A Staircase-Approximation Time-Domain Matrix Integral Equation Approach to the Computation of One-Dimensional Transient Propagation through an Inhomogeneous Slab*

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Abstract

A novel method based on the staircase-approximation time-domain matrix integral equation (SATDMIE) is proposed to solve one-dimensional transient propagation through an inhomogeneous slab. A numerical scheme is developed to compute the transient response iteratively. This scheme only needs to factorize a K by K matrix once, and requires $O(NK^2)$ storage and matrix element computations. Here N and K are the number of time steps and spatial divisions, respectively. The whole theory framework is quite general. It can be extended to solve 3D problems like the transient radiation and scattering of wire antennas.

Introduction

The staircase-approximation time-domain (SATD) method [1][2] has been successfully applied to transmission line transient problems. In order to extend it to general electromagnetic problems, the author tries the one-dimensional transient propagation through an inhomogeneous slab as the first step. Although this problem has been solved by time-domain integral equation [3][4], and could also be computed via taking the inverse Fourier transform of the frequency domain solution, the proposed new approach is efficient and easier to formulate as well as generalize.

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Problem

Consider a plane wave $\hat{x}E^i(z,t) = \hat{x}f(t-\frac{z}{c})$ incident upon a dielectric slab with dielectric constant $\varepsilon_r(z)$ in free space as shown in Fig. 1. The transient behaviors of the reflected field at $z=0$ and the transmitted field at $z=a$ are to be computed.

Matrix Integral Equation

Apply the SATD approach [1][2], as a special case of discrete-time electromagnetic theory [5], to expand the total field

$$E(z,t) = \sum_{n=0}^{N-1} E_n(z)h_n(t) = [E_n(z)]^T[h_n(t)] \tag{1}$$

where the time-domain rectangular pulse basis function $h_n(t) = \begin{cases} 1, & n\Delta t < t \le (n+1)\Delta t \\ 0, & otherwise \end{cases}$, and to manipulate the Maxwell equations to obtain the following matrix equation for the scattered field $E^s(z,t) = E(z,t) - E^i(z,t)$

$$\frac{d^2}{dz^2}[E_m^s(z)] = \mu_0\varepsilon_0[D_{mn}]^2[E_n^s(z)] + \mu_0\varepsilon_0(\varepsilon_r(z)-1)[D_{mn}]^2[E_n(z)] \tag{2}$$

Here we assume that the incident field and the scattered field have also been expanded in the form of (1) and $[D_{mn}]$ the matrix corresponding to differentiation in time

$$[D_{mn}] = \frac{2}{\Delta t}\begin{bmatrix} 1 & 0 & 0 & 0 & \cdots \\ -2 & 1 & 0 & 0 & \cdots \\ 2 & -2 & 1 & 0 & \cdots \\ -2 & 2 & -2 & 1 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix} \tag{3}$$

A matrix Green's function corresponding to (2) satisfying

$$\frac{d^2}{dz^2}[G_{mn}(z,z')] = \frac{1}{c_0^2}[D_{mn}]^2[G_{mn}(z,z')] - \delta(z-z')[\delta_{mn}] \tag{4}$$

is solved as $[G_{mn}(z,z')] = \frac{c_0}{2}e^{-\frac{1}{c_0}[D_{mn}]|z-z'|}[D_{mn}]^{-1}$. With this Green's function and (2), the scattered field can be written as

$$[E_n^s(z)] = -\int_0^a [G_{mn}(z,z')]\frac{1}{c_0^2}(\varepsilon_r(z)-1)[D_{mn}]^2[E_n(z')]dz' \tag{5}$$

A matrix integral equation is then deduced as

$$[E_m(z)] = [E_m^i(z)] - \int_0^a (\varepsilon_r(z') - 1)[g_{m-n}(|z - z'|)][E_n(z')]dz' \quad (6)$$

where $[g_{m-n}(|z-z'|)] = \frac{1}{2c_0} e^{-\frac{1}{c_0}[D_{mn}]|z-z'|}[D_{mn}]$ depends on $m-n$ only.

Numerical Solution Scheme

To solve the matrix integral equation (6), we expand $E_n(z) = \sum_{k=0}^{K-1} E_n^{(k)} b_k(z)$, where the spatial basis function $b_k(z) = \begin{cases} 1, & k\Delta z < z \le (k+1)\Delta z \\ 0, & \text{otherwise} \end{cases}$, with $\Delta z = a / K$. Substituting this expansion into (6) and matching the equation at $z_\ell = (\ell + 1/2)\Delta z$, $\ell = 0, 1, 2, \cdots, K-1$, we achieve the following recurrence formula

$$[E_m^{(k)}] = ([\delta^{(\ell,k)}] + [P_0^{(\ell,k)}])^{-1} \left\{ [S_m^{(\ell)}] - \sum_{n=0}^{m-1} [P_{m-n}^{(\ell,k)}][E_n^{(k)}] \right\}, \quad m = 0, 1, 2, \cdots, N-1 \quad (7)$$

where $[\delta^{(\ell,k)}]$ is an identity matrix of order K, $S_m^{(\ell)} = E_m^i(z_\ell)$, and $P_{m-n}^{(\ell,k)} = \int_0^a (\varepsilon_r(z') - 1) g_{m-n}(|z_\ell - z'|) b_k(z') dz'$. Through this recurrence equation we solve $[E_m^{(k)}]$ iteratively in time. Note that in (7) the matrix inversion needs only be factorized once by LU decomposition. This is in contrast to the frequency domain approach, where N matrix inversions must be done. On the other hand, computing the matrix elements in the frequency domain approach requires $O(NK^2)$ computation and $O(K^2)$ storage. For the SATDMIE approach, we have to save $[P_{m-n}^{(\ell,k)}]$, whose computational and storage complexities are both $O(NK^2)$. Note also that $[g_{m-n}(|z_\ell - z'|)]$ can be calculated efficiently using the fast algorithm in [2].

Verification

The above scheme has been implemented as a C++ program. Figure 2 is the computed transient behaviors for a homogeneous slab with $\varepsilon_r = 5$, $a = 8c_o\Delta t$, $\Delta z = c_o\Delta t$, $f(t) = (e^{-t/100\Delta t} - e^{-t/50\Delta t})U(t)$ and $U(t)$ the unit step function. The three 512-point curves agree with those obtained by taking the inverse FFT of the analytic frequency-domain solution. They were computed through a notebook PC with a Pentium III 1.13 GHz CPU and a 256 M memory in 49 seconds.

References

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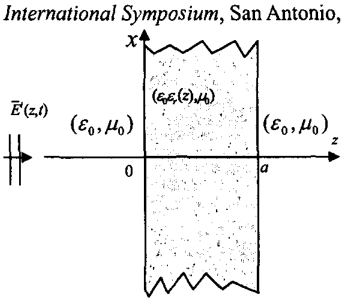

Fig. 1. Transient propagation Problem

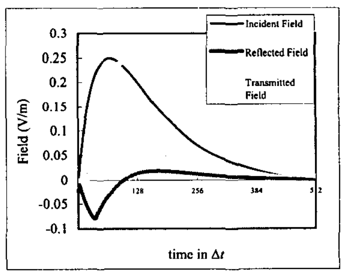

Fig. 2. Incident, Reflected, and Transmitted fields