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Ultimate boundedness control of linear systems with band-bounded nonlinear actuators and additive measurement noise ☆

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Abstract

For linear systems driven by band-bounded nonlinear actuators, a set of linear matrix inequality (LMI) based sufficient conditions are derived for finding state feedback controllers which assure ultimate boundedness of state trajectories. Besides actuator nonlinearity, it is assumed that additive noise exists when state variables are measured for feedback. The purpose is to minimize the ultimate boundedness region while tolerating noise of the largest magnitude. When a state feedback controller is determined for a given system by solving the LMI conditions or by any other means, a less conservative LMI condition is given for further examination of the resultant ultimate boundedness region and tolerable noise magnitude.
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1. Introduction

Actuators in control systems inevitably have nonlinear characteristics. For example, no actuators have an infinite operation range. Thus saturation is the most often discussed nonlinearity for actuators [1,2]. However, depending on the underlying physical principles and manufacturing precision, most actuators have nonlinear characteristics other than saturation. Designing control systems without taking these characteristics into account may produce an undesirable system response. The sector-bounded nonlinearity [12] has long been used to represent a quite general class of actuator characteristics, but with only a few exceptions, such as [10]; related works seem to focus more on the analysis problems, such as the circle and Popov criteria for the famous Lur'e problem. This may be an indication of the difficulty of the controller design problem in the presence of nonlinear actuators. As to still other actuator nonlinearities like deadzone, hysteresis, backlash, and quantization which cannot be accommodated by the sector-bounded nonlinearity, general discussions are rarely seen. Individual results usually cover specific types of nonlinearity or use approximations. For instance [15] treats the hysteresis characteristic by converting it into a passive operator, and [5] uses the describing function method to approximate the nonlinearity in a controller synthesis problem. Here we intend to deal

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with all of the above non-sector-bounded nonlinearities by viewing them as members of another class called the band-bounded nonlinearity. A precise definition of this term will be given in Section 2.

Just like the actuator nonlinearity, measurement noise is also an ever-present challenge to controller designers. Many authors have studied the problems of noise suppression or disturbance rejection [7,8,11,13] using various methods, or have discussed related problems under the framework of H_2/H_∞ theory [16]. However, consideration of measurement noise seems scarce in the works that treat actuator nonlinearities.

In this paper continuous-time linear systems driven by the band-bounded nonlinear actuators are to be regulated by state feedback controllers. It is assumed that the state variables of the linear systems are measurable, but contaminated by some additive measurement noise. Due to the existence of actuator nonlinearity and measurement noise, the control objectives have to bring the state trajectories from any initial conditions to a small neighborhood of the origin of the state-space, i.e., to guarantee that all state trajectories will eventually enter an ultimate boundedness region, and to minimize the "size" of the region. In the mean time, it is hoped that the closed-loop systems can tolerate the measurement noise of the largest magnitude. In the following sections, we start by presenting a rigorous formulation of the problem, and proceed with the help of the linear matrix inequality (LMI) approach. The first result is a set of LMI based sufficient conditions for the existence of the feasible state feedback control law. These LMI conditions can be augmented by an appropriate objective function to form a convex optimization problem, which reflects our quest of state feedback controllers that minimize the ultimate boundedness region and tolerate the largest measurement noise. After solving the very tractable convex optimization problem, we propose a generalized eigenvalue problem (GEVP) to further examine the optimal state feedback controller, because it is derived based on a set of sufficient conditions. The examination may produce less conservative estimates of the ultimate boundedness region "size" and tolerable measurement noise magnitude. In fact, the examination procedure may also be applied to any state feedback controller designed by other methods. All proposed methods are applied to a numerical example so that all application results may be compared on the same basis.

2. Problem formulation

Before we start, some notations are defined first. For any positive integer k and $M,N \in \mathbb{R}^{k\times k}, M \geqslant N$ means that M,N are symmetric and $M-N$ is positive semi-definite. Similar notations apply to symmetric positive/negative definite matrices. The notation I_k represents the $k\times k$ identity matrix, $\lambda_{\max}(M)$ denotes the largest eigenvalue of the symmetric matrix M, and $\mathrm{diag}(M_1,\dots,M_k)$ is a block diagonal matrix with the blocks $M_1,\dots,M_k$ on the diagonal position.

Let us consider the feedback control system in Fig. 1, which has the mathematical model

$$\begin{aligned} \dot{x}(t) &= Ax(t)+Bu(t), \quad x(0)=x_o, \\ \tilde{u}(t) &= K[x(t)+n(t)], \\ u(t) &= [\beta_1(t,\tilde{u}_1(t)) \ \cdots \ \beta_m(t,\tilde{u}_m(t))]^{\mathrm{T}}. \end{aligned} \tag{1}$$

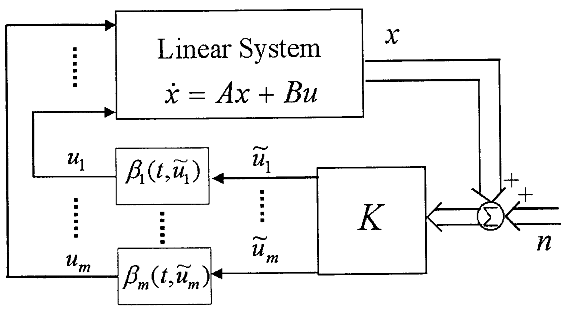

Fig. 1. A feedback control system with nonlinear actuators and additive measurement noise.

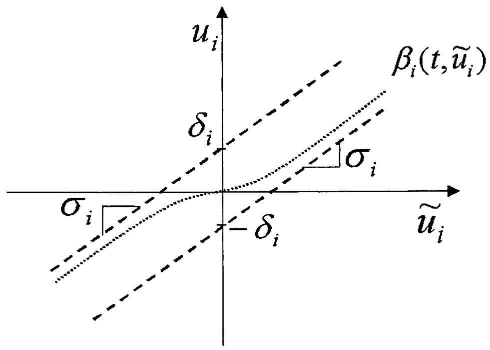

Fig. 2. Actuator input–output characteristic.

In (1) the first linear state equation describes the plant to be controlled, where $x(t) \in \mathbb{R}^{\ell}$ is the state vector, and $u(t) \in \mathbb{R}^m$ is the control input vector. The second equation represents the state feedback controller, where $n(t) \in \mathbb{R}^{\ell}$ is the additive measurement noise vector, bounded by $n(t)^{\mathrm{T}}n(t) \leqslant \rho^2$ for all $t \geqslant 0$ and some $\rho > 0$, $K \in \mathbb{R}^{m\times\ell}$ is the state feedback gain matrix, and $\tilde{u}(t) \in \mathbb{R}^m$ is the controller output vector. Finally, the third equation stands for the nonlinear characteristics of the actuators, where $\tilde{u}_i(t)$ is the ith component of $\tilde{u}(t)$, and $\beta_i(\cdot,\cdot):[0,\infty)\times\mathbb{R}\to\mathbb{R}$ denotes the ith nonlinear characteristic, illustrated in Fig. 2. Note that the dotted curve (or curves, as the relationship may be time-varying) of $\beta_i(\cdot,\cdot)$ is only known to be confined to the band between the two parallel dashed lines. Hence we call it the band-bounded nonlinearity. Mathematically, it is equivalent to say that $u_i(t)$ and $\tilde{u}_i(t)$ satisfy the algebraic inequality

$$[u_i(t) - \sigma_i\tilde{u}_i(t) + \delta_i][u_i(t) - \sigma_i\tilde{u}_i(t) - \delta_i] \leqslant 0, \tag{2}$$

for $t \geqslant 0$ and $i = 1,2,\ldots,m$, where $u_i(t)$ is the ith component of $u(t)$, and σ_i, δ_i are positive constants.

Clearly the band-bounded nonlinearity can accommodate characteristics such as backlash, hysteresis, dead-zone, and quantization, which are not included by the sector-bounded nonlinearity [12]. As to the measurement noise $n(t)$, our formulation allows one to handle the sensor-introduced noise with only the worst case information, or the quantization error introduced in the analog-to-digital conversion process [3]. Facing these nonlinearity and noise factors, we want to find state feedback controllers to reach the ultimate boundedness control [6,9,14] goal, which basically means making every state trajectories of (1) enter a neighborhood (the ultimate boundedness region) of the origin of the state-space eventually. To be more specific, we consider the ellipsoidal neighborhood $\mathbf{E}_c = \{x^{\mathrm{T}}Px \leqslant c\}$ for some $P > 0$ and $c > 0$. Of course, it is best if the "size" (with any acceptable definition) of $\mathbf{E}_c$ can be guaranteed to the smallest while ρ is as large as possible.

We end this Section by presenting two useful lemmas subsequently.

Lemma 1. *For any $\ell\times\ell$ matrix $Q > 0$ and scalar $\tau > 0$, $-\tau Q^2 \leqslant (1/\tau)I_\ell - 2Q$.*

Proof.

$$-(\sqrt{\tau}Q - (1/\sqrt{\tau})I_\ell)^{\mathrm{T}}(\sqrt{\tau}Q - (1/\sqrt{\tau})I_\ell) \leqslant 0 \Leftrightarrow -\tau Q^2 \leqslant (1/\tau)I_\ell - 2Q. \qquad \square$$

Lemma 2. *For any $F \in \mathbb{R}^{p\times p}$, $G \in \mathbb{R}^{q\times q}$, $H \in \mathbb{R}^{p\times q}$, and R, S of appropriate dimensions, where p, q are positive integers, we have*

$$\begin{bmatrix} F - HG^{-1}H^{\mathrm{T}} & R \\ R^{\mathrm{T}} & S \end{bmatrix} < 0, \quad G < 0 \;\Leftrightarrow\; \begin{bmatrix} F & R & H \\ R^{\mathrm{T}} & S & 0 \\ H^{\mathrm{T}} & 0 & G \end{bmatrix} < 0. \tag{3}$$

Proof. This is a special case of the Schur complement [4]. $\square$

3. Ultimate boundedness control via the LMI method

For $\mathbf{E}_c = \{x^{\mathrm{T}}Px \leqslant c\}$ to be an ultimate boundedness region of the closed-loop system (1), where $P \in \mathbb{R}^{\ell\times\ell}$ is positive definite and $c > 0$, it is sufficient that the derivative of the Lyapunov function $v(x) = x^{\mathrm{T}}Px$ be negative along all state trajectories of (1) outside $\mathbf{E}_c$. This is exactly the expression

$$\dot{v}(x) = x^{\mathrm{T}}(A^{\mathrm{T}}P + PA)x + x^{\mathrm{T}}PBu + u^{\mathrm{T}}B^{\mathrm{T}}Px < 0, \tag{4}$$

for all $x \notin \mathbf{E}_c$ and u satisfying (2). Note that in (1) and (2) $\tilde{u}=K(x+n)$, where $n^{\mathrm{T}}n \leqslant \rho^2$. Also, the condition $x \notin \mathbf{E}_c$ may be implied by a stronger requirement $x \notin \mathbf{B}_\varepsilon = \{x^{\mathrm{T}}x \leqslant \varepsilon\}$, where $\sqrt{\varepsilon}$ equals the length of the shortest principal axis of $\mathbf{E}_c$. By the S-procedure [4], a sufficient condition for (4) to hold for all x outside B_ε and u satisfying (2) is the existence of positive constants τ_i, $i = 1,\dots,m+2$, such that

$$\begin{aligned} &x^{\mathrm{T}}(A^{\mathrm{T}}P + PA)x + x^{\mathrm{T}}PBu + u^{\mathrm{T}}B^{\mathrm{T}}Px - \sum_{i=1}^{m}[\tau_i(u_i - \sigma_i\tilde{u}_i + \delta_i)\cdot(u_i - \sigma_i\tilde{u}_i - \delta_i)] \\ &\quad -\tau_{m+1}(\varepsilon - x^{\mathrm{T}}x) - \tau_{m+2}(n^{\mathrm{T}}n - \rho^2) < 0, \end{aligned} \tag{5}$$

for all $x, n \in \mathbb{R}^{\ell}$ and $u \in \mathbb{R}^m$. Letting $\Sigma=\mathrm{diag}(\sigma_1,\sigma_2,\dots,\sigma_m) > 0$, $T=\mathrm{diag}(\tau_1,\tau_2,\dots,\tau_m) > 0$, $\delta=[\delta_1\ \delta_2\ \cdots\ \delta_m]^{\mathrm{T}}$, and $\hat{\varepsilon} = \tau_{m+1}\varepsilon > 0$, we can re-arrange (5) as

$$\begin{aligned} &x^{\mathrm{T}}(A^{\mathrm{T}}P + PA + \tau_{m+1}I_\ell - K^{\mathrm{T}}\Sigma T\Sigma K)x + x^{\mathrm{T}}(PB + K^{\mathrm{T}}\Sigma T)u \\ &\quad + u^{\mathrm{T}}(B^{\mathrm{T}}P + T\Sigma K)x - n^{\mathrm{T}}K^{\mathrm{T}}\Sigma T\Sigma Kx - x^{\mathrm{T}}K^{\mathrm{T}}\Sigma T\Sigma Kn - u^{\mathrm{T}}Tu \\ &\quad + n^{\mathrm{T}}K^{\mathrm{T}}\Sigma Tu + u^{\mathrm{T}}T\Sigma Kn - n^{\mathrm{T}}(K^{\mathrm{T}}\Sigma T\Sigma K + \tau_{m+2}I_\ell)n \\ &\quad + \tau_{m+2}\rho^2 + \delta^{\mathrm{T}}T\delta - \hat{\varepsilon} \\ &\qquad = [x^{\mathrm{T}}\quad n^{\mathrm{T}}\quad u^{\mathrm{T}}\quad 1]\begin{bmatrix} \Psi & 0 \\ 0 & \psi \end{bmatrix}[x^{\mathrm{T}}\quad n^{\mathrm{T}}\quad u^{\mathrm{T}}\quad 1]^{\mathrm{T}} < 0, \end{aligned} \tag{6}$$

where

$$\Psi = \begin{bmatrix} \begin{pmatrix} A^{\mathrm{T}}P + PA + \tau_{m+1}I_\ell \\ -K^{\mathrm{T}}\Sigma T\Sigma K \end{pmatrix} & -K^{\mathrm{T}}\Sigma T\Sigma K & PB + K^{\mathrm{T}}\Sigma T \\ -K^{\mathrm{T}}\Sigma T\Sigma K & -K^{\mathrm{T}}\Sigma T\Sigma K - \tau_{m+2}I_\ell & K^{\mathrm{T}}\Sigma T \\ B^{\mathrm{T}}P + T\Sigma K & T\Sigma K & -T \end{bmatrix} \tag{7}$$

and

$$\psi = \tau_{m+2}\rho^2 + \delta^{\mathrm{T}}T\delta - \hat{\varepsilon}. \tag{8}$$

Therefore a sufficient condition for ultimate boundedness control is that for some diagonal $T > 0$, $\tau_{m+1} > 0$, $\tau_{m+2} > 0$ and $\hat{\varepsilon} > 0$, the inequalities $\Psi < 0$ and $\psi < 0$ hold. The remaining task is to convert the condition to LMIs with respect to suitable variables.

First of all, multiply Ψ in (7) on the left- and right-hand sides by the nonsingular matrix $\mathrm{diag}(P^{-1}, P^{-1}, I_m)$, and introduce the new variable matrices $Q = P^{-1} > 0$, $Y = KQ$. The result can be used to transform $\Psi < 0$ into

$$\begin{bmatrix} \begin{pmatrix} QA^{\mathrm{T}} + AQ + \tau_{m+1}Q^2 \\ -Y^{\mathrm{T}}\Sigma T\Sigma Y \end{pmatrix} & -Y^{\mathrm{T}}\Sigma T\Sigma Y & B + Y^{\mathrm{T}}\Sigma T \\ -Y^{\mathrm{T}}\Sigma T\Sigma Y & -Y^{\mathrm{T}}\Sigma T\Sigma Y - \tau_{m+2}Q^2 & Y^{\mathrm{T}}\Sigma T \\ B^{\mathrm{T}} + T\Sigma Y & T\Sigma Y & -T \end{bmatrix} < 0. \tag{9}$$

It follows from the Schur complement [4] pivoting on the third diagonal block $-T$ that when $T > 0$, (9) is equivalent to

$$\begin{bmatrix} QA^{\mathrm{T}} + AQ + BT^{-1}B^{\mathrm{T}} + Y^{\mathrm{T}}\Sigma B^{\mathrm{T}} + B\Sigma Y + \tau_{m+1}Q^2 & B\Sigma Y \\ Y^{\mathrm{T}}\Sigma B^{\mathrm{T}} & -\tau_{m+2}Q^2 \end{bmatrix} < 0. \tag{10}$$

With the help of Lemma 2 twice, (10) may be changed to

$$\begin{bmatrix} QA^{\mathrm{T}} + AQ + Y^{\mathrm{T}}\Sigma B^{\mathrm{T}} + B\Sigma Y & B\Sigma Y & Q & B \\ Y^{\mathrm{T}}\Sigma B^{\mathrm{T}} & -\tau_{m+2}Q^2 & 0 & 0 \\ Q & 0 & -\tau_{m+1}^{-1}I_\ell & 0 \\ B^{\mathrm{T}} & 0 & 0 & -T \end{bmatrix} < 0, \tag{11}$$

which is implied by

$$\begin{bmatrix} QA^{\mathrm{T}} + AQ + Y^{\mathrm{T}}\Sigma B^{\mathrm{T}} + B\Sigma Y & B\Sigma Y & Q & B \\ Y^{\mathrm{T}}\Sigma B^{\mathrm{T}} & \tau_{m+2}^{-1}I_\ell - 2Q & 0 & 0 \\ Q & 0 & -\tau_{m+1}^{-1}I_\ell & 0 \\ B^{\mathrm{T}} & 0 & 0 & -T \end{bmatrix} < 0 \tag{12}$$

due to Lemma 1. As to ψ in (8), usage of the Schur complement shows that when $\tau_{m+2} > 0$ the inequality $\psi < 0$ is identical to

$$\begin{bmatrix} \delta^{\mathrm{T}}T\delta - \hat{\varepsilon} & \rho \\ \rho & -\tau_{m+2}^{-1} \end{bmatrix} < 0. \tag{13}$$

Thus, we have obtained two LMIs (12) and (13) with respect to the variables $Q > 0$, $Y, T > 0$, $\rho > 0$, $\hat{\varepsilon} > 0$, $\tau_{m+1}^{-1} > 0$, and $\tau_{m+2}^{-1} > 0$. This means that given the plant information $\{A, B\}$ and actuator nonlinearity information $\{\Sigma, \delta\}$, we can search for state feedback controllers to reach the ultimate boundedness control goal. Feasible solutions of the two LMIs not only give the state feedback gain (via Q and Y), but also the ultimate boundedness region $\mathbf{E}_c$ (via Q, τ_{m+1} and $\hat{\varepsilon}$), and the tolerable noise magnitude ρ, which may also be specified beforehand.

Naturally, it would be best if we could set an objective function on top of the two LMIs (12) and (13) to form an optimization problem, enabling us to find the state feedback controller which results in the smallest ultimate boundedness region while tolerating the largest measurement noise. However, compromises must be made in the choice of the objective function because some quantity, such as ε, directly related to the "size" of the ultimate boundedness region is not a variable in (12) and (13), and because it is advantageous to form convex objective functions compatible with the LMI formulation. After making some trade-off, we choose the following optimization problem:

$$\min_{Q>0,\ Y,T>0,\ \tau_{m+1}^{-1}>0,\ \tau_{m+2}^{-1}>0,\ \rho>0,\ \hat{\varepsilon}>0} \hat{\varepsilon} + \tau_{m+1}^{-1} + \lambda_{\max}(Q) - \rho$$
$$\text{subject to (12) and (13).} \tag{14}$$

In the objective function, the first two terms represent an indirect effort to minimize the length of the shortest principal axis of $\mathbf{E}_c$, which is

$$\sqrt{\varepsilon} = \sqrt{\tau_{m+1}^{-1}\hat{\varepsilon}} \leqslant \frac{\hat{\varepsilon} + \tau_{m+1}^{-1}}{2}. \tag{15}$$

The third term represents the attempt to minimize the length of the longest principal axis of $\mathbf{E}_c$, which is proportional to $\lambda_{\max}(Q)$. Finally, the last term reflects our desire to tolerate noise of the largest magnitude. Of course, suitable weighting factors can be multiplied to each term if necessary. This kind of an optimization problem may be solved easily by using software tools like [8].

Example. Consider the system (1) with

$$A = \begin{bmatrix} 0.2 & -1 \\ 1 & 0.2 \end{bmatrix}, \quad B = \begin{bmatrix} 0.5 \\ 1 \end{bmatrix}, \quad \sigma_1 = 1, \quad \delta_1 = 0.4,$$

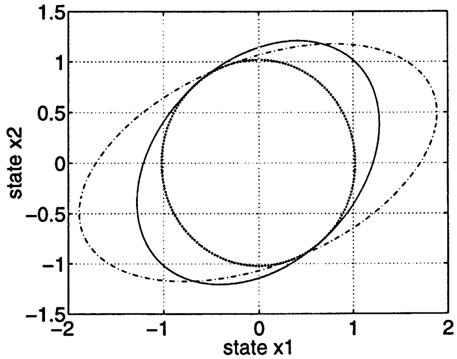

Fig. 3. Ultimate boundedness regions of the Example.

which is open-loop unstable. By solving the convex optimization problem (14), we obtain (hereafter the superscript * means the optimal value)

$$K^* = [-0.0715 \quad -1.963], \quad Q^* = \begin{bmatrix} 0.233 & 0.071 \\ 0.071 & 0.208 \end{bmatrix}, \quad \tau_1^* = 7.437$$

$$(\tau_2^*)^{-1} = 0.874, \quad (\tau_3^*)^{-1} = 4.211 \times 10^{-4}, \quad \varepsilon^* = 1.042 \quad \text{and} \quad \rho^* = 6.612 \times 10^{-4}.$$

The corresponding ultimate boundedness region $\mathbf{E}_{c^*}$ is the interior of the ellipse depicted with a solid line in Fig. 3, where the ball $\mathbf{B}_{\varepsilon^*}$ is also shown with a dotted line. The numerical results show that ε is of the same order as the width of the nonlinearity band ($2\delta_1 = 0.8$), but the tolerable noise magnitude ρ seems very small. As a remedy, we multiply a weighting factor $w \geqslant 1$ to ρ in (14) and solve the modified optimization problem. It is found that for $w \leqslant 5$, the effect is mild only. Corresponding to ultimate boundedness regions that are compromised marginally in size, the resultant ρ's do become larger, though not significantly. For instance, when $w = 5$ the bound ρ is about twice as large as the original ρ^*, and the corresponding ultimate boundedness region $\mathbf{E}_c$ is not so different from $\mathbf{E}_{c^*}$. For $w > 5$, the growth of ρ is more evident, but the expansion of $\mathbf{E}_c$ is equally noticeable. In other words, meaningful gain in the tolerable noise bound has to be obtained at the expense of accepting a larger $\mathbf{E}_c$.

4. A GEVP for conservativeness reduction

From the example in Section 3 it is suspected that there is conservativeness in the estimation of the tolerable noise magnitude. Furthermore, a comparison of (11) and(12) reveals that the usage of Lemma 1 may require that τ_{m+2}^{-1} be unnecessarily small, which in turn limits ρ in (13) too much. Therefore we seek to reduce this potential conservativeness. It is observed from (7) that $\Psi < 0$ is an LMI with respect to the variables $P > 0$, $T > 0$, $\tau_{m+1} > 0$, and $\tau_{m+2} > 0$ when K is known. Also, $\psi < 0$ can be written as

$$\delta^{\mathrm{T}} T \delta - \hat{\varepsilon} < -\rho^2 \tau_{m+2}, \tag{16}$$

which together with $\Psi < 0$ forms a GEVP with respect to the variables $P > 0$, $T > 0$, $\tau_{m+1} > 0$, $\tau_{m+2} > 0$, $\hat{\varepsilon} > 0$, and the "generalized eigenvalue" variable $-\rho^2$. Substituting the optimal solution K^* of the problem (14) into Ψ in (7) and solving the GEVP with the help of [8] will give new estimations of $\mathbf{B}_\varepsilon$, $\mathbf{E}_c$ and ρ. In fact, any state feedback gain K such that $A + BK$ is strictly Hurwitz may be tested by the GEVP to estimate the corresponding $\mathbf{B}_\varepsilon$, $\mathbf{E}_c$ and ρ. However, there are two details to notice. The first one is that if $\{P, T, \tau_{m+1}, \tau_{m+2}, \hat{\varepsilon}, \rho\}$ is a set of the solution of the GEVP, then $\{\gamma P, \gamma T, \gamma\tau_{m+1}, \gamma\tau_{m+2}, \gamma\hat{\varepsilon}, \rho\}$ is also a solution set, where γ is an arbitrary positive constant. Theoretically this nonuniqueness says that scaling the Lyapunov function $v(x) = x^{\mathrm{T}} P x$ by γ does not affect the analysis results ρ, $\mathbf{B}_\varepsilon$, and $\mathbf{E}_c$, since $\varepsilon = \hat{\varepsilon}/\tau_{m+1}$ and $c = \varepsilon\lambda_{\max}(P)$, but numerically it may cause some trouble. For example, our numerical experience shows that the software

[8] always tries to find the solutions of the GEVP with extremely small magnitudes. Fortunately, this difficulty may be overcome easily by adding constraints like $P \geqslant I_\ell$ or similar ones which fix a lower bound for the variables. The second detail is that the GEVP, due to its nature, tends to give very large ρ as its answer, which is accompanied by very large $\mathbf{B}_\varepsilon$ and $\mathbf{E}_c$, and means that in the trade-off between tolerating large noise and getting small ultimate boundedness region, it is biased to the former. When the GEVP is used to evaluate the effectiveness of a given state feedback controller, extra constraints to be introduced in the sequel may be needed to prevent this preference.

Suppose a state feedback gain K^*, along with its corresponding $\mathbf{E}_{c^*}$ and $\mathbf{B}_{\varepsilon^*}$, are found by solving (14). To use the above GEVP to evaluate its noise tolerance capability, it is suggested that the following two LMIs be added:

$$\hat{\varepsilon} \leqslant \varepsilon^* \tau_{m+1}, \tag{17}$$

$$\frac{P^*}{\lambda_{\max}(P^*)} \leqslant P \leqslant I_\ell, \tag{18}$$

where $P^* = (Q^*)^{-1}$. The purpose is to make sure that when a new estimation of ρ is obtained, it is not obtained at the expense of a larger $\mathbf{E}_c$, since (17) and (18) imply $\lambda_{\max}(P) = 1$ and $P^*/\varepsilon^* \lambda_{\max}(P^*) \leqslant P/\varepsilon$, which in turn imply that $\mathbf{E}_c \subseteq \mathbf{E}_{c^*}$.

Example. For the system and state feedback controller obtained in the example of Section 3, we may apply the above procedure to derive a new estimation of its noise tolerance capability. Substituting K^* into Ψ in (7), placing ε^* and $P^* = (Q^*)^{-1}$ into (17) and (18), respectively, and solving the GEVP which consists of (16), $\Psi < 0$, (17), and (18), we obtain

$$P^{\text{new}} = \begin{bmatrix} 0.710 & -0.243 \\ -0.243 & 0.797 \end{bmatrix}, \quad \tau_1^{\text{new}} = 0.548, \quad \tau_2^{\text{new}} = 0.170,$$

$$\tau_3^{\text{new}} = 2.085, \quad \varepsilon^{\text{new}} = 1.042 \quad \text{and} \quad \rho^{\text{new}} = 0.208.$$

Within our computational precision, it is seen that the newly estimated ultimate boundedness region $\mathbf{E}_c^{\text{new}}$ can be regarded as equal to $\mathbf{E}_{c^*}$ obtained previously, and $\mathbf{B}_\varepsilon^{\text{new}} = \mathbf{B}_{\varepsilon^*}$ in the same sense. However, here a much less conservative estimation $\rho^{\text{new}} = 0.208$ is derived, compared with $\rho^* = 6.612 \times 10^{-4}$ obtained in the example of Section 3.

Next, assume that there is an alternative state feedback gain K^a for the system (1), which is designed to make $A + BK^a$ strictly Hurwitz by using, for example, the eigenvalue assignment method. Then the GEVP consisting of (16), $\Psi < 0$ with K in (7) replaced by K^a, $P \geqslant I_\ell$, and $\hat{\varepsilon} \leqslant \varepsilon^\circ \tau_{m+1}$ may be utilized to find ρ and $\mathbf{E}_c$ corresponding to K^a and a pre-determined $\mathbf{B}_{\varepsilon^\circ}$.

Example. Let us continue to consider the above Example. Suppose a $K^a = [-0.608 \quad -2.096]$ is chosen to place eigenvalues of $A + BK^a$ at $\{-1, -1\}$, which have about the same magnitude as those of $A + BK^*$. If in the last GEVP ε° is set to $\varepsilon^{\text{new}} = 1.042$, the solution is

$$P = \begin{bmatrix} 2.070 & -1.390 \\ -1.390 & 5.322 \end{bmatrix}, \quad \tau_1 = 3.983, \quad \tau_2 = 1.368,$$

$$\tau_3 = 15.309, \quad \varepsilon = 1.042 \quad \text{and} \quad \rho = 0.227.$$

Note that although the tolerable noise magnitude is larger ($\rho = 0.227 > \rho^{\text{new}} = 0.208$) in this case, the corresponding $\mathbf{E}_c$ with approximately the same short axis length as $\mathbf{E}_c^{\text{new}}$ ($\varepsilon = 1.042 = \varepsilon^{\text{new}}$) is larger in area. This is shown in Fig. 3, where the dashed ellipse is the boundary of the $\mathbf{E}_c$ for K^a.

5. Conclusion

A state feedback controller design method for linear systems driven by band-bounded nonlinear actuators is proposed. The controller is to be operated under the influence of additive measurement noise, and should assure ultimate boundedness of state trajectories. It is possible for the designer to make a trade-off between tolerating larger noise and obtaining a smaller ultimate boundedness region. Finally, a GEVP is proposed for the designer to make less conservative analysis of any state feedback controllers regarding their noise tolerance capability and sizes of the corresponding ultimate boundedness region.

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