

# 行政院國家科學委員會專題研究計畫成果報告

計畫名稱：時間數列同期性的新檢定方法

(New Tests for Synchronized Time Series)

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## 一、中文摘要

本研究中提出時間可逆性的新檢定方法。這個檢定方法針對分配對稱性加以檢定，極限上具有常態分配以及一致性；更重要的是此方法不受資料中動差性質的影響，因此適用於總體以及財務市場資料。實證分析中也顯示相較於傳統方法，新檢定法能對模型設定上提供相當不一樣的結論。

**關鍵詞：**分配對稱性，高斯分配特性，線性，時間可逆性

## Abstract

In this project we propose a class of new tests for time reversibility. It is shown that this test has an asymptotic normal distribution under the null hypothesis and non-trivial power under local alternatives. A novel feature of this test is that it does not have any moment restriction, in contrast with other time-reversibility and linearity tests. Our simulations also confirm that the proposed test is very robust when data do not possess proper moments. An empirical study of stock market indices is also included to illustrate the usefulness of the new test.

**Keywords:** distribution symmetry, Gaussianity, linearity, time reversibility

## 二、緣由與目的

A leading approach to representing stationary time series is based on Gaussian ARMA models. Unfortunately, many empirical studies suggest that this class of models is unable to capture many interesting

dynamic behaviors of data. For example, a Gaussian ARMA model cannot describe macroeconomic variables that have an asymmetric pattern during the periods of expansion and contraction, nor can it characterize financial time series that exhibit “amplitude-frequency dependence” (e.g., clustering of volatility). Hydrological data and other well known data sets such as Wolf's sunspot data and the Canadian lynx data are also frequently adopted to demonstrate nonlinearity and/or non-Gaussianity.

To accommodate various dynamic structures in data, there is now a growing interest in nonlinear time series models and non-Gaussian distributions such as stable distributions. See e.g., Tong (1990), Brockwell and Davis (1991), and Granger and Terasvirta (1993) for examples and more detailed discussion. Numerous diagnostic tests are also proposed in the literature, e.g., Hinich (1982), McLeod and Li (1983), Keenan (1985), Weiss (1986), Tsay (1986,1989), Petruccioli and Davis (1986), Luukkonen et al. (1988), Lee et al. (1993), and Wong (1997). These tests focus on either linearity or Gaussianity but not both and hence may yield misleading results. For instance, a linearity test that ignores potential non-Gaussianity (e.g., distributions without proper moments) may perform quite poorly, as shown in de Lima (1997). An alternative approach is to consider tests for time reversibility, as did in Ramsey and Rothman (1996) and Hinich and Rothman (1998).

A stationary time series is said to be time reversible if its finite dimensional distributions are all invariant to the reversal of time indices; otherwise, it is time irreversible. In particular, sequences of i.i.d.

random variables and stationary Gaussian processes are time reversible. On the other hand, a linear, non-Gaussian process is time irreversible in general, except when its coefficients satisfies certain constraints. For nonlinear processes, Tong (1990) also argued that: “time irreversibility is the rule rather than the exception when it comes to nonlinearity” (p. 197). As such, testing for time reversibility may be viewed as a joint test for linearity and Gaussianity and therefore can serve as a useful diagnostic check in model building, as noted in Cox (1981). Rejecting the null hypothesis suggests that standard Gaussian ARMA models are inappropriate; more efforts are needed to formulate proper models. Empirically, Rothman (1994) and Ramsey and Rothman (1996) also employed such a test to evaluate the random walk hypothesis in financial data and business cycle asymmetry in macroeconomic data.

### 三、研究結果

Our first result shows that if a time series  $\{Y_t\}$  is time reversible, then its differenced series  $\{Y_t - Y_{t-k}\}$  must have symmetric marginal distribution for each  $k$ . Based on this implication, we propose a class of new tests for distribution symmetry. By contrast, the test of Ramsey and Rothman (1996) focuses only on the third moment of these distributions which is not equivalent to distribution symmetry. We show that the proposed test has an asymptotic normal distribution under the null hypothesis and non-trivial power under local alternatives. A novel feature of this test is that it tests whether the imaginary part of the characteristic function is zero and therefore does not have any moment restriction, whereas the tests of Ramsey and Rothman (1996) and Hinich and Rothman (1998) require that the sixth moment of data exists. Note that most of linearity tests also have similar requirements. This concern is indeed practically relevant. Jansen and de Vries (1991) and de Lima (1997) found that

the maximal moment exponents of many financial time series do not exceed four; see also Mandelbrot (1963).

In this project we also suggest two weighting functions for the proposed test which lead to convenient statistics. We discuss why weighting functions of this type is useful in constructing operational test statistics. Our simulations confirm that these tests outperform the existing moment-based test in most of cases considered and that they are very robust when data do not possess proper moments. In particular, our tests all maintain proper sizes regardless of the moment properties of data. While the third-moment-based test virtually has no power when data lack proper moments, our tests still have very high power. This feature makes the proposed test a useful complement to existing diagnostic tests.

In our empirical analysis of six stock indices, our tests also lead quite different results. For example, the Ljung-Box test may show that a series is Gaussian white noise so that no further modelling would help, whereas our test indicates that it is time irreversible and therefore should be modelled using nonlinear models and/or non-Gaussian distributions. All these results suggest that the proposed test can lead to very different conclusions than do the traditional tests. Therefore, the proposed test should serve as an important diagnostic test in practice.

### 四、成果自評

本計畫是以研究時間數列的同期性為出發的。但是經過深入分析，發現無法針對一般的時間數列發展出同期性的檢定方法。一個可能的做法是考慮特定屬性的時間數列並加以分析。本研究於是著重在具有時間可逆性的數列加以分析，而且先針對可逆性本身提出新的檢定方法。在這方面來說可以說成果豐碩，因為本研究從不同的觀念出發而設計出新的檢定方法。

本研究所提出的檢定方法，不僅僅可以用來檢定時間可逆性，而且也可做為一般的分配對稱性檢定；在資料的適用性上，這些方法均能用在總體和財務市場資料，而不受其動差特性干擾。所以這個檢定方法有廣泛的適用性，應可成為時間數列分析的基本檢定方法之一。本研究的結果已寫成論文寄送國外期刊 *Journal of Econometrics*。

## 五、參考文獻

Brockwell, P. J. and R. A. Davis (1991), *Time Series: Theory and Methods*, New York: Springer-Verlag.

Cox, D. R. (1981), Statistical analysis of time series: Some recent developments, *Scandinavia Journal of Statistics*, 8, 93-115.

de Lima, P. J. F. (1997), On the robustness of nonlinearity tests to moment condition failure, *Journal of Econometrics*, 76, 251-280.

Granger, C. W. J. and T. Terasvirta (1993), *Modelling Nonlinear Economic Relationships*, Oxford: Oxford University Press.

Hallin, M., C. Lefevre and M. Puri (1988), On time-reversibility and the uniqueness of moving average representations for non-Gaussian stationary time series, *Biometrika*, 75, 170-171.

Hinich, M. J. (1982), Testing for Gaussianity and linearity of a stationary time series, *Journal of Time Series Analysis*, 3, 169-176.

Hinich, M. J. and R. Rothman (1998), A frequency domain test of time reversibility, *Macroeconomic Dynamics*, forthcoming.

Jansen, D. W. and C. G. de Vries (1991), On the frequency of large stock returns: Putting booms and busts into perspective, *Review of Economics and Statistics*, 73, 18-24.

Li, X. and J. M. Morris (1991), On measuring asymmetry and the reliability of the skewness measure, *Statistics and Probability Letters*, 12, 267-271.

Luukkonen, R., P. Saikkonen, and T. Terasvirta (1988), Testing linearity against smooth transition autoregressive models, *Biometrika*, 75, 491-499.

Mandelbrot, B. (1963), The variation of

certain speculative prices, *Journal of Business*, 36, 394-419.

McLeod, A. I. and W. K. Li (1983), Diagnostic checking ARMA time series models using squared-residual autocorrelations, *Journal of Time Series Analysis*, 4, 269-273.

Ord, J. K. (1968), The discrete Student's  $t$  distribution, *Annals of Mathematical Statistics*, 39, 1513-1516.

Petrucelli, J. D. and N. Davies (1986), A portmanteau test for self-exciting threshold autoregressive-type nonlinearity in time series, *Biometrika*, 73, 687--694.

Ramsey, J. B. and P. Rothman (1996), Time irreversibility and business cycle asymmetry, *Journal of Money, Credit, and Banking*, 28, 3-20.

Rothman, P. (1992), The comparative power of the TR test against simple threshold models, *Journal of Applied Econometrics*, 7, 187-195.

Rothman, P. (1994), The time reversibility test with application to financial data, in *Business Cycles: Theory and Empirical Methods*, W. Semmler ed., 389-403, Boston: Kluwer Academic Publishers.

Tong, H. (1990), *Nonlinear Time Series-A Dynamic System Approach*, Oxford: Oxford University Press.

Tsay, R. S. (1986), Nonlinearity tests for time series, *Biometrika*, 73, 461-466.

Weiss, G. (1975), Time reversibility of linear stochastic processes, *Journal of Applied Probability*, 12, 831-836.

Wong, W. (1997), Frequency domain tests of multivariate Gaussianity and linearity, *Journal of Time Series Analysis*, 18, 181-194.