

DYNAMIC GAME-BASED ANALYSIS FOR A HYBRID MULTI-AGENT ROBOTIC SYSTEM

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ABSTRACT

In this paper, a novel design and analysis of the dynamic game of the Hybrid Multi-Agent Robotic System (HMARS) is proposed for dealing with the formation problem of multiple robots with leaders and followers. Each autonomous agent is a wheeled mobile robot (WMR) with switching behaviors including target approach and obstacle avoidance. Under the well designed controller, an agent can perform formation tasks together with other agents in order to accomplish a cooperative mission. The top-down design method deals with not only logical conflicts but also optimal solution of the game at Nash equilibrium point. On the other hand, the bottom-up design method searches for the best performance on the local controller over continuous trajectory. In addition, *reachability* is verified at the logical level. The contributions of this paper can be split into three parts: first, Lyapunov style formation control of a HMARS is developed based on reachability, existence and stability issues under the hybrid approach; second, a dynamical cooperative game for HMARS is proposed and analyzed for the stability with Constrained Logic; third, we offer a framework to implement cooperation of HMARS with the innate characteristics from discrete event space to continuous state space.

Keywords: Multi-Agent, Dynamic Game Theory, Hybrid Control System, Stability

1. INTRODUCTION

An important goal in the development of the Hybrid Multi-Agent Robotic System (HMARS) is to design a distributed management infrastructure to perform their tasks over a problem-solving period without human supervision. The HMARS must be capable of dealing with dynamic changes over time, including incremental variables in their own performance capabilities or real-time responses for cooperative tasks. From an engineering standpoint, the problem of how to prescribe the relationship among cooperative agents through application of local interactions is of real and practical interest. However, global outcome of such a behavior-based system is often difficult to predict analytically. It is hence important to construct a reasonable multi-agent cooperation framework for dealing with the issue. The topic of multi-agent cooperation framework has been extensively studied in distributed problem solving [6]. Multi-Agent systems have been applied to problems involved in acting in the physical world, such as cooperative team decision control [12,5], task-level planning and control [19,25], the design of

robotic systems [4,17], and logic-based model on formation and action of WMRs [9]. Nevertheless, using Multi-Agent systems to control robotic devices in unconstrained environment presents special challenges due to the tight coupling between the computational agents and their embedding physical space.

In fact, the modeling of hybrid systems is the basic condition for all further investigations and requires the availability of a well defined modeling language. Therefore, one important goal is to use hybrid system theory, which is able to deal with systems with both continuous and discrete states, to model our agent system. The cooperation/coordination problems with hybrid systems are still under investigation, e.g. [13,20] and [22].

Evaluating the HMARS problem from the view-point of Distributed Artificial Intelligence (DAI), there are several programming tools for constructing agents. Examples include Voyager [16], Angles [14] and Odyssey [7]. Such tools provide the framework for agent communication and knowledge sharing by defining ontologies, communication protocols and infrastructures, as well as interaction protocols. On the other hand, it was not until recent

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years that several independent industrial and research groups started to pursue the standardization of multi-agent technology, such as the Object Manager Group (OMG), the Foundation for Physical Agent (FIPA), the Knowledge-able Agent-oriented System (KAoS) group, and the General Magic group. Nevertheless, existing modeling methods do not support efficient HMARS applications, which demand both the precision design in discrete state and accuracy control in continuous state.

The theory of dynamic games is concerned with multi-person decision making, in which differential equations play an important role. Dynamic game theory, whose development started in the early fifties, is versatile by adopting characteristics from both game theory and optimal control theory. In this paper, we formulate the HMARS problem as team decision. It should be emphasized that we permit any kind of communication and agreement among the decision makers. The introduction and analysis for team decision theory and information system can be found in [12] and [11]. In [11], a logic-based formalism is proposed to model the dialogues between intelligent and autonomous software agents, building on a theory of abstract dialogue games.

In this paper, a novel design and analysis of the dynamic game of a HMARS is proposed for dealing with the formation problem of a HMARS with leader and followers. In the next section, we start by formulating the dynamic game for Multi-Agent system. Section 3 presents the hybrid Multi-Agent system of a WMR, defines the execution based on its continuous trajectory, and describes the switching behaviors. The task design is proposed in section 4. In Section 5, the top-down game-theoretic model is analyzed. Finally, conclusion and future work are summarized in section 6.

2. PROBLEM FORMULATION

This paper proposes a dynamic game-based hybrid Multi-Agent system control that aims to combine the advantages of an optimal robotic control given a complete continuous model and the powerful logical inference of high level tasks in the discrete domain. Each autonomous agent is a WMR with two basic behaviors, *target approach* and *obstacle avoidance*. Under the carefully designed controller, an agent can perform formation tasks together with other agents in order to accomplish a cooperative mission. The top-down design method deals with not only logical conflicts but also optimal solution of the game at Nash equilibrium point. On the other hand, the bottom-up design method searches for the best performance on the local controller and continuous trajectory. The system is necessarily complex, and we make the following assumptions regarding the underlying robot control.

Assumption 1: Agents work in an indoor environment, and the application terrain is confined to a 2D plane.

Assumption 2: Each WMR is subject to the non-holonomic constraint: sliding and slipping do not occur while the WMR moves in the indoor terrain.

Let's further assume that every agent knows its position through some self-localization mechanism. The agents can freely communicate with each other via wireless LAN, e.g., IEEE 802.11g. However, communication happens over time. In other words, the Multi-Agent system has complete but possibly non-synchronous information during execution.

A sample scenario of a Multi-Agent system in an indoor environment is illustrated in Figure 1. There are 3 WMRs bound as a subgroup to elect their local leader when a new task is received. Several problems need to be addressed. First of all, it is necessary to analyze the existence of any suboptimal solution of the hybrid system with dynamic game. If it exists, the reachability of the logic level has to be verified by the theoretical method. On the other hand, in the continuous domain, the optimal solution is produced according to searching for possible combinations of logical inference with dynamic game. In addition, the existence of a unique solution on a continuous domain has to be evaluated after the game.

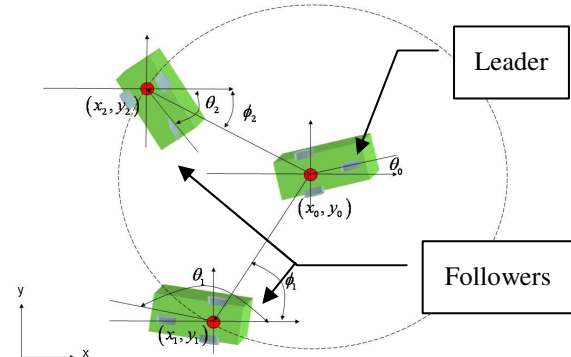


Figure 1: A scenario of HMARS formation problem

Where $\theta_i \in R, i=0 \sim 2$ denotes the heading angle of a WMR; $\phi_i \in R$ denotes the relative angle from i to $i+1$; (x_i, y_i) draws the position with regard to a global coordinate. In figure 1, each agent is not immortal, namely, the playing role may change if agents accomplish their different missions. The dynamically elected local leader of each subgroup creates a coordination mechanism, and leads the followers to track a well designed path defining a fixed formation with initial configurations.

Figure 2 demonstrates the scenario for applications in a 2D indoor environment. There are three agents executing tasks in different initial postures and locations. Therefore, after selecting a

local leader, it decouples the given task into subtasks and propagates these subtasks to its members. To gain flexibility in the overall system, each agent in the subgroup can be re-assigned to another subgroup after completing the task.

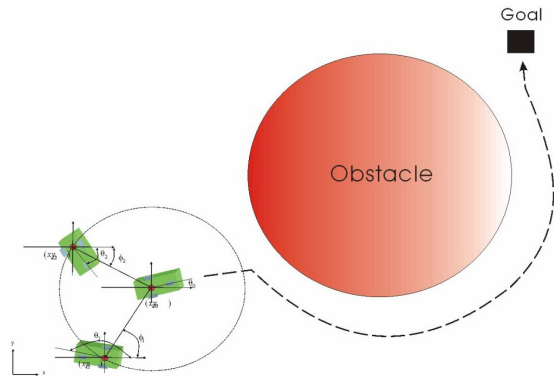


Figure 2: The scenario of multi-Agent formation system at indoor environment

Note that for quite a few applications it also makes sense to approximate purely continuous system with discrete models. The hybrid system, which combines interacting discrete and continuous dynamics, helps model Multi-Agent system with autonomous WMRs. Figure 3 depicts a hybrid system for a WMR with two behaviors.

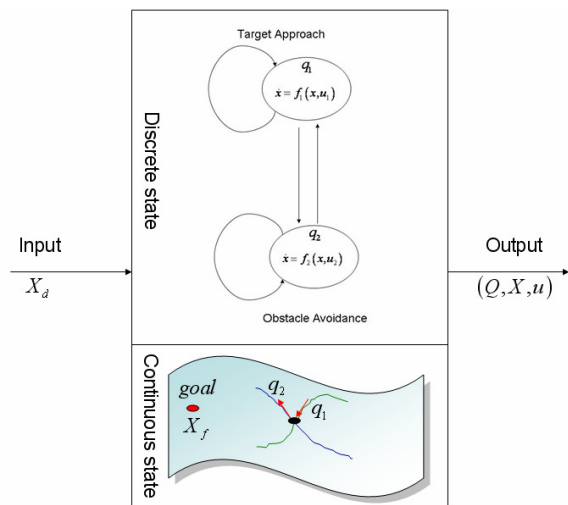


Figure 3: A hybrid system with the target approach and obstacle behavior

Where f_1 and f_2 both are vectors. q_1 and q_2 are discrete event state. The input $X_d \in R^3$ is the desired continuous state and output is the state of discrete Q , continuous state X and admissible control variable $u \in R^2$. More precisely, we want to give a desired path and final goal from the continuous state. It is a difficult challenge due to the physical limitation in an environment. A HMARS needs to not only make decisions to respond to discrete events

within a control slot to finish the given task in the continuous state, but also make a team decision cooperatively. Here, we propose a dynamic game based control to reach the goal. To simplify the problem, we assume that each agent in a group trusts one another during task execution completely and has full state information from the Multi-Agent system.

3. SYSTEM ARCHITECTURE

Each agent has two well-designed basic behaviors [9], namely, target approach (TA) and obstacle avoidance (OA).

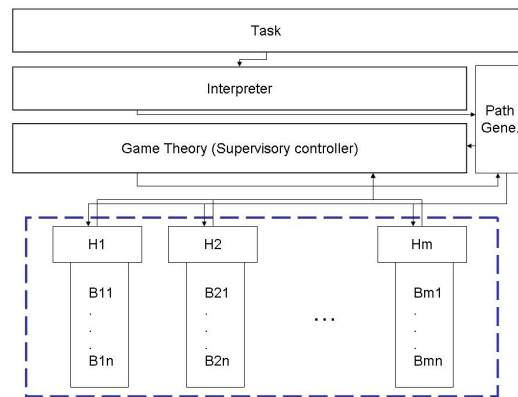


Figure 4: HMARS with a game-based cooperative supervisory system

Where H_i denotes the hybrid system for the i th WMR, for $i = 1, 2, \dots, m$. B_{ij} 's are behaviors. When a new task is decomposed by the interpreter as shown in Figure 4, it translates into a subtask. The Path Generator (PG) receives a series of subtasks and translates them into physical paths in continuous state. There has been research involving these topics such as [15] and [21]. In this paper, the mechanism of a PG is established on the elected local leader, which can receive full information from all other agents.

The supervisory controller with game theory plays the role of a coordinator. Apparently, it is dedicated to evaluate the performance of each of agents. According to the performance, all players coordinate their efforts to generate nearly optimal strategies on the Multi-Agent formation problem dynamically. After the process, configurations of a PG will be modified with these strategies. In figure 4, the feedback control is applied on each of an agent. For handling the discrete event from game controller of Multi-Agent system and feedback control of an agent system, the hybrid system is designed to be a platform implementing our control algorithm in such a complex system. Besides, the relationship between game controller and feedback controller of a continuous state was analyzed in this paper.

Essentially, an agent does not wait for a complete sequence to be generated before a sequence is executed. A partial plan may be executed to meet real-time constraints or when portions of the generated sequences violate a priori determined safe transition. It generates the short term objective and mode sequences to accomplish this objective using the process with the knowledge planner and the algorithms to be discussed later.

3.1 General Nonlinear Kinematics Model

To simplify the problems, the Drift-Free Control System is adopted to formulate the kinematics model of a WMR. We consider a control system of the following form.

$$\dot{x}_i = g(x_i)u_i \quad (1)$$

where $x_i \in R^n, i=1, \dots, m$ denotes the system state with n being the degree of freedom of a WMR, and $g(x_i)$ denotes the vector fields associated with the mobile platform and the manipulator respectively, which are well defined all the time. It can be shown that there exists an admissible control $u: [0, T] \mapsto R^n$ such that the system can be steered from x_i^0 to x_i^1 with $x_i(t) \in V$, a compact set, for all $t \in [0, T]$ if the system is *locally controllable*.

On the other hand, when the continuous trajectory is generated by the navigator, it is necessary to check the *closure* property of the reachable set.

Definition 1 (Reachable set): Given a control system as define in (1), the reachable set from x_i^0 in T seconds is defined by

$$R^v(x_i^0, T) = \left\{ \begin{array}{l} x_i \in R^n \mid \forall x_i(t) : \dot{x}_i(t) = g(x_i(t))u_i \\ , i = 1, \dots, m, t \in [x_i^0, T] \end{array} \right\} \quad (2)$$

Also, these definitions enable us to characterize local controllability as being equivalent to

$$R^v(x_i^0, \leq T) = \cup_{0 \leq \tau \leq T} R^v(x_i^0, \tau) \quad (3)$$

Proposition 1: Considering the system (1) with a locally controller satisfies the Definition 1~5. In discrete plane, there exists a safety set $F \subseteq Q \times X$ which is a map from start to the end while executing a task. We can have a safety property, denoted by $\square F$. If the hybrid system is reachable in hybrid automata such that

$$F(\mathcal{X}) = \begin{cases} True, & \text{if } \forall t \in \tau, (q(t), x(t)) \in F \\ False, & \text{otherwise} \end{cases} \quad (4)$$

When we think about the variation of the physical trajectory on the moving plane of each WMR, the continuous property attracts our attention while switching a state from another one. Intuitively, it should be continuous by our observing. However, it is a problem that how do we manuscript this original idea. As with mapping between manifolds, we can define vector bundle mappings.

Definition 2: Let two discrete state, q and q' , correspondence to two vector bundles E and E' respectively. A map $g: E \rightarrow E' \exists q \rightarrow q'$ is called a C^r vector bundle mapping(isomorphism) when for each $\zeta \in E$ and each admissible local bundle chart (P, ψ) of E' for which $g(\zeta) \in P$ there is an admissible bundle chart (L, φ) with $g(L) \in P$ such that the local representative $g_{\varphi\psi}: \varphi \circ g \circ \psi^{-1}$ is a C^r local vector bundle mapping(isomorphism).

This definition makes sense only for local vector bundle charts and not for all manifold charts. Also, such a L must be guaranteed by the continuity of g . Thus, when we consider a hybrid system with discrete transition, $E \rightarrow E' \exists q \rightarrow q'$, which satisfies the Definition 1 and Definition 2, the trajectory in the working space of a WMR is compact, continuous and connected.

3.2 Hybrid System of a WMR

A hybrid automaton is a dynamical system that describes the evolution in time of the valuations of a set of discrete and continuous variables. The stability analysis issues of an agent have been addressed in [23]. Hereafter, the point of the hybrid system in this paper is to play the role as interactive media under the state switching condition of the Multi-Agent dynamics w.r.t. the designed behaviors, target approach and obstacle avoidance. First of all, we will make the following definition to describe the HMAS with mathematical tool.

Definition 3(Hybrid automation): ybrid automation H is a collection

$$H = (Q, X, U, Y, Init, g, Inv, E, G, R) \quad (5)$$

where

- $Q = \{S, T_a, O, E\}$: set of edges with S being “start”, T_a being “target approach”, O being “obstacle avoidance”, and E being “end”;
- $X = \{x_i \in R^n \mid i = 1, \dots, m\}$: finite set of continuous variables;

- $U = \{X_d\}$: a set of input variables with X_d is the desired state which is generated by PG;
- $Y = \{Q, X, u\}$: a set of output variables;
- $g: Q \times X \rightarrow X$: a vector field;
- $Init \subseteq Q \times X$: set of initial states;
- $Inv: Q \rightarrow 2^X$: a domain (invariant set) under which the dynamics of the mobile manipulator hold;
- $E = \{(Q_i, Q_{i+1}) | (Q_i \rightarrow Q_{i+1}) \subset Q \times Q, 0 < i < m\}$: a set of events;
- $G: E \rightarrow 2^X$: guard-set function;
- $R: E \times X \rightarrow 2^X$: reset relation.

When transition in discrete event state occurs, we define this process so called an execution.

Definition 4: n execution of a hybrid automaton H_i is a collection $\chi = (\tau, q, x)$, where τ is a hybrid time trajectory, $q \langle \tau \rangle \rightarrow Q$ is a discrete variable, and $\chi_i = \{x^i; i \in \langle \tau \rangle\}$ is a collection of dynamics such that

- $(q(0) \ x^0(0)) \in Init$;
- for all $t \in [\tau_i, \tau'_i)$, $\dot{x}^i = f(q(i), x^i(t))$ and $x^i(t) \in Inv(q(i))$;
- for all $i \in \langle \tau \rangle \setminus \{N\}$, $e = (q(i), q(i+1)) \in E$, $x^i(\tau'_i) \in G(e)$, and $x^{i+1}(\tau_{i+1}) \in R(e, x^i(\tau'_i))$.

In addition, for checking the reachability property on a discrete event state, the set of executions of a Multi-Agent system is written as Definition 4.

Definition 5:The collection of an execution of a HMARS is the set of execution of the constituent agents. That is,

$$\xi_H^* = \left\{ \bigcup_{i=1}^N \chi_i \mid \chi = (\tau, q, x) : X \rightarrow f(X) \right\} \quad (6)$$

A hybrid automaton and Multi-Agent execution, after our definition, are described clearly. Naturally, it brings out the question whether each of the state can be touched. Accordingly, it should be checked the property of reachability of hybrid automata before analysis. Here, we define the set of states reachable by H , R_H , as

$$R_H = \left\{ \left(\hat{q}, \hat{x} \right) \in Q \times X : \left(\left[\tau, \tau' \right]_{i=0}^N, q, x \right) \in \xi_H^* \left(\begin{array}{l} \left(q(N), x^N(\tau'_N) \right) = \left(\hat{q}, \hat{x} \right) \end{array} \right) \right\} \quad (7)$$

In hybrid automaton, each of the state presents a kind of behavior draws from human's experiments or a specific well designed autonomous system. In this paper, we, also, concentrate the stability of a

hybrid system of a Multi-Agent with controllers at a state.

3.3 Behavior of an Agent

In this section, an implementation of a hybrid automaton is designed for Multi-Agent behaviors. The fuzzy system has been proposed to inference the modified parameter which controls the turning rate of the heading angle. Besides, a vision system cooperatives with laser system also captures the location of the target object [3].

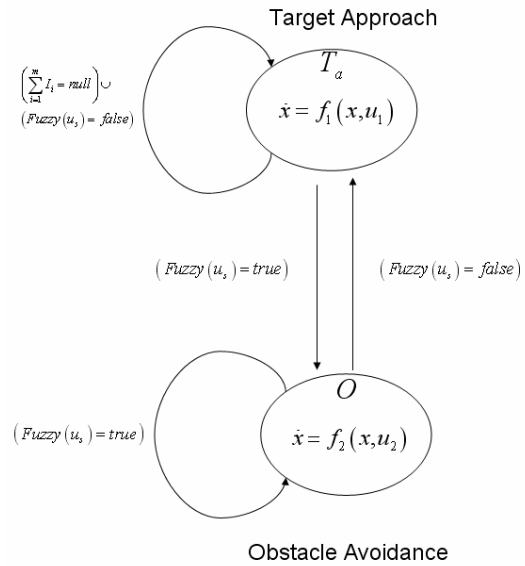


Figure 5: Ybrid state diagram

Where $\sum_{i=1}^m I_i$ denotes the processing result of a visual tracking system. $Fuzzy(u_s)$ is the sensing result form Ultrasound u_s . Each state, namely, start, target approach, obstacle avoidance, and end, is a set denotes as $\{\{q\} | q \in Q\}$, whose definition can be describe as

1. **Start** $S = \{True, False\}$: It's an entry point of the hybrid automaton. The initialize behavior of the system will be taken while entering the state.
2. **Target Approach** $T_a = \{True, False\}$: The Target Approach is an basic behavior in a Multi-Agent system. Basically, the agent will reach the target by dead-reckoning system, ultra-sound system and visual tracking $I = \{I_A, I_u, I_v\} \in R^{3 \times 1}$ when we select a proper closed-loop controller along a desired line path.
3. **Obstacle Avoidance** $O = \{True, False\}$: When visual tracking signal is null and the fuzzy based ultrasonic signal is true, the state will switch to obstacle avoidance.

4. End $E = \{True, False\}$: When all the grasping tasks are finished, the end state will be triggered.

The state flow chart has been depicted in Figure 5 which has 2 states totally (the state of start and end is hidden here) in the automaton. The behavior begins with start state which is assigned all variables with initial configurations. After finishing the initialized step, it transfers to the target approach mode. The obstacle guarding flag, simultaneously, becomes true to guide the WMR navigating freely in the unknown environment. Here, we define $u(i) \in R^{2 \times 1}, i=1,2$ with $u(1)$ being the predicted distance between obstacle and camera, $u(2)$ being the size information of an object.

4. MTUTI-AGENT TASK DESIGN

A general nonlinear framework of a WMR is adopted. In our opinion, it is extensible to other agents if each of them has the similar system configurations then we may analyze the hybrid system applied on those agents by induction method. So far, the second step is to look for the basic property of an agent. At the same time, the definitions on the controllability and the reachability are arisen to be the preparation for further discussion. Finally, a general form of a multi-agent system with supervisory controller can be designed by the proposed analysis tools.

4.1 Task Oriented Domain

The tasks exist in a Task Oriented Domain (TOD) which describes a certain class of scenarios for Multi-Agent encounters. They are given by a task scheduling system. Intuitively, it is a domain that is cooperative, agent welcomes the existence of other agents, for they can only benefit from one another.

Definition 6: Task Oriented Domain (TOD) is a tuple $\langle T, A, J \rangle$ where T denotes the set of all possible tasks, $A = \{A_1, A_2, \dots, A_n\}$ is an ordered list of agents. $J \in R^+$ denotes a convex and monotonic function.

For each finite set of tasks $X \subseteq T$, $J(X)$, a monotonic function, is the cost of executing all the tasks in X by a single agent. There are several useful properties in a serious of tasks [2]:

- Subadditivity: $J(X \cup Y) \leq J(X) + J(Y)$ with $X, Y \subseteq T$.
- Concavity: $J(Y \cup Z) - J(Y) \leq J(X \cup Z) - J(X)$ with $X \subseteq Y$ and $Z \subseteq T$.

- Modularity:

$$J(X \cup Y) = J(X) + J(Y) - J(X \cap Y) \quad \text{with} \\ X, Y \subseteq T.$$

When a group receives a task $\langle T, A, J \rangle$, the local leader will be selected by the shortest distance d from the agent to target. After that, the local leader splits the task $\langle T, A, J \rangle$ into $\langle T, \{A_1, A_2, \dots, A_m\}, J \rangle$ with m being the number of members of the group. Actually, J will be designed as a cost function for the Multi-Agent cooperative game design.

4.2 Formation Task

Formation problem of a Multi-Agent is an essential one associated to cooperative behavior of WMRs. It is not only extending abilities of a Multi-Agent system but also reducing complexity of the task executing. There is a generation task $\langle T, A, J \rangle$. Hence, the formation function is defined:

Definition 7(Formation Function): Considering the Multi-Agent system, there exists a formation function Now, let x_{id} be a desired trajectory. $\tilde{x}_i = x_{id} - x_i$ is the vector of state errors.

$$F_i(x_i) = \frac{1}{2} \tilde{x}_i^T \Gamma \tilde{x}_i \in R \mid_{i=1, \dots, m} \quad (8)$$

where $x_i = [p_{ix} \ p_{iy} \ p_{i\theta}]^T$ denotes a system state of an agent i . Γ is a positive matrix. Now, it interests to connect it with local Lyapunov based controller with formation function.

Theorem 1 (Multi-Agent Formation Control): Considering Multi-Agent system which is composed by agents as system (5), if there exists class K functions, α_3 and proper selected controls

$$u = \varphi(t, x) + d \mid_{\varphi, v \in R^{n \times m}} \quad (9)$$

which satisfied the equation (2) such that

$$\frac{\partial F}{\partial t} \geq -\alpha_3 \|x\| \geq \left(\frac{\partial F}{\partial t} + Wd \right) \quad (10)$$

where $d \in R^m$ is the system disturbance and $W \in R^{n \times m}$ can be obtained as bellow

$$W = \frac{\partial F}{\partial x} g(x)$$

Now, we define a formation function of a HMARS as

$$F(x) = \sum_{i=1}^m \beta_i F_i(x_i), i=1, \dots, m \quad (11)$$

where $x = [x_1 \ \dots \ x_m]^T \in R^{m \times n}$ denotes the continuous state. β_i is a scalar vector and $\sum_{i=1}^m \beta_i = 1$.

If a proposed control, $u = [u_1 \ \dots \ u_m]^T \in R^{m \times n}$ is chosen, the HMARS is asymptotically stable.

Proof: See [4].

By way of Theorem 1, we can design an autonomous heterogeneous Multi-Agent control regarding with tracking problem. Obviously, this system can not undertake complex task without different reactive behaviors design. We try to solve this problem using dynamic cooperative game approach to come to our purpose.

5. DYNAMIC GAME CONTROL OF A HMARS

Game-theoretic modeling starts with an idea related to some aspect of the interaction of decision-makers. For handling the mechanism of a HMARS control, a game theory is applied in this paper to help us to make the optimal decision on the Nash equilibrium point at logic level. In the Multi-Agent problem solving, the cooperative mode under game based supervised system is very important for reflecting the real state. Through the way, we chose a joint costs mode such that it minimizes a cost function and then finally we will design an example of a HMARS dynamic game with only two players. More detail will be discussed in this section.

Remark 1: Considering an Multi-Agent system composed with m WMRs which have a cost function $J_i(x_i, u_i, d_i)$. If the players have the options to cooperative and not to cooperative, and for cooperation either joint cost function method is chosen, then only cooperation takes place. There will be no switches to the noncooperative mode.

Proof sketch: According the definition of weights $\gamma_i, i = 1, \dots, m$,

$$\begin{aligned} J^* &= J_1(x_1^*, u_1^*, d_1) + \dots + J_m(x_m^*, u_m^*, d_m) \\ &\leq J_1(x_1, u_1, d_1) + \dots + J_m(x_m, u_m, d_m) \end{aligned}$$

For arbitrary u_1, u_2 and therefore also for the controls u_1^N, u_2^N with N steps which constitute the Nash equilibrium. Hence, $J^* = J_1^N + \dots + J_m^N$ denotes that it is always advantageous for the players to cooperative.

5.1 Cost Function Design

For applying optimal strategies, the cost

function is selected carefully to evaluate the i^{th} agent performance.

$$J_i = \int_{t=0}^T \left(F_i(x_i, d_i) + \frac{1}{2} u_i^T(x_i, d_i) \Lambda u_i(x_i, d_i) \right) dt \quad (12)$$

Where $F_i(x_i, d_i)$ is written as equation (8). u_i represents the well-designed control on the continuous state as equation (9).

Theorem 2 (Existence of a HMARS) Consider an agent that is continuous differentiable and satisfies the Lipschitz condition. Then, the Multi-Agent system as equation (1) with $x_i(t_0) = x_{0i}$ has the unique solution over $[t_0, t_0 + \delta]$.

Proof: By integration

$$x_i(t) = x_{0i}(t) + \int_{t_0}^t g(x_i(s)) u_i ds \quad (13)$$

We can write equation (11) as

$$x_i(t) = (Px)(t) \quad (14)$$

The existence of equation (14) can be established with contraction mapping theorem. This requires defining a Banach space $\mathcal{X} = \{x_i \mid i = 1, 2, \dots, m\}$ and a closed set $S \subset \mathcal{X}$ such that P maps S into S and is a contraction over S . Let $\mathcal{X} = C[t_0, t_0 + \delta]$ with norm $\left\| \sum_{i=1}^m x_i \right\|_C = \max_{t \in [t_0, t_0 + \delta]} \left\| \sum_{i=1}^m x_i(t) \right\|$ and $S = \left\{ \sum_{i=1}^m x_i \mid x_i \in \mathcal{X}, \|x_i - x_{i0}\| \leq r_i \right\}$. where r is the radius of the ball B and δ is a positive constant to be chosen. Notice that $\|x_i(t)\|$ denotes a norm on R^n , while $\|x\|_C$ denotes a norm on \mathcal{X} . To show that it maps S into S , write

$$\begin{aligned} x_i(t) - x_{0i} &= \int_{t_0}^t \left[g(x_i(s)) u_i(s) - g(x_{0i}(s)) u_i(s) + g(x_{0i}(s)) u_i(s) \right] ds \quad (15) \end{aligned}$$

Using the Lipschitz condition and the fact that for each $r \in S$,

$$\begin{aligned} (Px_i - x_{i0})(t) &= \sum_{i=1}^m \max_{t \in [t_0, t_0 + \delta]} \left\| (Px_i)(t) - x_{i0} \right\| \leq \delta(Lr + h) \quad (16) \end{aligned}$$

where $h = \sum_{i=1}^m \max_{t \in [t_0, t_1]} \|g(x_i(t))u_i(t)\|$.

Hence, choosing $\delta \leq r/(Lr+h)$ ensure that P maps S into S . To show that P is a contraction mapping over S , let x and $y \in S$ and consider

$$\begin{aligned} & \| (Px)(t) - (Py)(t) \| \\ &= \left\| \int_{t_0}^t [g(x_i(s))u_i(s) - g(y_i(s))u_i(s)] ds \right\| \quad (17) \\ &\leq \int_{t_0}^t ds \cdot L \|x - y\|_C \end{aligned}$$

Therefore $\|Px - Py\|_C \leq \delta \|x - y\|_C$ for $\delta \leq \rho/L$.

Choosing $\rho < 1$ and $\delta \leq \rho/L$, by contraction mapping theorem, we can conclude that if $\delta \leq \rho/L$ is chosen to satisfy

$$\delta \leq \min \left\{ t_1 - t_0, \frac{r}{Lr+h}, \frac{\rho}{L} \right\} \quad (18)$$

then we have a unique solution in S .

Theorem 3 (Stability): The HMARS which satisfies Theorem 2 is *asymptotically stable* if and only if we choose a cost function as equation (11).

Proof: The forward proof is easy to proof with optimal control theory. On the contrast, in equation 10, the control is selected as equation (9). For solving the optimal problem, the nonlinear equation (1) should be linearized using Taylor expansion and written as

$$\dot{x}_i = G(x_i)u_i \quad (19)$$

where $G(x_i) = \frac{\partial g(x_i)}{\partial x_i}$. Hence, the controller u_i

takes the agents to any where with a limit time and a bounded speed which is proved by Theorem 1 if the formation function satisfied equation (11). By equation (12), the Lagrange equation is obtained as

$$L = \sum_{i=1}^m J_i(x_i, u_i) + \lambda \sum_{i=1}^m (G(x_i)u_i) \quad (20)$$

Naturally, it is an optimal problem to solve an optimal control u^* . No doubt, the system is asymptotically stable by optimal control theory if the solution exists via evaluating of Theorem 2.

However, the reachability of the HMARS on the logic level has to be contented. It is, actually, an important issue whether the solution satisfies

Multi-Agent system especially the logic is constrained by the interaction of agents.

5.2 Two Agent Game with Constrained Logic

The model of the two agents with two behaviors is drawn as Figure 6. Both of the agents are autonomous and homogenous. The behavior $q_{ij}, i=1,2, j=1,2$ is a set of collections from each of an agent at the same group which is defined as

$$Q = \{q_{11}, q_{21}, q_{12}, q_{22} | q_{11}, q_{21} \subseteq T_a, q_{12}, q_{22} \subseteq O\} \quad (21)$$

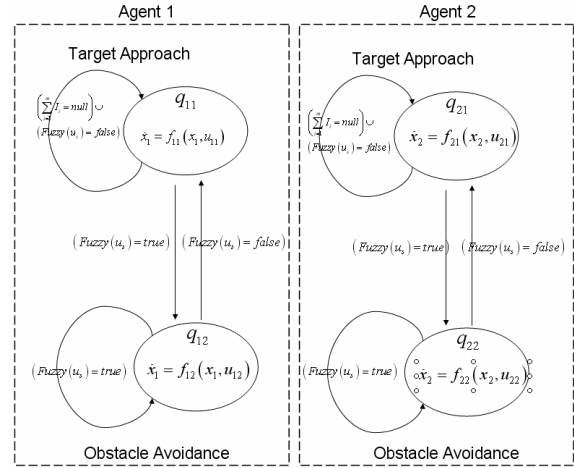


Figure 6: Hybrid automaton of two agent systems

The hybrid automaton of two autonomous agents shows as Figure 6. From system point of view, it is a homogenous Multi-Agent system. In addition, the strategies of a two agent game is formulate as Table 1.

Table 1: The table of a game of a HMARS

$A_1 \backslash A_2$	q_{11}	q_{12}
q_{21}	$(J_1(\tilde{x}_1, u_{11}), J_2(\tilde{x}_2, u_{21}))$	$(J_1(\tilde{x}_1, u_{12}), J_2(\tilde{x}_2, u_{21}))$
q_{22}	$(J_1(\tilde{x}_1, u_{11}), J_2(\tilde{x}_2, u_{22}))$	$(J_1(\tilde{x}_1, u_{12}), J_2(\tilde{x}_2, u_{22}))$

In Table 1, the Nash equilibrium point can be derived by the hybrid automaton with 2 agents. It is to say, there exists a game solution for the both of the strategy, i.e., (q_{12}, q_{22}) . If both of agents are competition with each other, it is the solution what we want. However, it is not reasonable that we have such a strategy for minimize one agent's cost and maximize another one's cost. Hence, this game we prefer to have a mixed strategies solution otherwise it is a cooperative game which minimizes both of the cost and can be defined as

$$\min_{J_1, J_2} f(J_1, J_2) \tag{22}$$

where $f(J_1, J_2) = \gamma_1 J_1 + \gamma_2 J_2$ depicts the cooperative function with being $\gamma_1 + \gamma_2 = 1$. Plainly, f is a convex function. If we overview the game with dynamic property with time then we has the following definition for the optimal response.

Definition 7: In a two-person nonzero-sum game, let the minimum of $f(J_1, J_2)$ with respect to $J_2 \in X \times U_2$ be attainable for each $J_1 \in X \times U_1$. Then, the set

$$R(u_1, u_2) = \left\{ \begin{array}{l} \xi \in X \times U \mid f(J_1(x, u_1), \xi) \\ \leq f(J_1(x, u_1), J_2(x, u_2)) \\ , \forall u \in U \end{array} \right\} \tag{23}$$

is called the optimal response or rational reaction set of Agent 2. If $R(u_1, u_2)$ is a singleton for every $u_1 \in U_1$, the it is called reaction curve or reaction function of Agent 2, and is denoted by $l_2(u_1)$. The reaction set $R(u_1, u_2)$ and curve $l_1(u_2)$ of Agent 1 are similarly defined. For a large class of problems, the Pareto-optimal solutions can be obtained by minimizing the scalar criterion

$$J_i = \mu_{i1} J_{i1} + \mu_{i2} J_{i2} + \dots + \mu_{iN} J_{iN} \tag{24}$$

for some scalar coefficients μ_{ij} satisfying $\sum_{j=1}^N \mu_{ij} = 1, \mu_{ij} > 0$. Here the cost function J_{ij} , defined on R^n are assumed to be complex and twice differentiable with respect to J_{ij} for $i \in m, j \in N$. Graphically, the set of Pareto-optimal solutions is illustrated the curve A and B in Figure 7.

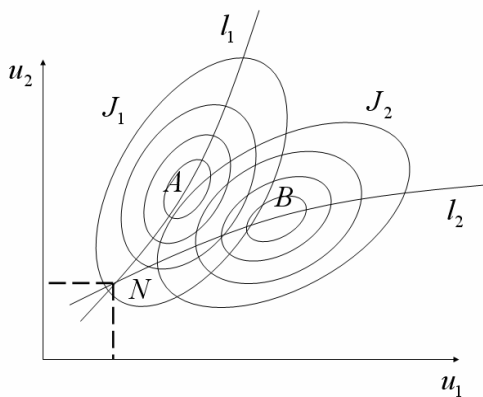


Figure 7: Pareto-optimal solution of a two person game

Now we turn back to review the properties under cooperation game of a HMARS.

Lemma 1 (Uniqueness of The Game): Considering a system with a two agents game as equation (5), there is a task $\langle T, A, J \rangle$ such that a cost function can be chosen as equation (22). For $x_i \in G_i(q_j, q'_j)$, there exists $(q_j, q'_j) \in E$ such that $\chi_k \rightarrow \chi_{k+1}$ with j and k being the jth and kth execution. If there exist an execution of Multi-Agent system ξ_H^* then an unique dynamic game exists.

Proof: The task $\langle T, A, J \rangle$ is assigned dynamically to each of agents. The execution ξ_H^* of a Multi-Agent system is occurred with $(q_j, q'_j) \in E$ such that $\chi_k \rightarrow \chi_{k+1}$. By Theorem 2, there exists unique cost function for HMARS. It is to say, there exist a unique game for the system.

From lemma 1, we understand an event triggers the HMAS dues to a dynamic game for each of agents at the same group. Nevertheless, there are some specific properties happened while executing the game with HMAS.

Lemma 2 (Constraint Logics (CL) in HMARS): Considering a HMAS with a controlling time ΔT , $x_i \in G_i(q_j, q'_j)$, there does exist $(q_j, q'_j) \in E$ such that $\chi_k \rightarrow \chi_{k+1}$. If the CL occurs, the state is forced to remain itself: $q_j \rightarrow q_j$, all $(\tau, q_j, x_j) \in \xi_H(q_{j0}, x_{j0})$.

Proof: When agent 1 detects an obstacle, that is to say, $\left(\sum_{i=1}^m I_i = null \right) \cup (Fuzzy(u_s) = true)$. The agent 1 can not switch to another state while $Fuzzy(u_s) = true$. On the other word, the state is forced to remain itself: $q_j \rightarrow q_j$, all $(\tau, q_j, x_j) \in \xi_H(q_{j0}, x_{j0})$.

Corollary 1(Unbalanced Game with Constrains Logic): Considering a system (5), for $x_i \in G_i(q_j, q'_j)$, there exists $(q_j, q'_j) \in E$ such that $\chi_k \rightarrow \chi_{k+1}$ with j and k being the jth and kth execution. If it satisfied Lemma 2 during an execution then it is an unbalanced dynamic game.

Proof: We assume agent 1 occurs an CL which leads the state stays at q_{i2} . According to Table 1, the competition of the game is constrained to the right part of the table. It is an unbalanced game with CL.

Corollary 1 emphasizes that the significant step for executing the assigned task is to evaluate the constrained logic. Moreover, the dynamic game

becomes unbalanced when specific event trigger the guard set persistently.

6. CONCLUSION

In this paper, a novel design and analysis of a dynamic game of a Hybrid Multi-Agent System (HMARS) control is proposed. Under the well designed controller, each autonomous agent is composed of a WMR with switching behaviors, target approach, and obstacle avoidance. The agents, subjected to the flow over a hybrid model, can perform formation with other agents to successfully accomplish cooperative missions. The contributions of this paper consist of three parts: first, Lyapunov based formation control of a HMARS is developed subjected to the reachability, existence and stability issues under the hybrid approach; second, the dynamic cooperative game of a HMARS is proposed and analyzed for the stability with Constrained Logic; third, this research offers a reasonable framework to implement cooperation problem of a HMARS with the innate characteristics from discrete event space to continuous state space.

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