex structure under a relatively low pressure difference between the vortex core and the ambient flow; and (2) the recirculation zone shown in Fig. 8f, which develops in the later stage of vortex detachment. In the downward jet impingement stage, the inner cell can be observed by the fluid marked by the yellow dye, as shown in Fig. 9b–d, while the lower portion of the outer cell marked by the red dye is also clearly observed in Fig. 9b–d. Finally, in the detachment stage of the inner cell, the detachment of the inner cell as well as the formation of the cup-like recirculation zone are observed in Fig. 9h–j. Moreover, the flow phenomena associated with these three stages of the vortex evolution are similar to those shown in Fig. 8. Figure 10 shows the results of the PTV measurement of the velocity distributions in a horizontal plane at a height of $\bar{z} = 0.19$ from the tank bottom plate of the vortex at various time instants, while Fig. 10c shows the decrease in $\bar{r}_{max}$ in the siphoning stage. The increase in $\bar{r}_{max}$ in the inner cell detachment stage is similar to that shown in Fig. 3c for the case of $Ro \approx 18$. The PTV measurement of vorticity distributions in the same horizontal plane at three different time instants are also shown in Fig. 10d–f. Although the intrusive method used in the flow visualization shown in Fig. 9 may disturb the development of the air core after the removal of the siphoning orifice, we have demonstrated that at this Rossby number, $Ro \approx 8$, all the flow behaviours remain essentially similar to those presented in the case of $Ro \approx 18$. Therefore, in the following section, only the case of $Ro \approx 18$ will be discussed with regard to the three stages of vortex evolution in the core region of the swirling vortex.

3.2 Three stages of vortex evolution

3.2.1 The siphoning stage

Flow visualization by the particle-tracking technique on a horizontal plane, as shown in Fig. 11, is used to estimate the thickness of the boundary layer during the later stage of siphoning or at $\bar{t} = 521$. Figure 11a shows an image

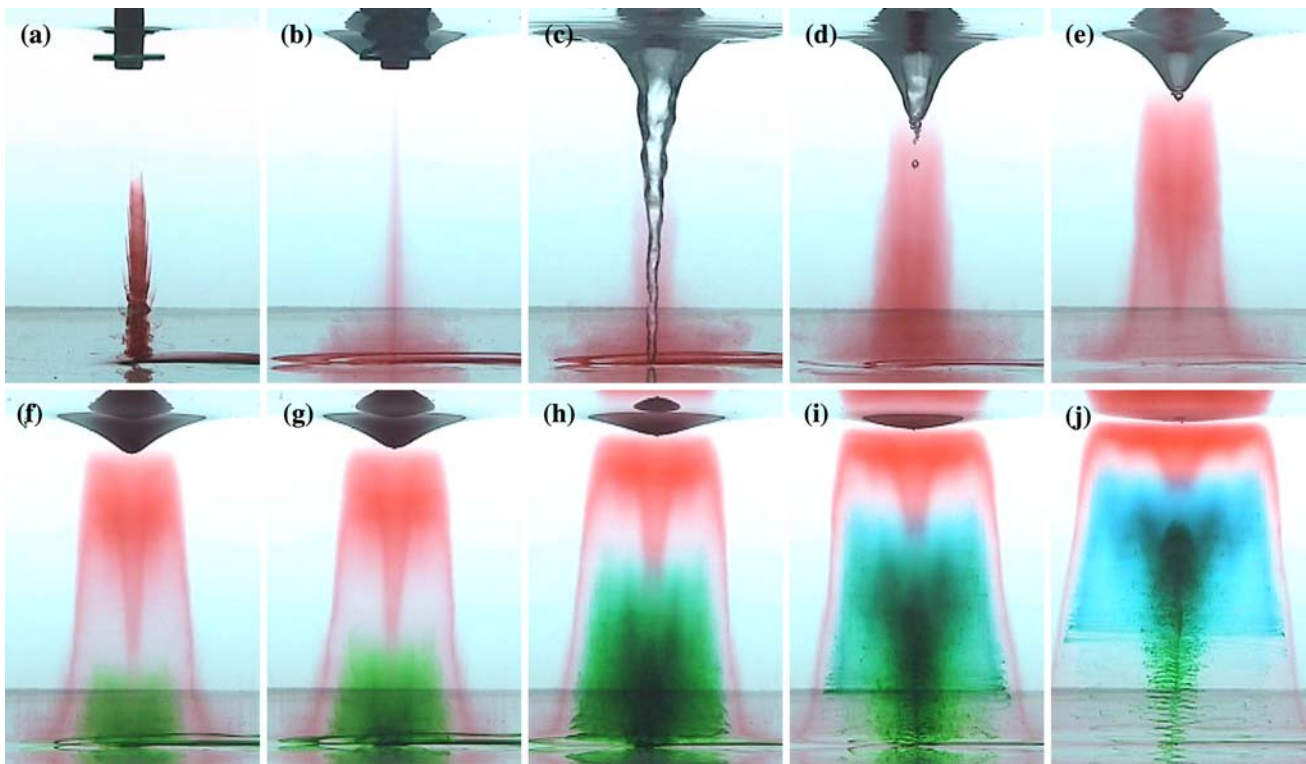


Fig. 7 This figure shows a sequence of the side-view snapshots of the swirling vortex with the $Ro \approx 18$, where Ro is measured at the time instant $\bar{t} = 658$ or at 1 s prior to the siphoning orifice is removed. The dimensionless time instants are respectively: **a** 356, **b** 658 (the siphoning stage); **c** 685, **d** 726, **e** 767 **f** 858, **g** 904 (the downward jet impingement stage); and **h** 1,110, **i** 1,274, **j** 1,605 (the detachment

stage of the inner-cell). The time instants are counted from the beginning of the siphoning stage. It should be noted that some shadows are also shown in both the very top and bottom portions of images due to reflections. The whole sequence of movie is shown in the Supplementary Video 1

located 2 mm ($\bar{z} = 0.015$) above the bottom plate, which is apparently above the boundary layer since most particles in this image encircle the core region. In contrast, the image shown in Fig. 11b is positioned 1 mm above the bottom plate ($\bar{z} = 0.0075$) and exhibits the characteristic of the boundary layer, since the particles are observed to spirally flow into the core region. The thickness of the horizontal light sheet is less than 1 mm. Therefore, the thickness of the boundary layer is estimated to be around 1.5 mm. This is close to that estimated using the Ekman layer thickness, $\delta = \sqrt{2\nu/f} \approx 1$ mm. Due to the occurrence of violent rotations, as described above, the intense inward flow is constrained in the thin boundary layer; however, at a small distance from the boundary layer, this inward flow is insignificant. Due to the strong shear flow over the bottom plate, a substantial vorticity is generated that is convected by the intense inward flow, and is thus concentrated in the vortex core. Figure 12 shows the velocity component in the radial direction at different times on a horizontal plane that is 1.5 mm above the bottom plate ($\bar{z} = 0.011$). In the siphoning stage, it is apparent that the inward flow strengthens towards the vortex centre. However, near and

inside the core, it is difficult to obtain the velocity data due to the strong axial flow perpendicular to the measurement plane. Figure 13 shows the time evolution of the axial velocity, $\bar{v}_z = v_z/U$, near the core axis at elevations $\bar{z} = 0.19, 0.48$ and 0.78 , where the flow conditions are the same as those shown in Figs. 3 and 7. In the siphoning stage, the acceleration of the axial velocity near the core axis is apparent. In general, the velocity is greater at a higher elevation at a certain time instant. This is consistent with the observations in Fig. 14e or f, where the core region, marked by the green colouring, becomes thinner as the elevation increases. With the present suction rate, the measured axial velocity during the later stages of siphoning is approximately $\bar{v}_z = 10$. The concentrated vorticity will be enhanced rapidly through the strengthening by this forced upward flow in the central core. Therefore, this suction mechanism will cause a vigorous and concentrated vorticity. The estimated flow rate of the liquid sucked into the pipe-like structure from the boundary layer at the later stage of siphoning is approximately 50% of the total suction rate at the orifice. This estimation is based on the 1.5 mm thickness of the boundary layer measured at a

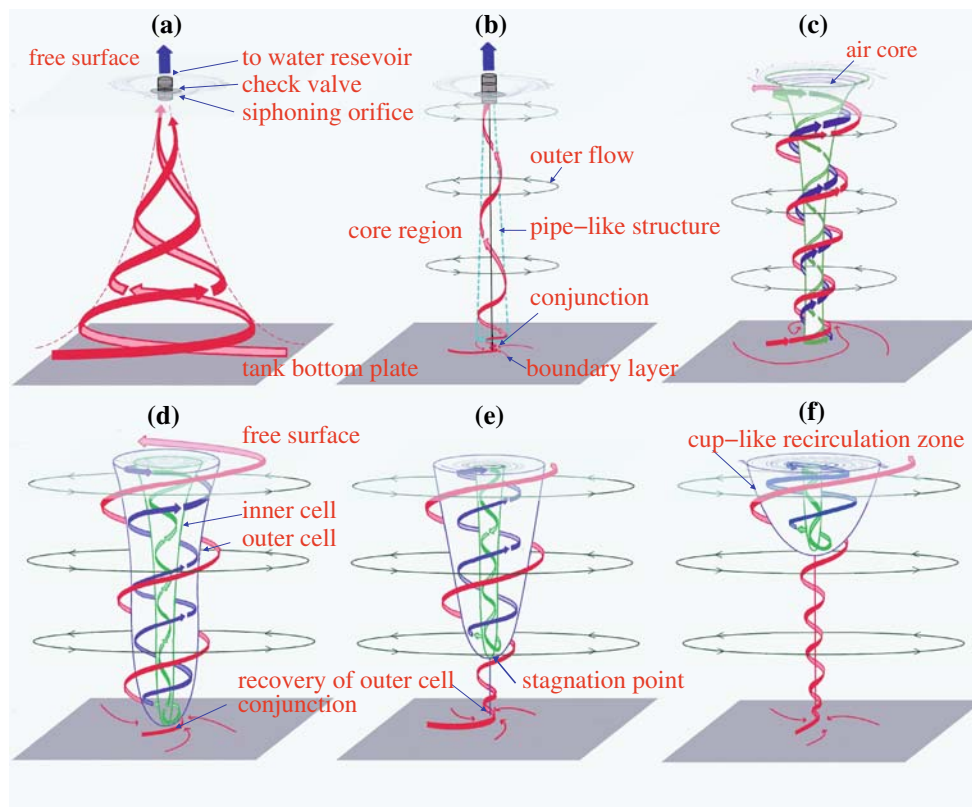


Fig. 8 This figure shows the sketches depicting the 3-D conceptual flow structures corresponding to different stages shown in Fig. 7. They are **a** the early stage of siphoning; **b** the one-celled structure;

c the downward jet impingement stage; **d** the two-celled structure, **e** the detachment stage of the inner cell; and **f** the quasi-two-dimensional vortex

distance of 5 cm ($\bar{r} = 4.5$) away from the centre of the core. The rest of the fluid sucked into the orifice was from the region near the orifice.

Figure 14 shows a sequence of dye-injection images taken under the same conditions as those in Fig. 7 in order to identify the formation process of the one-celled structure in the core region. Before siphoning (see Fig. 14a), red colouring is freely released beneath the free surface close to the siphoning orifice, while green colouring is also pre-released at the bottom of the tank. Once the siphoning commences, the red colouring is sucked rapidly into the orifice (see Fig. 14b). Due to the Coriolis force, the fluid moves cyclonically around the orifice and is then sucked into the orifice. This strong suction-driven motion initially occurs near the orifice and rapidly extends downwards to cover the entire fluid layer, as indicated by the concentrically rotating colouring sheets (see Fig. 14b–f). Furthermore, it has been shown that unlike in the non-rotating environment, the momentum transports effectively from top to bottom in a rotating environment. Additionally, the strong inward flow in the boundary layer (depicted by the green colouring) that is driven by the imbalanced inward pressure-gradient-force near the vortex center ejects radially outward and then turns vertically upwards to erupt

into an intense helical flow to form the core region of the one-celled structure (see Fig. 14c–f). The inward fluid in the boundary layer takes the form of a high-velocity, rapidly swirling jet that immediately jumps to a new diameter with lower axial and swirl velocities. In fact, the supercritical eruption of the boundary layer fluid transits to a subcritical state through the vortex jump phenomenon, which was previously discussed in Maxworthy (1972, 1982). Figure. 14g shows a close-up of the eruption near the bottom plate of the tank.

Although the Rossby number exists in the present study in the background vorticity, a relatively high Rossby number reduces the importance of the Coriolis effect in the core region. The evolution of the interface of the core region and outside cyclostrophic region resembles an upside-down funnel shrinking from the region close to the orifice, as indicated in Figs. 8a and 15a, which then propagates towards the bottom of the tank to form a pipe-like structure, as shown in Figs. 8b and 15b–d. The images shown in Fig. 15 show the tracks of the particles on the meridional plane that are illuminated by a sheet of laser light. The image shown in Fig. 15a was obtained by overlapping 30 frames, while those shown in Fig. 15b and c were obtained from 10 frames; the image shown in

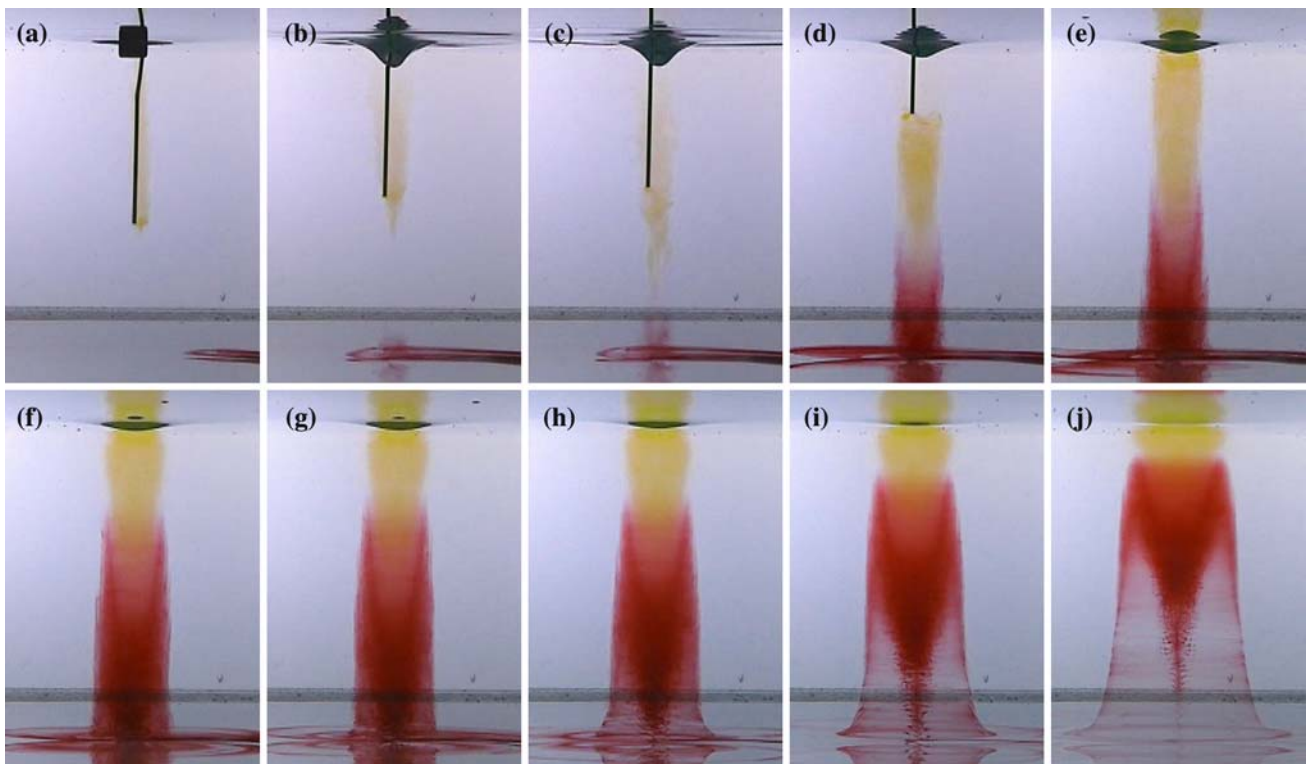


Fig. 9 The snapshots of a swirling vortex with $Ro \approx 8$ at time instants: **a** $\bar{t} = 306$ (the later stage of siphoning); **b** $\bar{t} = 312$, **c** $\bar{t} = 314$, **d** $\bar{t} = 328$, **e** $\bar{t} = 352$, **f** $\bar{t} = 377$ (the downward jet impingement

stage); and **g** $\bar{t} = 389$, **h** $\bar{t} = 413$, **i** $\bar{t} = 475$, **j** $\bar{t} = 621$ (the detachment stage of inner cell). The whole sequence of movie is shown in the Supplementary Video 2

Fig. 15d was obtained in a single snapshot. The flow conditions of Fig. 15 are the same as those shown in Fig. 7, illustrating the formation process of the pipe-like structure. The vortex genesis is due to the pre-existing background vorticity in the entire tank of fluid, allowing it to be easily rotated and transported downward until it reaches the bottom plate. The core will be further intensified by a mechanism that is very similar to the effect of the dynamic pipe described in Trapp and Davies-Jones (1997). The radius of the core decreases while the Rossby number increases in the siphoning stage, as indicated in Fig. 3c. As shown in Fig. 16a, the nondimensional azimuthal velocity, $\bar{v}_\theta$, exhibits a solid body rotation for the small core region in addition to a potential vortex outside the core. These results are very similar to those of the steady Burgers' vortex (1948). Further, we use the steady Burgers' exact solution in the nondimensional form to compare our measured results for the one-celled vortex at different time instances. The fitting curve is similar to the Gaussian function; however, the circulation, Γ , and axially deformed motion, a , must be determined first. Due to the continuous siphoning in this stage, the one-celled vortex will gradually increase its circulation, and its comparison with Burgers' vortex reveals close agreement (see Fig. 16b).

3.2.2 The downward jet impingement stage

Recall that the siphoning stage produces the upwelling one-celled vortex, which rotates counterclockwise and is surrounded by an environment that is in approximately cyclostrophic balance. In the downward jet impingement stage, the major phenomenon is the formation of the two-celled vortex structure. The formation process can be summarized briefly. Immediately after the removal of the siphoning mechanism, the pressure difference induces a downwards jet with an air core, piercing the central part of the one-celled vortex and impinging upon the bottom of the tank. The air core is soon filled up with fluid from the top such that the pierced-through one-celled vortex is folded inward at the free surface, flowing downwards in the core. This results in an inner vortex. The remaining part of the original one-celled vortex spirals upward round the inner vortex and is now called the outer cell. A two-celled vortex is thus formed.

In the downward jet impingement stage (see Fig. 7c–g), as soon as the siphoning orifice is lifted off the water, a whirlpool-like downward jet stream with an air core (denoted as the *inner cell*) suddenly forms and rapidly pierces the existing vortex core or the *outer cell*, resulting

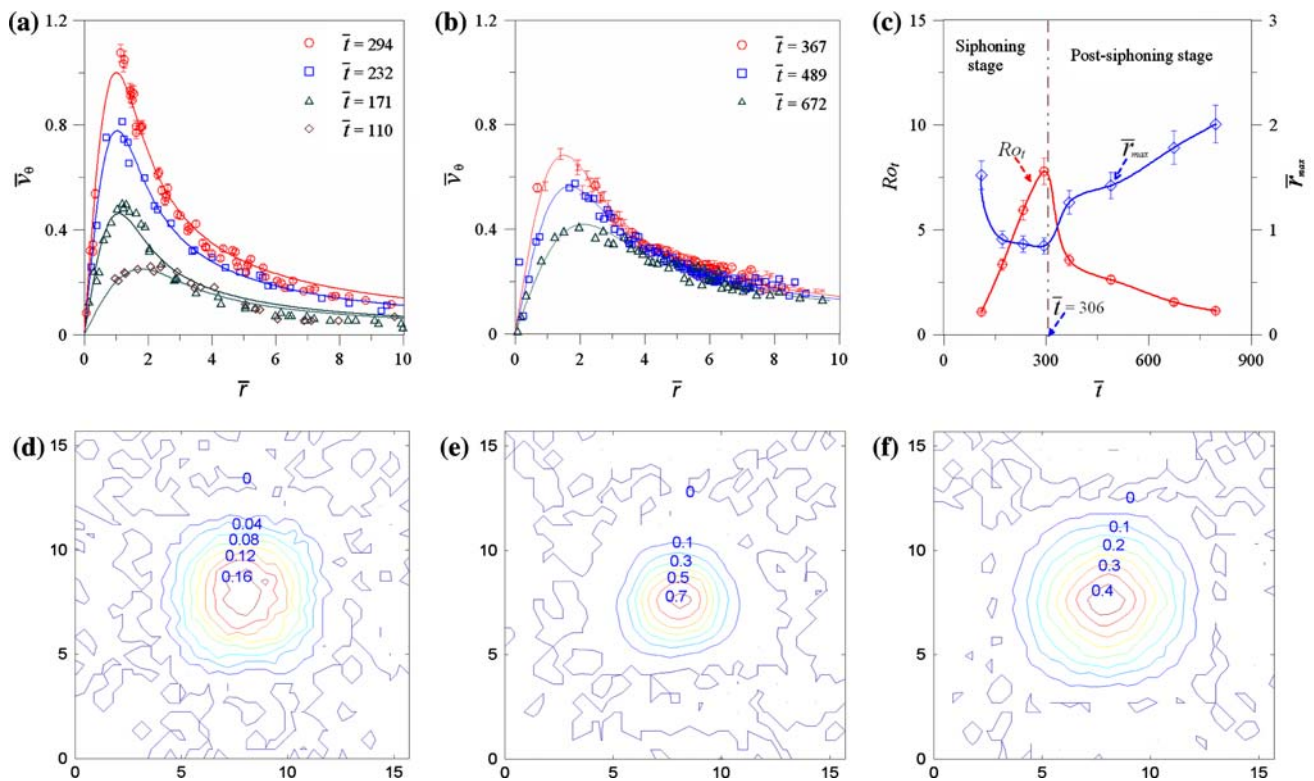


Fig. 10 The results of PTV measurement of velocity distributions in a horizontal plane of $\bar{z} = 0.19$ from the tank bottom plate of the vortex at various time instants: **a** before the siphoning orifice is removed; **b** after the siphoning orifice is removed; and **c** the evolution

of $\bar{r}_{max}$ and Ro_t ($L = 0.94$ cm and $U = 12.50$ cm/s), where the error bars associated with each measurement point is also shown in the figure, **d–f** express the nondimensional vorticity distributions at $\bar{t} = 110, 232$ and 367 respectively

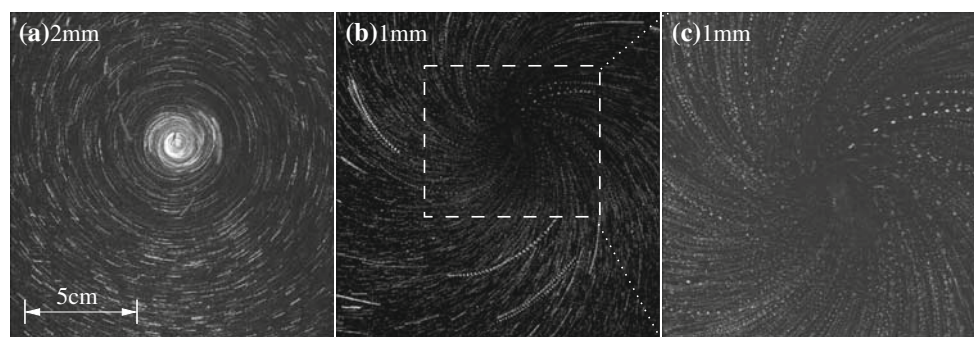


Fig. 11 This figure shows the boundary layer during the later stage of siphoning. **a** Image located at 2 mm or $\bar{z} = 0.0015$, above the bottom plate, which is apparently outside the boundary layer, since particles in this image encircle the core region, and **b** positioned at 1 mm

above the bottom plate, which shows the characteristic of boundary layer, as particles are observed to spirally flow into the core region. This image was obtained by overlapping 30 frames of successive images over 1 s. **c** is the close-up of **b** in the *dashed area*

in the formation of a two-celled vortex structure, as shown in Fig. 8c and d. The inner cell is formed due to the downward pressure–gradient force. Once the siphoning orifice is lifted off the water, low pressure is maintained at the bottom of the tank due to the cyclostrophic balance, which no longer exists at the top. Therefore, the air core, along with the inner cell, is actuated from above. As shown in Fig. 13, the downward velocity of the tip of the air core is $\bar{v}_z \approx 10$ in the moment of downward impingement. The

momentary downward bubble jet results in a long-lasting downward motion. The air core is then elevated from the bottom plate due to the decrease in the downward pressure–gradient force as a result of the outward diffusion of the angular momentum. The initial upward velocity of the air core is $\bar{v}_z \approx 1$, and it abruptly decreases to $\bar{v}_z \approx 0.03$ within $\Delta\bar{t} = 55$. Although the siphon has been removed, the updraft of the outer cell, which arises from the converging boundary layer, continues to pump the fluid

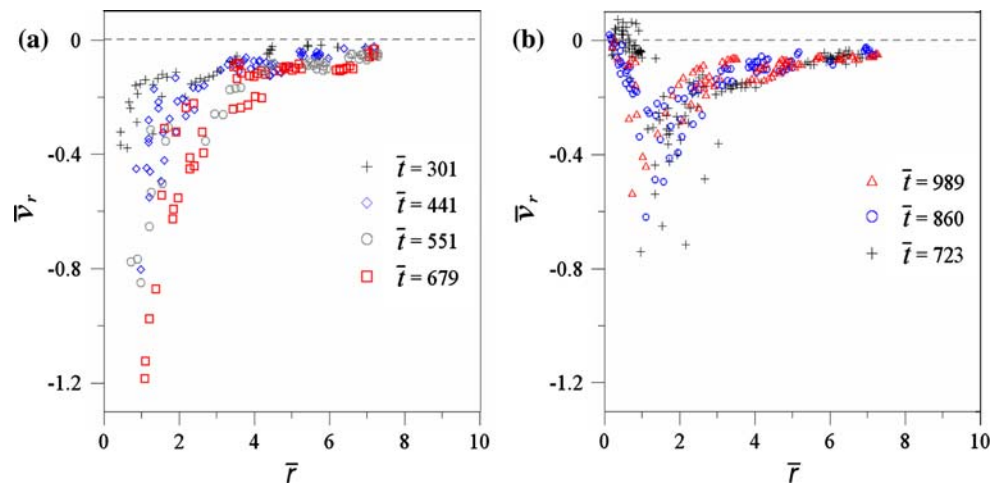


Fig. 12 This figure shows the radial velocity distribution in the boundary or $\bar{z} = 0.011$ above the bottom plate, at different times, **a** in the siphoning stage; and **b** in the post-siphoning stage

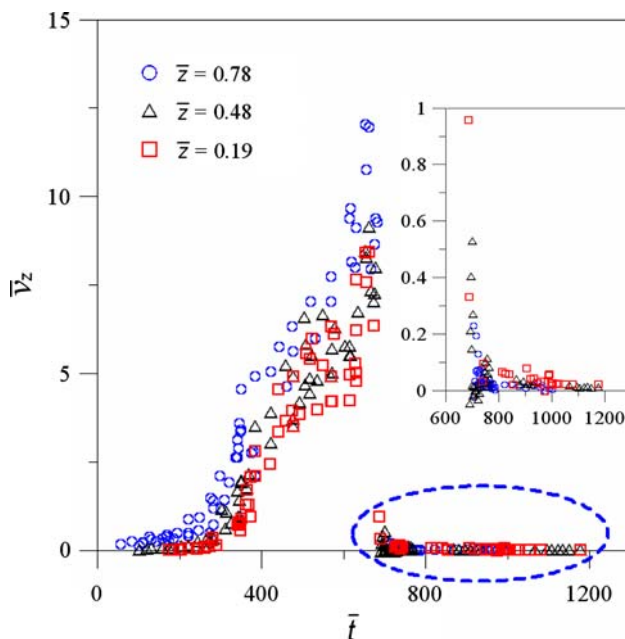


Fig. 13 The vertical velocities are measured near the core axis at different times and different elevations

upwards. However, the inward flow in the boundary layer cannot extend into the inner cell region; instead, it spirals, as shown in Fig. 8c and d.

Both the inner and outer cells have the same sense of rotation as that of the background vorticity, as shown in Fig. 8c and d. Figure 17 shows a sketch of the meridional cross-section corresponding to those shown in Fig. 8c and d. The inner cell can be considered to be the inward-folding of the outer cell moving upwards against the free surface. Since the vortex core is instantaneously enlarged by the air bubble, the maximum azimuthal velocity shown in Fig. 3b

decreases suddenly. Even if the air bubble is lifted away, there is a short period of time during which the two-celled vortex structure is most visible, accompanying a weak downward motion. However, this motion will soon change into an upward motion in a short time. Although the longitudinal stretch caused by the lifted air bubble will increase the vortex strength, it is still dominated by the vorticity diffusion and decays gradually. Figure 13 shows that at a height of $\bar{z} = 0.48$ from the bottom plate, the velocity near the core axis is negative (downward) from $\bar{t} = 712 - 767$. With regard to the upward motion of the outer cell after the air bubbles are lifted away from the bottom plate, the upward velocity can be up to $\bar{v}_z \approx 1$, following which it abruptly decreases to $\bar{v}_z \approx 0.16$ within $\Delta\bar{t} = 82$; it then slows down further to $\bar{v}_z \approx 0.07$ at $\bar{t} = 822$, and gradually to $\bar{v}_z \approx 0.03$ at $\bar{t} = 1,096$. Most of the updraft in the outer cell flows outward near the free surface, as shown in Fig. 8d, after the air bubbles are lifted away from the bottom plate. The outward flow in the boundary layer at $\bar{t} = 723$ and $\bar{t} = 860$, as shown in Fig. 12b, also confirms the characteristics of the two-celled structure, as shown in Fig. 17b. This also roughly indicates the detachment of the inner cell.

In order to confirm the two-celled vortex structure, we performed a bottom-view flow visualization by releasing a fluorescent dye from the bottom plate. Figure 18 presents the images on a horizontal plane, which is at a height of $\bar{z} = 0.48$ above the tank bottom plate; it is illuminated by a horizontal laser light sheet emitted from a 1-W argon-ion laser. It should be noted that the fluorescent dye was released 2 s after the removal of the siphoning orifice, or roughly at $\Delta\bar{t} = 55$. The bright portion shown in Fig. 18a–c indicates the region of the outer cell arising from the boundary layer fluid, which extends upwards towards the

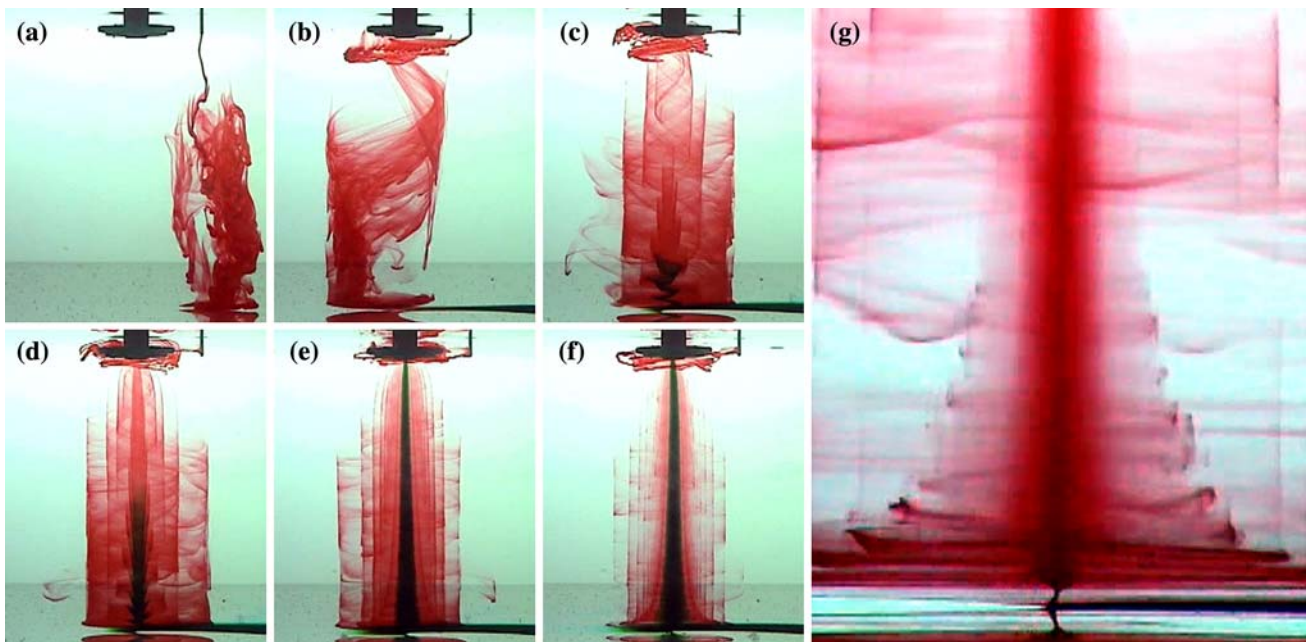


Fig. 14 The snapshots of the formation of the swirling vortex in the siphoning stage. The flow conditions are the same as those of Fig. 7, except the location of dye-injection. The red coloring is injected near the orifice underneath the free surface, while the green coloring is released near the vortex center at the bottom of the tank. A certain

amount of red coloring has been pre-injected into the water tank as shown in **a** before the siphoning is turned on. The nondimensional time instants are respectively: **a** 0, **b** 137, **c** 301, **d** 392, **e** 419, and **f** 521 counted from the beginning of siphoning. **g** A close-up of the ‘eruption’ near the bottom plate

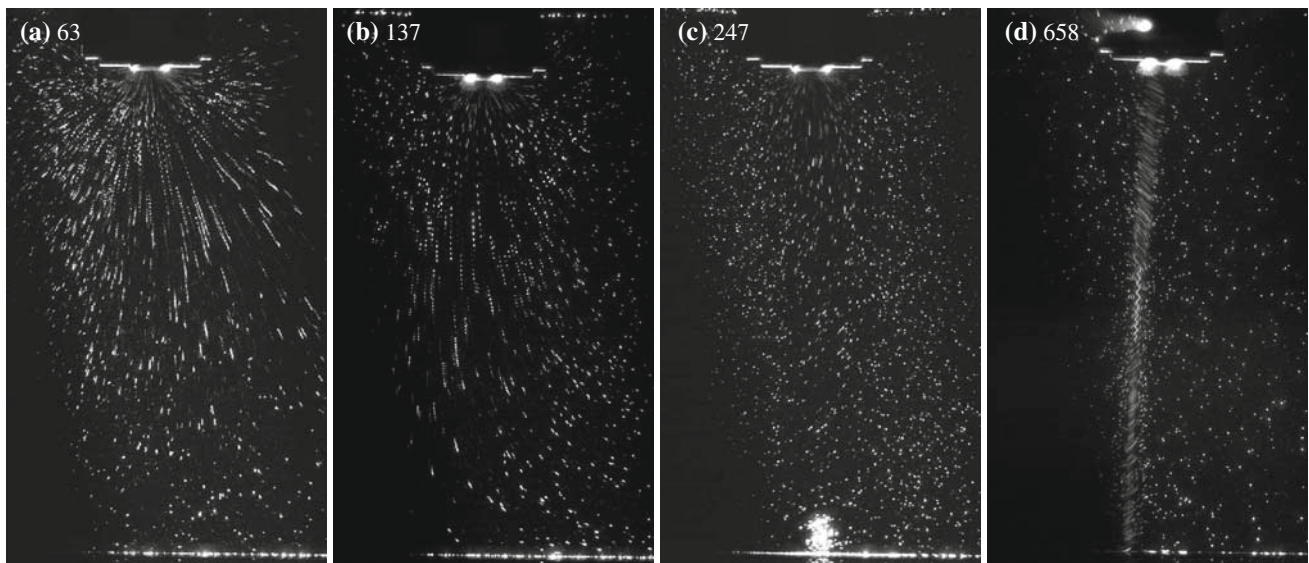


Fig. 15 The image shown in **a** was obtained by overlapping 30 frames, while those shown in **b** and **c** were obtained from 10 frames; the image shown in **d** was obtained in a single snapshot

free surface, while the central dark portion or the fluid without fluorescent dye marks the region of the inner cell that is pushing downwards against the bottom plate. Simultaneously, the outer flow remains in the cyclostrophic balance encircling the core region (the inner cell plus the outer cell) of the vortex. Although the external siphoning mechanism is removed, the updraft of the outer cell does not

cease due to the inertia of the swirling vortex. Therefore, the two-celled structure can be maintained for a short time.

Figure 19 shows the comparison between the present measured data and the steady (2.9) and unsteady (2.13) exact two-celled solutions of Sullivan (1959) and Bellamy-Knights (1970), respectively. The initial time is set at $\bar{t} = 822$, and the corresponding nondimensional parameter is

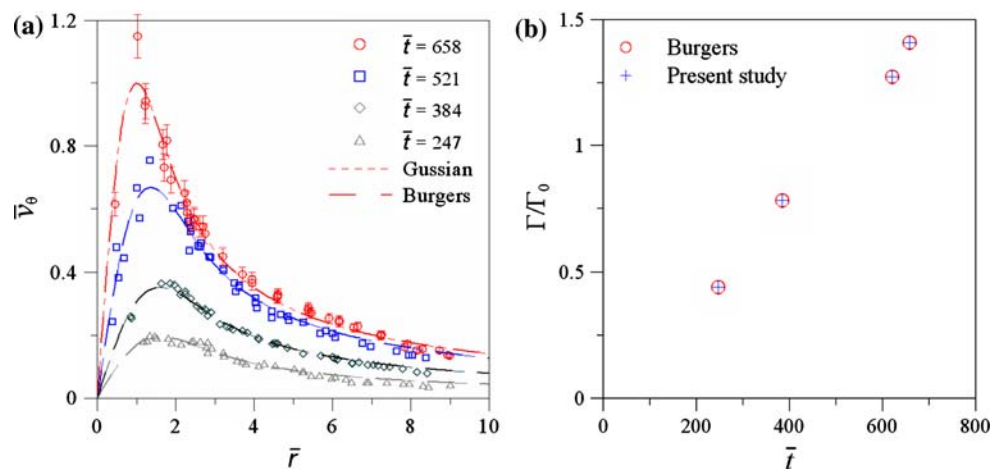


Fig. 16 **a** Comparing to the Burgers' one-celled solution, and **b** the relation between circulation and time

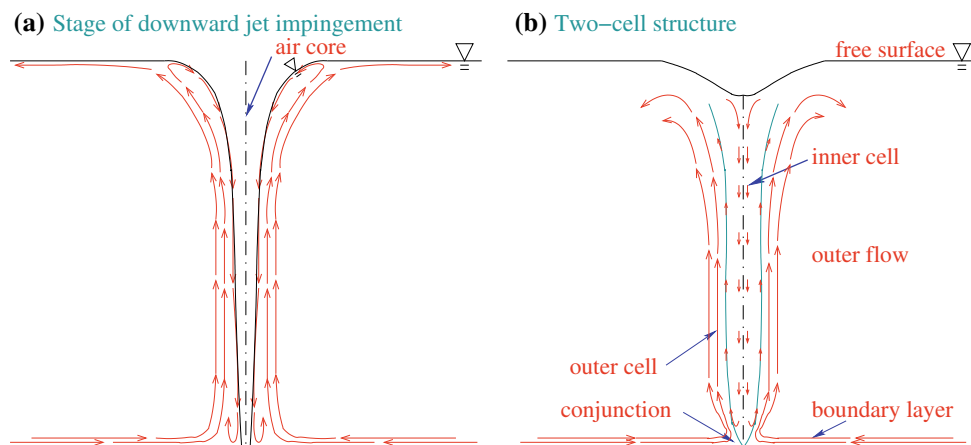


Fig. 17 The sketches show the early stage of downward jet impingement in the meridional plane, where **a** and **b** are corresponding to Fig. 8c and d

determined and maintained constant. The figure shows that the curve fitting differs slightly from the measured data outside the core region due to the different generation process and time-varying boundary conditions. The present data exhibits a flow pattern that is different from the existing two-celled models outside the core region, although they all behave in a manner similar to that of the potential vortex. The flow region shown in Fig. 8d includes the boundary layer, the core region comprising the inner and outer cells, the conjunction region, and the outer cyclostrophic flow encircling the core region. This two-celled flow structure is also similar to the conceptual picture of a two-celled tornado proposed in Snow (1982) and Davies-Jones (1986).

3.2.3 The detachment stage of the inner cell

Next, the two-celled structure gradually spins down to form a quasi-two-dimensional vortex; we observe the

detachment stage of the inner cell in Figs. 7h–j and 8e. Due to the diffusion of momentum near the bottom surface, the inner cell cannot resist the pressure-gradient forces from the ambient inward flow, and it therefore detaches from the bottom surface. As a result, the fluid coming from the bottom boundary layer takes over the bottom part of the inner cell, as shown in Fig. 8e. Meanwhile, the inner cell develops into a cup-like recirculation zone pushed up by the updraft secondary flow from the bottom. The stagnation point at the bottom of the cup-like recirculation zone oscillates and moves up gradually with a velocity of $\bar{v}_z \approx 0.03$. The oscillations are presumably caused by the inertial or centrifugal waves travelling along the core (R.P. Davies-Jones, private communication). The filling-up of the central region of the image plane shown in Fig. 18c and d also depicts the take-over process by the outer cell in the core region. Note that the bright portion in Fig. 18a and c indicates the fluid of the outer cell in which the fluorescent dye was

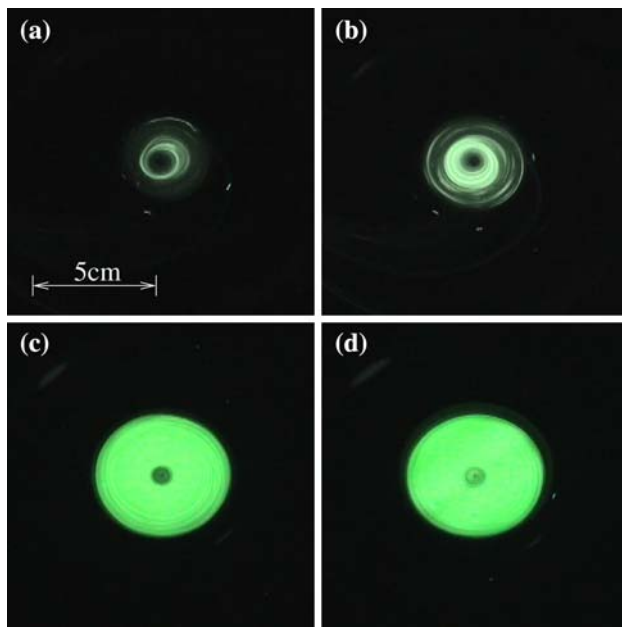


Fig. 18 This figure presents a sequence of images of top-view flow visualization having the same flow conditions as that of Fig. 7a and b, except the green colouring was replaced by a fluorescent one released from the bottom plate. The image plane is at $\bar{z} = 0.48$ above the bottom plate and the nondimensional time instants for each photo are, respectively, **a** 852, **b** 948, **c** 975 and **d** 1,110, corresponding to those shown in Fig. 7a and b

illuminated by a horizontal light sheet. Gradually, the updraft flow pushes the cup-like recirculation zone to a confined region near the free surface, as shown in Figs. 7i and j and 8f. Figure 20a–c show the images of the flow structure of the meridional cross-section in the detachment stage of the inner cell, which has the same flow conditions as those shown in Fig. 7. A two-celled structure is clearly observed in the core region as the downward-moving inner cell and updrafting outer cell in (a). The cup-like secondary circulation is also shown in (b), while (c) shows the image of the meridional cross-section of a mature barotropic vortex. The bright portion beneath the free surface indicates the cup-like recirculation zone wherein the fluid with fluorescent colouring was originally released into the inner cell in the early stage of downward jet impingement. The orange fluorescent colouring indicates the outer cell that was originally released into the boundary layer and then became a part of the outer cell. Figure 20d shows an image with only the green fluorescent colouring in order to emphasize the cup-like recirculation zone in the later stage of inner cell detachment. The bright portion above the free surface is an image of the cup-like recirculation zone. Finally, we observe that the entire vortex system attains the state of a quasi-two-dimensional flow.

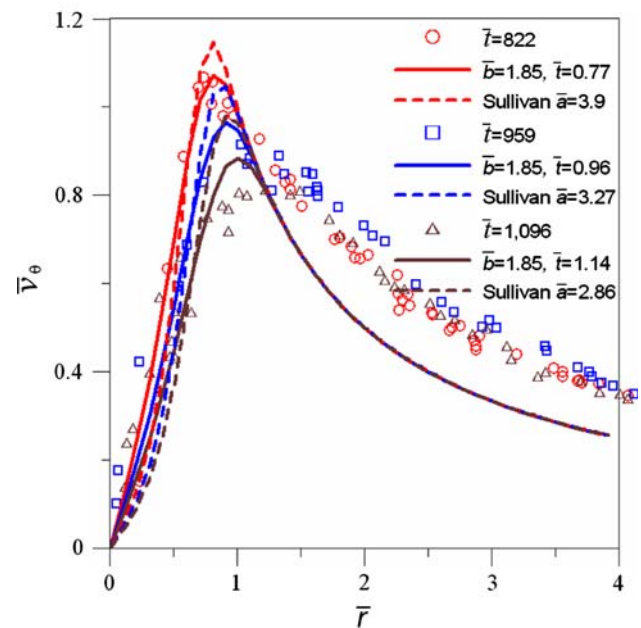


Fig. 19 This figure shows a comparison between the present measured data and the exact two-celled solutions of Sullivan (1959) and Bellamy-Knights (1970). The *solid lines* are the exact Bellamy-Knights' solutions at different times. The *dashed lines* are fitted by Sullivan's solution

3.2.4 Similarity to a tornado vortex

At the end of the siphoning stage, the following flow regions are observed, as indicated in Figs. 7b and 8b: (1) the bottom boundary layer; (2) the one-celled core region, or the pipe-like structure; (3) the conjunction region connecting the boundary layer and the core region; and (4) the large-scaled cyclonic motion, or outer flow encircling the core region under the cyclostrophic balance. Those four regions resemble the boundary layer, core, corner flow, and outer flow, respectively, in the conceptual model of a one-celled tornado, as proposed in Snow (1982) and Davies-Jones (1986). Furthermore, during the later stage of siphoning, the generated swirling vortex has a horizontal length scale of 50 cm ($\bar{r} = 45$), which is approximately two orders of magnitude greater than that of the core. With regard to the vorticity, in the later stage of siphoning, the estimated maximum relative vorticity in the core region obtained by differentiating the fitted velocity profiles is approximately 100 s^{-1} , which is significantly higher than that of the background vorticity of 1.57 s^{-1} . Therefore, the entire vortex presumably models a 'mesocyclone' along with a spirally updrafting one-celled tornado in its core region. In contrast, in the downward jet impingement stage, the flow region shown in Fig. 8d includes the boundary layer, the core region comprising the inner and outer cells, the conjunction region, and the outer cyclostrophic flow

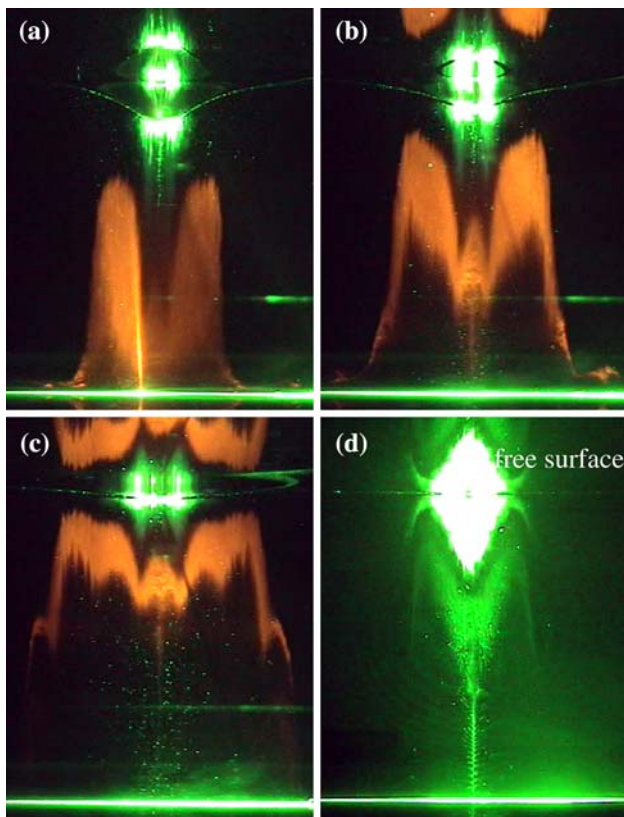


Fig. 20 This figure shows the images of the flow structure of the meridional cross-section in the later stages of downward jet impingement and inner-cell detachment, where **a** $\bar{t} = 849$, **b** $\bar{t} = 1096$, and **c** $\bar{t} = 1,507$ are images by fluorescent two-colorings, which **d** $\bar{t} = 1,644$ purposely showing the recirculation zone. It should be noted that the bright portion underneath the free surface indicates the cup-like recirculation zone

encircling the core region. This two-celled flow structure is also similar to the conceptual image of a two-celled tornado as proposed in Snow (1982) and Davies-Jones (1986).

4 Concluding remarks

In the past, one- and two-celled vortices have been studied as separate subjects. In this paper, we have presented a siphoning mechanism of vortex generation that enables us to observe the transition of a swirling vortex from the one-celled to the two-celled structure in a single experiment. The strength of the swirling vortex measured by its maximum azimuthal velocity is found to be linearly proportional to $\sqrt{Qf/h_0}$. The whole life cycle of the swirling vortex has been divided into three stages: (1) a siphoning stage, (2) a downward jet impingement stage, and (3) a detachment stage of the inner cell. The siphoning stage produces the upwelling one-celled vortex, which rotates in the positive z -direction and is surrounded by an

environment which is in approximately cyclostrophic balance. Immediately after the removal of the siphoning mechanism, the pressure difference induces a downwards jet with an air core, piercing the central part of the one-celled vortex and impinging upon the bottom of the tank. The air core is soon filled up with fluid from the top such that the pierced-through one-celled vortex is folded inward at the free surface, flowing downwards in the core. This results in an inner vortex. The remaining part of the original one-celled vortex is spiraling upward round the inner vortex and is now called the outer cell. This is the process of formation of the two-celled vortex. In the later stage of flow development, the touched-down inner cell is beginning to detach from the bottom of the tank. This is followed by a boundary layer flow which sneaks into the bottom core, developing a spiral upward stream, pushing backward the downward jet stream, and thus supporting a cup-like region of rotating fluid. It has also been shown that in the siphoning stage, the flow can be well modeled by Burgers' vortex for the one-celled structure. In the post-siphoning stages, the measured data show a flow pattern different from the two-celled vortex models of Sullivan and Bellamy-Knights. The one-celled and two-celled flow structures are also similar to the conceptual images of the one-celled and two-celled tornadoes proposed in the literature.

Acknowledgments The work was supported in part by the National Science Council of the Republic of China (Taiwan) under contract No's NSC 94-2111-M-002-016 and 93-2119-M-002-008-API. The authors would like to appreciate the helpful comments by Professor R. P. Davies-Jones, National Severe Storms Laboratory, NOAA, on an early draft of the paper.

References

- Andersen A, Bohr T, Stenum B, Rasmussen JJ, Lautrup B (2003) Anatomy of a bathtub vortex. *Phys Rev Lett* 91:104502
- Bellamy-Knights PG (1970) An unsteady two-celled vortex solution of the Navier–Stokes equations. *J Fluid Mech* 41:673–687
- Burgers JM (1948) A mathematical model illustrating the theory of turbulence. *Adv Appl Mech* 1:197–199
- Capart H, Young DL, Zech Y (2002) Voronoï imaging methods for the measurement of granular flows. *Exp Fluids* 32:121–135
- Davies-Jones RP (1986) Tornado dynamics. In: Kessler E (ed) *Thunderstorm morphology and dynamics*, 2nd edn. University of Oklahoma Press, Norman, pp 197–236
- Davies-Jones RP, Trapp RJ, Bluestein HB (2001) Tornadoes and tornadic storms. In: Doswell CA III (ed) *Severe Convective Storms*. Meteor Monogr Am Meteor Soc 28:167–221
- Flór JB, Eames I (2002) Dynamics of monopolar vortices on a topographic beta-plane. *J Fluid Mech* 456:353–376
- Hopfinger EJ, van Heijst GJF (1993) Vortices in rotating fluids. *Annu Rev Fluid Mech* 25:241–289
- Lugt HJ (1995) *Vortex flow in nature and technology*. Krieger, Malabar
- Maxworthy T (1972) On the structure of concentrated, columnar vortices. *Astronaut Acat* 17:363–374

- Maxworthy T (1982) The laboratory modeling of the atmospheric vortices: a critical review: intense atmospheric vortices. Springer, Berlin, pp 229–246
- Nezlin MV, Snezhkin EN (1993) Rossby vortices, spiral structures, solitons: astrophysics and plasma physics in shallow water experiments. Springer, Berlin
- Snow JT (1982) A review of recent advances in tonado vortex dynamics. *Rev Geophys Space Phys* 20:953–964
- Stegner A, Zeitlin V (1998) From quasi-geostrophic to strongly nonlinear monopolar vortices in a paraboloidal shallow-water-layer experiment. *J Fluid Mech* 356:1–24
- Sullivan RD (1959) A two-cell vortex solution of the Navier–Stokes equations. *J Aero/Space Sci* 26:767–768
- Trapp RJ, Davies-Jones RP (1997) Tornadogenesis with and without a dynamic pipe effect. *J Atmos Sci* 54:113–133
- Trieling RR, Linssen AH, van Heijst GJF (1998) Monopolar vortices in an irrotational annular shear flow. *J Fluid Mech* 360:273–294