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Weighted least-square estimation of demand product mix and its applications to semiconductor demand

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Estimation of demand product mix is important for effective production plans. Unlike most research in the literature where the product mix is either given or treated as a decision variable in optimization of the production efficiency, this paper focuses on the product mix itself and how to estimate it from the market demand. With more accurate information on the demand product mix, aggregate production plans for product families can be disaggregated into quality detailed plans for individual product items. In this paper, least-square estimates of demand product-mix proportions are first derived. To take into account the effect of the product life cycle, dynamic weighting schemes are then developed to improve the accuracy of the product-mix estimates. For applications, we concentrate particularly on semiconductor demand where new generations of semiconductor products emerge at the pace of every six months, as manifested by the celebrated Moore's laws. The proposed methodologies will be tested with simulated DRAM demands and actual semiconductor demands of different technology generations.

Keywords: Least-square estimation; Product-mix estimate; Demand disaggregation; Exponential weighting

1. Introduction

Results of demand planning serve as the basis of every planning activity in the supply network and ultimately determine the effectiveness of manufacturing/logistic operations in the network. Because demand forecast at the level of individual product items has become more and more difficult, a common practice of production planning is to plan at the aggregate level first for the product families. The detailed scheduling for individual product items is then determined by disaggregating the aggregate plans. Such a planning practice is commonly known as hierarchical production planning (HPP). A major difficulty of HPP is to uphold the consistency between the aggregate plan and the detailed plan (Özdamar *et al*. 1996, Zäpfel 1996, Özdamar and Yazgaç 1999). A coherent detailed plan requires accurate estimation of the individual product demand from the demand of the product family. A demand planning approach is therefore to make a forecast at the aggregate level and then break down (disaggregate) the forecast statistically and/or judgmentally into

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the individual forecasts. This top-down demand planning approach is known to be an effective means for better forecasting because the aggregated product-family demand is observed to fluctuate less and easier to make a forecast (Grunfeld and Griliches 1960, Schwarzkoph *et al.* 1988, Ilmakunnas 1990, Kahn 1998, Zotteri *et al.* 2005). In the literature of supply chain planning, demand aggregation is also known as a 'risk-pooling' strategy to reduce demand fluctuation for more effective material/capacity planning (Simchi-Levi *et al.* 2000). This approach, however, requires an accurate product-mix prediction to break down the aggregate demand forecast into individual product forecasts. Demand product-mix can be expressed as individual demand proportions in a product family. For example, demand proportions of 64, 128 and 256 M DRAM form the product-mix of the DRAM product family. Thus, a disaggregation method is required for the demand product-mix estimation to break down the aggregate demand. That is, to estimate the demand product-mix, we need to estimate the proportions of individual demands in a product family.

Gross and Sohl (1990) have investigated and compared twenty-one disaggregation methods to estimate the proportion of each product item in the product family. Most of the methods have a common root in the literature of forecasting combination (Mahmoud 1984). However, from their empirical study the two most effective methods are based on the simple average of a product item's share in the family over a period of time (Makridakis and Winkler 1983). One of the simple-average methods (method F in Gross and Sohl 1990) is to calculate the mean product demand and the mean total demand per period first and then calculate the proportion of the mean product demand in the mean total demand. This method is referred to as a *mean-proportion* estimate:

$$\hat{P}_i = \frac{\sum_{t=1}^{n} d_{it}/n}{\sum_{t=1}^{n} D_t/n}, \tag{1}$$

where d_{it} is the demand of product i at period t; n is the number of historical demand periods; $D_t = \sum_{i=1}^{k} d_{it}$ is the total demand at period t; k is the number of products in the product family; and $\hat{P}_i$ is the mean-proportion estimate of product i. The other simple-average method (method A in Gross and Sohl 1990) is to calculate the proportion of each product for each period first and then take the average of the proportions over a period of time:

$$\hat{p}_i^* = \frac{\sum_{t=1}^{n} d_{it}/D_t}{n},$$

where $\hat{p}_i^*$ is referred to as a *proportion-mean* estimate. These two methods are both provided without theoretical development. One of this paper's objectives is thus to lay a theoretical foundation for development of the least-square estimates. It is found through our theoretical development that the proportion-mean estimate is actually one of the least-square estimates.

An effective estimate of the demand product-mix should also take into consideration characteristics of the market dynamics and the product life cycles. There are three important characteristics that will be considered in our proposed estimators:

1. Product life cycle (PLC) (Brockhoff 1967, Cox 1967, Thorelli and Burnett 1981): fast-paced technology development leads to fast PLC transition (Easingwood 1988).

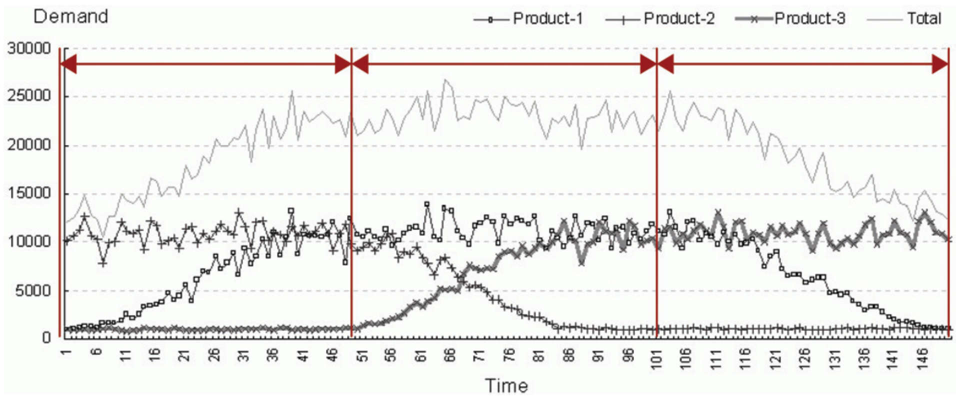

Figure 1. Simulated DRAM demands.

2. Demand switchover: the demand for new-generation products increasingly replaces the demand for products of old technologies.
3. Variability proportional to volume: the greater the demand volume, the more volatile the demand (Brown 1959, Heath and Jackson 1994).

In figure 1, we simulated the 128 Mb (product 2), 256 Mb (product 1) and 512 Mb (product 3) DRAM demands to resemble these three characteristics. Though we intend to simulate the weekly wafer demand for an IC company, the units of demand and time are not explicitly indicated in figure 1 to reflect the general DRAM demand trend regardless of the market specificity.

It can be seen that the switchover from the demand for phasing-out products to the demand for emerging products leads to a dramatic change in the product-mix. It is difficult to estimate the demand product-mix without considering the effects of PLC and demand switchover. The second objective of this research is then to accommodate these characteristics of dynamic demands in the proposed estimates.

This paper is organized into six sections. Following this introduction, we develop the demand product-mix estimates with least squared errors. The weighted least-square estimates are then derived in section 3. To capture the PLC effect for better product-mix estimation, the dynamic weighting schemes are proposed in section 4. In section 5, the proposed methods are tested with the simulated DRAM demands and the actual semiconductor demands of different technology generations. Finally, some concluding remarks are made in the last section.

2. Least-square estimates of demand product-mix

The first objective of this paper is to find a demand product-mix estimate that best fits the historical demand trends. The sum of squared errors (SSE) is usually used to evaluate the quality of the estimate – the lower the SSE, the better the estimate. A least-square estimate is obtained by minimizing the SSE. We calculate two types of SSE in this research, one is the sum of squared demand errors (SSEd) and the other is the sum of squared proportion errors (SSEp).

2.1 *Least-SSEd demand product-mix estimate*

First, we focus on minimizing the demand estimate error. Let p_i represent the expected proportion of product i. Then, the following model describes how the demand of product i is obtained through its proportion in the product family:

$$d_{it} = p_i D_t + \varepsilon_{it}, \tag{2}$$

where $\sum_{i=1}^{k} p_i = 1$ and ε_{it} is the demand error of product i at period t with $\sum_{i=1}^{k} \varepsilon_{it} = 0$.

Usually D_t can be modelled as a time series for demand forecasting. Though forecasting is a very important subject, it is not in our current research scope. In model (2), errors are assumed to cause the period-to-period demand deviations (ε_{it}) from its nominal share ($p_i D_i$) in the total family demand. The sum of squared demand errors can be calculated as:

$$\text{SSEd} = \sum_{t=1}^{n} \sum_{i=1}^{k} (d_{it} - p_i D_t)^2. \tag{3}$$

We want to find the estimate of p_i that minimizes SSEd subject to $\sum_{i=1}^{k} p_i = 1$. That is to minimize:

$$\text{SSEd} = \sum_{t=1}^{n} \sum_{i=1}^{k} (d_{it} - p_i D_t)^2 - \lambda \left(\sum_{i=1}^{k} p_i - 1 \right), \tag{4}$$

where λ is a Lagrange multiplier (see, for example, chapter 20 of Taha 2002). Since the partial derivatives of (4) with respect to p_i are:

$$\frac{\partial \text{SSEd}}{\partial p_i} = -2 \sum_{t=1}^{n} d_{it} D_t + 2p_i \sum_{t=1}^{n} D_t^2 - \lambda, \tag{5}$$

we obtain the least-SSEd estimate $\hat{p}_i$ by setting (5) equal to zero:

$$\hat{p}_i = \frac{\sum_{t=1}^{n} d_{it} D_t + (\lambda/2)}{\sum_{t=1}^{n} D_t^2}. \tag{6}$$

To satisfy $\sum_{i=1}^{k} \hat{p}_i = 1$, λ in (6) has to be 0. Therefore,

$$\hat{p}_i = \frac{\sum_{t=1}^{n} d_{it} D_t}{\sum_{t=1}^{n} D_t^2}. \tag{7}$$

$\hat{p}_i$ is the estimate of product i proportion that minimizes the sum of squared differences between $D_t \hat{p}_i$ and d_{it}. This demand proportion estimate is first proposed by this research and will be shown to be more accurate, in terms of demand errors, than the two most effective methods in Gross and Sohl's (1990) study.

2.2 *Least-SSEp product-mix estimate*

Now, our aim turns to minimize the proportion errors. Let each product i have an observed proportion d_{it}/D_t at period t. The following model is used to describe how this observed proportion deviates from the expected proportion p_i:

$$\frac{d_{it}}{D_t} = p_i + \varepsilon^*_{it}, \tag{8}$$

where ε^*_{it} is the proportion error of product i at period t and $\sum_{t=1}^{n} \varepsilon^*_{it} = 0$. Unlike model (2), model (8) assumes that errors cause the actual proportion (d_{it}/D_t) to deviate from its expected value (p_i). The sum of squared proportion errors can be calculated as:

$$\text{SSEp} = \sum_{t=1}^{n} \sum_{i=1}^{k} \left(\frac{d_{it}}{D_t} - p_i \right)^2. \tag{9}$$

Now we want to find the estimate of p_i that minimizes the SSEp subject to $\sum_{i=1}^{k} p_i = 1$. That is to minimize:

$$\text{SSEp} = \sum_{t=1}^{n} \sum_{i=1}^{k} \left(\frac{d_{it}}{D_t} - p_i \right)^2 - \lambda \left(\sum_{i=1}^{k} p_i - 1 \right), \tag{10}$$

where λ is the Lagrange multiplier. Since the partial derivatives of (10) with respect to p_i are:

$$\frac{\partial \text{SSEp}}{\partial p_i} = -2 \sum_{t=1}^{n} \frac{d_{it}}{D_t} + 2np_i - \lambda, \tag{11}$$

we obtain the least-SSEp estimate $\hat{p}^*_i$ by setting (11) equal to zero:

$$\hat{p}^*_i = \frac{\sum_{t=1}^{n} (d_{it}/D_t) + (\lambda/2)}{n}. \tag{12}$$

To satisfy $\sum_{i=1}^{k} p_i = 1$, λ in (12) has to be 0. Therefore,

$$\hat{p}^*_i = \frac{\sum_{t=1}^{n} d_{it}/D_t}{n}. \tag{13}$$

$\hat{p}^*_i$ is the proportion estimate that minimizes the sum of squared difference between d_{it}/D_t and $\hat{p}^*_i$. $\hat{p}^*_i$ is in effect the sample mean of observed proportions, d_{it}/D_t, and is exactly the same as the proportion-mean estimate mentioned in method A of Gross and Sohl (1990).

3. Weighted least-square estimates

In the previous section, demands are treated equally important in the least-square estimates regardless of their ages. To place varied emphases on demands at different periods of time, weights can be applied to different time periods in (1), (7), and (13) to obtain weighted demand product-mix estimates. First, the mean-proportion estimate in (1) now becomes a ratio of weighted average of historical demands:

$$\hat{P}_{i,n+1} = \frac{\sum_{t=1}^{n} W_{it} \cdot d_{it}}{\sum_{j=1}^{m} \sum_{t=1}^{n} W_{jt} \cdot d_{jt}}, \tag{14}$$

where $\hat{P}_{i,n+1}$ is the estimate of product i proportion for period $n+1$; and W_{it} is the weight applied to product i demand at time t and satisfies $\sum_{t=1}^{n} W_{it} = 1$.

Now, we apply the weights to the least-square proportion estimates in (7) and (13). For the least-SSEd estimate, different weights are given to the squared estimate errors of different products at different time periods. The weighted sum of squared demand errors (WSSEd) can be calculated as:

$$\text{WSSEd} = \sum_{t=1}^{n}\sum_{i=1}^{k} w_{it}(d_{it} - p_i D_t)^2,$$

where w_{it} is the weight given to the squared demand error of product i at period t. The WSSEd is then minimized subject to $\sum_{i=1}^{k} p_i = 1$. That is to find estimates of p_i minimizing:

$$\text{WSSEd} = \sum_{t=1}^{n}\sum_{i=1}^{k} w_{it}(d_{it} - p_i D_t)^2 - \lambda\left(\sum_{i=1}^{k} p_i - 1\right), \tag{15}$$

where λ is a Lagrange multiplier. Setting the partial derivatives of (15) with respect to p_i to zero:

$$\frac{\partial \text{WSSEd}}{\partial p_i} = -2\sum_{t=1}^{n} w_{it} d_{it} D_t + 2p_i \sum_{t=1}^{n} w_{it} D_t^2 - \lambda = 0,$$

we obtain the weighted least-SSEd product-mix estimate for period $n+1$:

$$\hat{p}_{i,n+1} = \frac{\sum_{t=1}^{n} w_{it} d_{it} D_t + (\lambda/2)}{\sum_{t=1}^{n} w_{it} D_t^2}, \tag{16}$$

where λ should be evaluated such that $\sum_{i=1}^{k} \hat{p}_i = 1$ is met. Let b_i be $\sum_{t=1}^{n} w_{it} d_{it} D_t$ and a_i be $\sum_{t=1}^{n} w_{it} D_t^2$. Then,

$$\lambda = \frac{2\left(\prod_{i=1}^{k} a_i - \sum_{j=1}^{k}\left(b_j \prod_{i=1}^{k} a_i\right)/a_j\right)}{\sum_{j=1}^{k}\left(\prod_{i=1}^{k} a_i\right)/a_j}. \tag{17}$$

Similarly, weighted least-SSEp estimate (WSSEp) can be calculated as:

$$\text{WSSEp} = \sum_{t=1}^{n}\sum_{i=1}^{k} w_{it}^*\left(\frac{d_{it}}{D_t} - p_i\right)^2,$$

where w_{it}^* is the weight given to the squared proportion error of product i at period t. The WSSEd is then minimized subject to $\sum_{i=1}^{k} p_i = 1$. That is to find estimates of p_i minimizing

$$\text{WSSEp} = \sum_{t=1}^{n}\sum_{i=1}^{k} w_{it}^*\left(\frac{d_{it}}{D_t} - p_i\right)^2 - \lambda^*\left(\sum_{i=1}^{k} p_i - 1\right), \tag{18}$$

where λ^* is a Lagrange multiplier. Set the partial derivative of (18) to zero:

$$\frac{\partial \text{WSSEp}}{\partial p_i} = -2\sum_{t=1}^{n} w_{it}^* \frac{d_{it}}{D_t} + 2p_i \sum_{t=1}^{n} w_{it}^* - \lambda^* = 0,$$

to obtain the weighted least-SSEp product-mix estimate for product i at period $n+1$:

$$\hat{p}^*_{i,n+1} = \frac{\sum_{t=1}^{n} w^*_{it}(d_{it}/D_t) + \lambda^*/2}{\sum_{t=1}^{n} w^*_{it}}, \tag{19}$$

where λ^* should be chosen such that $\sum_{i=1}^{k} \hat{p}^*_i = 1$ is satisfied. Let u_i be $\sum_{t=1}^{n} w^*_{it}(d_{it}/D_t)$ and v_i be $\sum_{t=1}^{n} w^*_{it}$. Then,

$$\lambda^* = \frac{2\left(\prod_{i=1}^{k} v_i - \sum_{j=1}^{k}\left(u_j \prod_{i=1}^{k} v_i\right)/v_j\right)}{\sum_{j=1}^{k}\left(\prod_{i=1}^{k} v_i\right)/v_j}. \tag{20}$$

Both weighted least-square estimates, (16) and (19), are first derived by this research. However, how to determine weights with consideration of product life cycle effects remains an issue to be addressed in the following section.

4. Exponential weights and PLC leading indicators

To capture the effects of PLC transition on the product-mix, various weights should be placed on demands of different ages. When the product demand is on the rise or decline, more weights should be placed on the most recent demands to reflect the heavier influence of the recent market transition. On the other hand, when the product demand is in a steady, mature phase, all demands in the steady phase should be equally accounted for in the estimate of the product-mix. In order to weigh the most recent demands more and to be able to adjust the weighting easily for different demand trends, we propose using the exponential weights for $i = 1, \ldots, k$ and $t = 1, \ldots, n$:

$$W_{it} = \frac{\beta_i(1-\beta_i)^{n-t}}{1-(1-\beta_i)^n} \tag{21}$$

$$w_{it} = \alpha_i(1-\alpha_i)^{n-t} \tag{22}$$

$$w^*_{it} = \alpha^*_i(1-\alpha^*_i)^{n-t} \tag{23}$$

with single smoothing constants β_i, α_i, and α^*_i for $\hat{P}_{i,n+1}$, $\hat{p}_{i,n+1}$ and $\hat{p}^*_{i,n+1}$, respectively. Using α_i as an example, figure 2 shows how the smoothing constant affects the weight distribution over different ages of demands.

How to choose appropriate values of β_i, α_i, and α^*_i in (21), (22), and (23) becomes critical for accurate product-mix estimation. β_i and α_i can be found such that the sum of squared demand forecast errors over s periods:

$$\mathrm{SSFEd}(s) = \sum_{\tau=t-s+1}^{t} \sum_{i=1}^{k} (\tilde{p}_{i,\tau} D_\tau - d_{i\tau})^2, \tag{24}$$

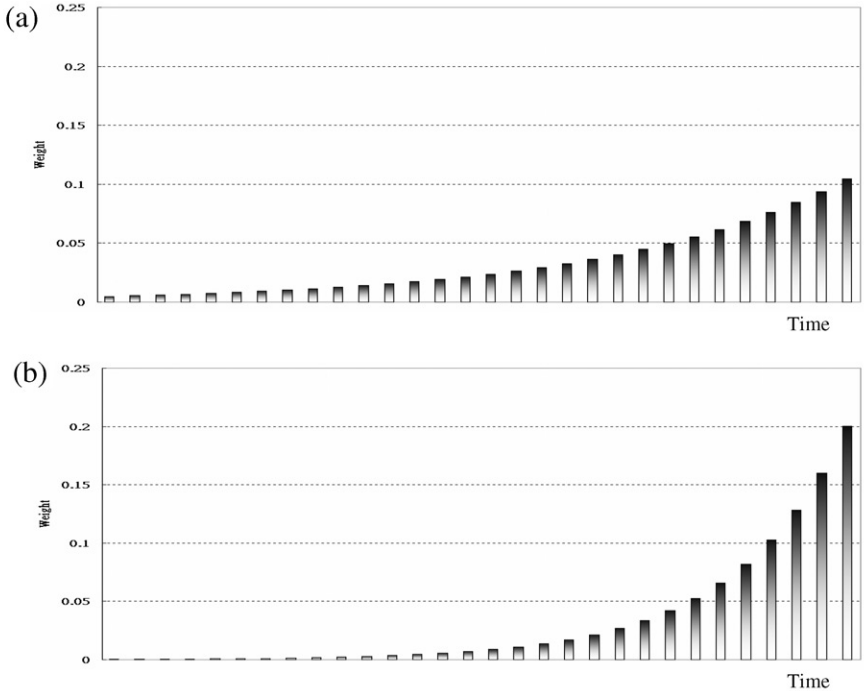

Figure 2. Exponential weight distributions controlled by the smoothing constant. (a) $\alpha_i = 0.1$, (b) $\alpha_i = 0.2$.

is minimized, where $\tilde{p}_{i,\tau}$ is the estimate for product i proportion using historical demands up to period $\tau - 1$ and is used as the forecast for period τ. $\tilde{p}_{i,\tau}$ can be estimated by $\hat{P}_{i,\tau}$ using (14) or by $\hat{p}_{i,\tau}$ using (16). Similarly, α_i^* can be found such that the sum of squared proportion forecast errors over s periods:

$$\text{SSFEp}(s) = \sum_{\tau=t-s+1}^{t} \sum_{i=1}^{k} \left(\hat{p}_{i,\tau}^{*} - \frac{d_{i\tau}}{D_\tau} \right)^2, \tag{25}$$

is minimized, where $\hat{p}_{i,\tau}^{*}$ is calculated using (19) with historical demands up to $\tau - 1$ and is used as the proportion forecast for period τ. Determination of smoothing constants is a very time consuming task. Take α_i as an example and suppose each α_i has 99 possible values ($\alpha_i = 0.01$–0.99). Then, there are 99^k possible $(\alpha_1, \alpha_2, \ldots, \alpha_k)$ candidates. The time to compute all candidates to search for the best $(\alpha_1, \alpha_2, \ldots, \alpha_k)$ to minimize (24) requires enormous computing power. The most common method is steepest-descent method (see, for example, chapter 21 of Taha 2000). The search for the smoothing constants can be seen as a k-variable $(\alpha_1 - \alpha_k)$ steepest-descent search with (24) as the objective function to minimize. Treat $(\alpha_1, \alpha_2, \ldots, \alpha_k)$ as a point α in the k-dimension coordinate system. Here, we simplify the steepest-descent search process to be a process that finds the point minimizing (24) by adjusting each α_i with a fixed step size r. There are three possible movements for each α_i: adding one step

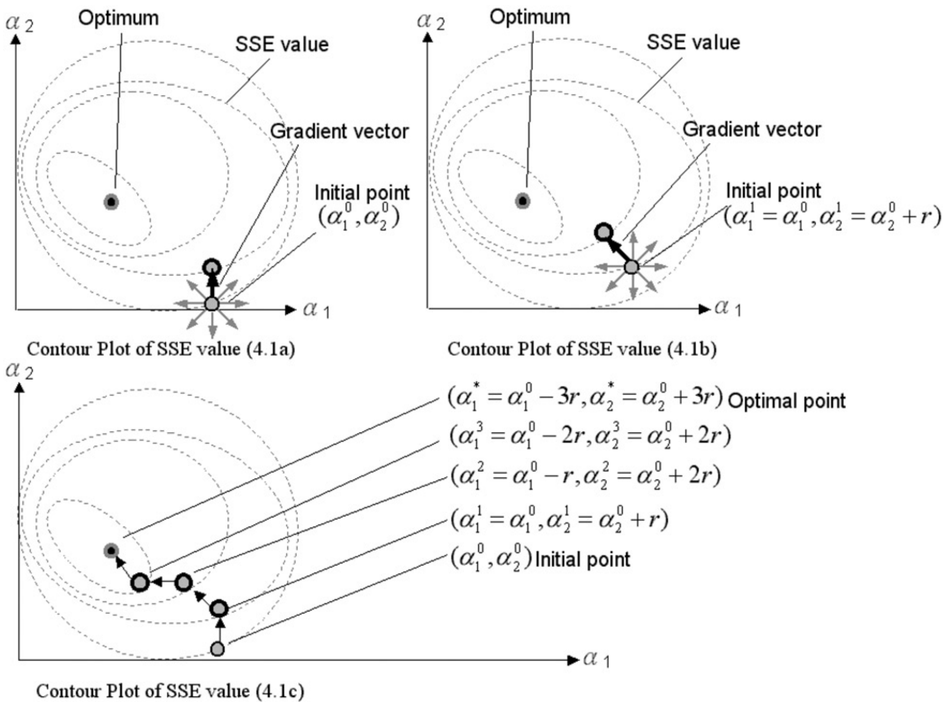

Figure 3. Simplified two-product steepest-descent search process.

size to α_i, i.e., $\alpha_i + r$, subtracting one step size from α_i, i.e., $\alpha_i - r$, or keep the α_i at the original position. The direction formed by successive points of α representing the most SSE-reduced direction which is called the 'gradient vector' while all the other possible directions are called 'candidate vectors'. Termination of the search occurs at the point where the gradient vector becomes null; i.e., the current α point has the smallest SSE value. An example of two-product steepest-descent search for a pair of smoothing constants is shown in figure 3.

The steepest-descent method is still a computation-intensive method. It would be much more efficient if we knew the direction toward the minimum. Taking the effect of changing demand variability into account, we propose a PLC transition leading indicator using the sample one-lag autocorrelation (SAC) statistic:

$$\mathrm{SAC}_{i,t} = \frac{1/s \sum_{\tau=t-1}^{t-(s-1)} (d_{i,\tau} - \bar{d})(d_{i,\tau+1} - \bar{d})}{1/s \sum_{\tau=t}^{t-(s-1)} (d_{i,\tau} - \bar{d})^2}, \tag{26}$$

where $\mathrm{SAC}_{i,t}$ is the sample one-lag autocorrelation of product i calculated at period t using the demand data set $\{d_{i,t\text{-}s+1}, \ldots, d_{i,t}\}$. SAC is a measure of stickiness between demands of successive periods and thus a measure of demand-trend significance. When the product is at the 'growth' or 'decline' phase; i.e., the product-mix proportion significantly rises or falls, the value of SAC will become larger because the market trend dominates the product-mix changes. If the product is mature in the market and its product-mix proportion is stable, the value of SAC will become smaller because only the noise dictates its proportion changes. When the value of SAC is large, the value of the smoothing constant should be large as well to weigh the most recent demands more and, thus, to capture the new market trend more effectively. Figure 4 shows the relationship among the SAC trend, the PLC stage, and the expected trend of the smoothing constant.

Sample size s in (26) determines how sensitive the SAC is to the PLC transition and to the demand noise. Figure 5 shows the SAC values with sample sizes $s = 15$, 25 and 50 for the simulated 256 Mb DRAM demand proportion. Since SAC is in the range of $[-1, 1]$ and the demand proportion is in the range of $[0, 1]$, we are able to directly superimpose the 256 Mb demand proportion onto the figure to show how the SAC responds to the proportion changes.

It can be seen that the SAC with a large sample size ($s = 50$) is too slow to reflect the PLC transition while the SAC with a small sample size ($s = 15$), though responsive to the PLC transition, is too sensitive to the demand noise. The sample

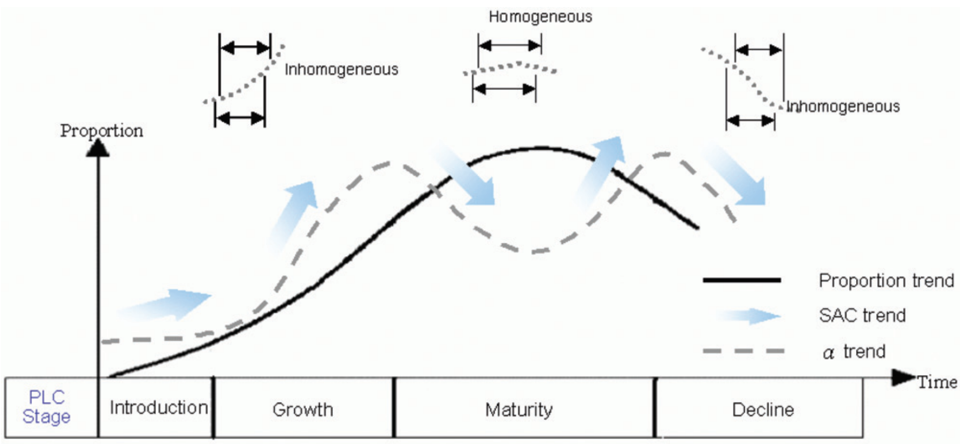

Figure 4. Relationship among α, SAC and PLC.

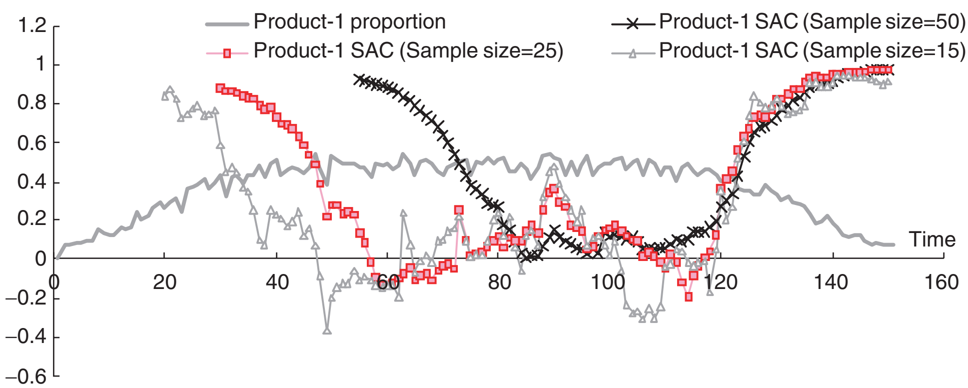

Figure 5. SAC calculated by different sample sizes.

size of 25, approximately one half of one PLC phase, gives the SAC a very good indication of the PLC transition.

The smoothing constant, in the range of [0, 1], estimated by the steepest-descent search (SDS), SAC, in the range of [−1, 1], calculated with $s = 25$ and the product-mix proportions, in the range of [0, 1], of the simulated DRAM data are directly superimposed together in figure 6. The SDS-estimated smoothing constant goes up when the trend is rising or declining but goes down in the maturity phase. The trend of SAC and the trend of SDS-estimated smoothing constants match very well all

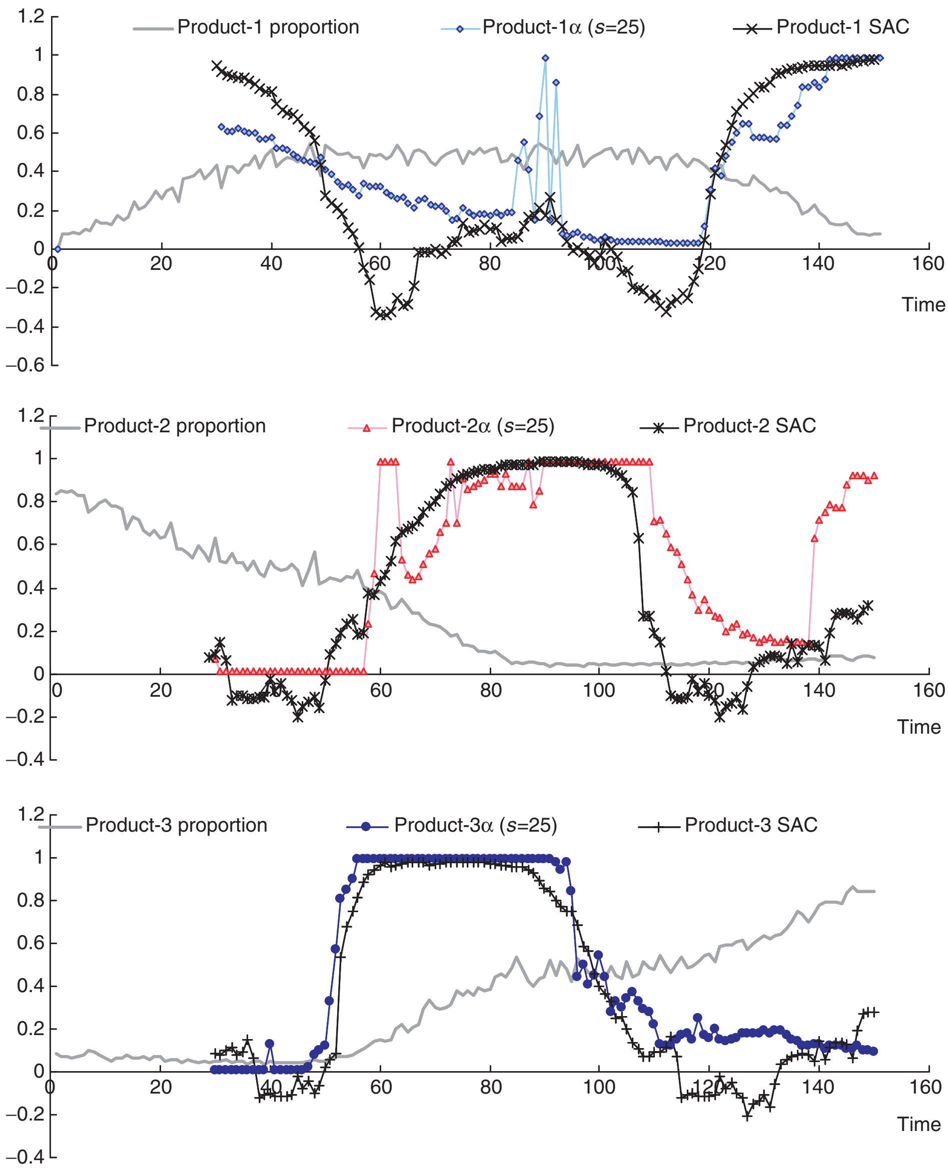

Figure 6. Smoothing constant estimates and $SAC_{i,t}$ (DRAM).

three products at three different PLC stages. SAC is thus a very good leading indicator to determine the changing trend of the smoothing constants. That is, the smoothing constant estimated at period $t+1$ should be higher than that at period t when $\mathrm{SAC}_{i,t+1}$ is higher than $\mathrm{SAC}_{i,t}$ and vice versa. Using the SAC as a leading indicator for the search for the best smoothing constants has been proven to cut the computing time to one fifty-fifth of that required by the steepest-descent search.

The smoothing constant estimate of product i at τ is denoted as $\hat{\alpha}_{i,t}$. The detailed steps are listed below:

1. Given historical demand data from period 1 to period $t-1$, calculate best-fitted initial values $\hat{\alpha}_{i,t}$ $(i=1,\ldots,k)$ using steepest-decent search at t.
2. Calculate the PLC leading indicator $\mathrm{SAC}_{i,t-1}$ for each product.
3. Calculate SSFEd or SSFEp.
4. New observations (period 't') of product demands are available.
5. Calculate new PLC indicators $\mathrm{SAC}_{i,t}$ $(i=1,\ldots,k)$.
6. Calculate the difference between the two successive indicators, $\mathrm{SAC}_{i,t\text{-}1}$ and $\mathrm{SAC}_{i,t}$, for each product to determine the gradient vector.
7. Move one step from the initial point $\hat{\alpha}_{i,t}$ $(i=1,\ldots,k)$ to a new point according to the gradient vector determined in step 5.
8. Recalculate SSFEd or SSFEp.
9. If reduction in SSFEd or SSFEp is found, go back to step 6.
10. Terminate the search and set values of $\hat{\alpha}_{i,t+1}$ equal to the final point when no further improvement can be found in the direction of the gradient vector.
11. Calculate the product-mix estimate for the next time period with the smoothing constant estimates $\hat{\alpha}_{i,t+1}$ $(i=1,\ldots,k)$.
12. Increase the value of t by 1 and go back to step 2.

5. Evaluation of proposed methodologies

To evaluate the performance of dynamic weighted least-SSEd and dynamic weighted least-SSEp product-mix proportion estimates, they are first tested with the simulated DRAM data (figure 1). The proportion estimates will start at the 31st period and end at the 150th period. Demand mean squared error (MSE) and proportion mean squared error (PMSE) are used as the performance measures. They are defined as:

$$\mathrm{MSE} = \frac{\sum_{\tau=t+1}^{t+n} \sum_{i=1}^{k} (\widehat{p}_{i,\tau-1} D_\tau - d_{i\tau})^2}{n \cdot k} \tag{27}$$

$$\mathrm{PMSE} = \frac{\sum_{\tau=t+1}^{t+n} \sum_{i=1}^{k} (\widehat{p}_{i,\tau-1} - d_{i\tau}/D_\tau)^2}{n \cdot k} \tag{28}$$

where $\widehat{p}_{i,\tau}$ is the proportion estimate for product i proportion at period τ made by methodologies developed above and two conventional methods, methods 1 and 2 of the following:

1. Mean-proportion estimate without dynamic weights:

$$\hat{P}_{i,t+1} = \frac{\left(\sum_{\tau=t-s+1}^{t} d_{i\tau}\right)/t}{\left(\sum_{\tau=t-s+1}^{t} D_\tau\right)/t},$$

where the sample size s is set equal to 25, same as the sample size used in the PLC leading indicator, SAC, for fair comparison;

2. Proportion-mean, i.e., least-SSEp estimate without dynamic weights:

$$\hat{p}^*_{i,t+1} = \frac{1}{t} \cdot \sum_{\tau=t-s+1}^{t} \frac{d_{i\tau}}{D_\tau},$$

where the sample size s is also set equal to 25 for fair comparison;

3. Least-SSEd estimate without dynamic weights:

$$\hat{p}_{i,t+1} = \frac{\sum_{\tau=t-s+1}^{t} d_{i\tau} D_\tau}{\sum_{\tau=t-s+1}^{n} D_\tau^2},$$

where the sample size s is also set equal to 25;

4. Dynamic weighted mean-proportion estimate in (14);
5. Dynamic weighted least-SSEp estimate in (19); and
6. Dynamic weighted least-SSEd estimate in (16).

Methods 1 and 2 are known to be the two most effective conventional methods in Gross and Sohl (1990). Method 3 is the least-SSEd method without dynamic weights. The smoothing constants of the exponential weights in (14), (16) and (19) are dynamically determined based on the PLC leading indicator, SAC, with a sample size $s = 25$.

5.1 *Performance evaluation with simulated DRAM demand data*

The performances of all product-mix proportion estimates are listed in table 1.

As shown in table 1, the two conventional methods (1 and 2) and the least-SSEd method without dynamic weights perform the worst in both PMSE and in MSE. With dynamic weights, the performances of all three product-mix estimation methods improve significantly. Among the dynamic weighted methods, the least-square methods perform better than the mean-proportion method. The least-SSEd estimate performs best in both measures. This reveals that model (2) is the most suitable model to describe the simulated demand data. The least-SSEp estimate does not have the best performance in PMSE in this case, but is very close to the best.

5.2 *Performance evaluation with actual semiconductor demand data*

Finally, the real semiconductor demands of different technology generations in a certain product group with the same metal-layer number are used to test the product-mix estimation methods. This will give us a comparison to see if the theoretically developed weighted least-square estimates are suitable for actual application.

There are six technology generations and 127 periods of demand data in this case. The demand trends for the six types of technologies are shown in figure 7. Since the demand data is the company's proprietary information, the demand volume and time are intentionally masked to protect the company's confidentiality. It is difficult

Table 1. MSE of different product-mix estimates for simulated DRAM demands.

Methods	PMSE	MSE
1. Mean-proportion	0.009017259	3 831 961
2. Least-SSEp	0.008947024	3 886 224
3. Least-SSEd	0.009127480	3 793 929
4. Dynamic wt. mean-proportion	0.0010772	528 851
5. Dynamic wt. least-SSEp	0.0009897	471 899
6. Dynamic wt. least-SSEd	0.0009811	466 602

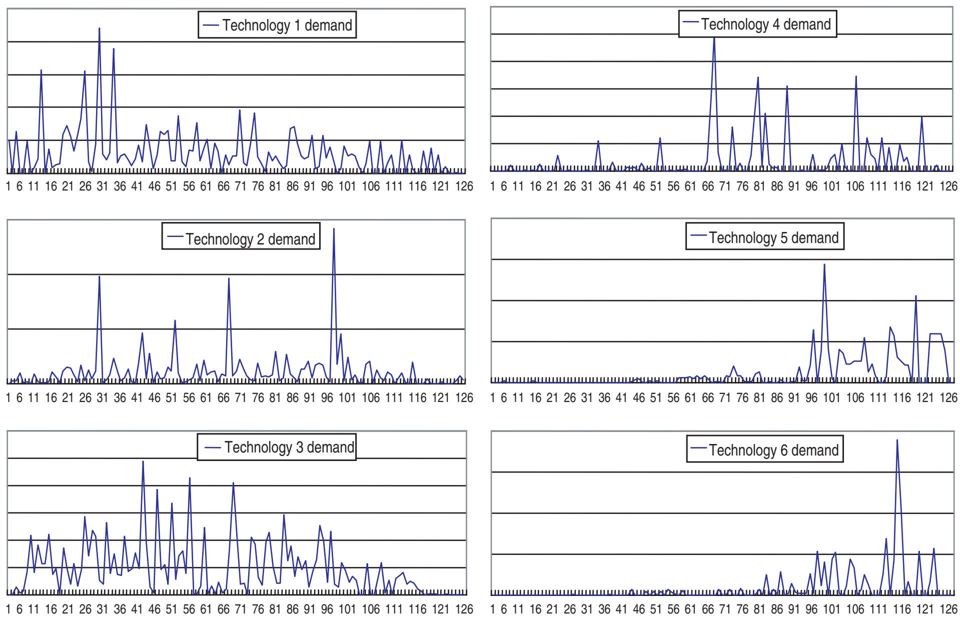

Figure 7. Demands of different semiconductor technologies.

to see the product demand trends from these raw demand data. Figure 8 shows the product-mix proportions of the six technologies. According to the proportions, demands for Technology 5 and Technology 6 gradually increase after about the 76th period. That indicates that the newly developed technologies are just introduced into the markets. Technology 3 gradually decreases after about the 86th period and represents a matured technology gradually phasing out from the market. The product-mix of different technologies can be seen clearly from this figure. The proportion of Technology 3 demand is much higher than the other five and dominates the total demand until about the 91st period.

Again, demands from period 1 to period 30 are used as the initial demand data. The product-mix estimate will start at period 31 and end at period 127. s is set to 25 periods. The performance comparison of all product-mix estimates is shown in table 2.

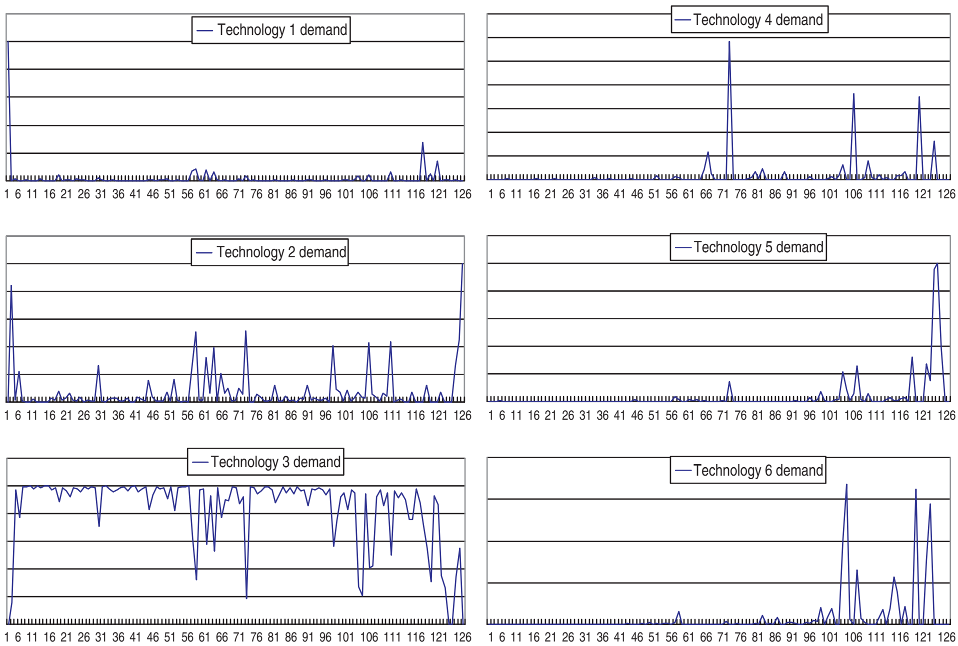

Figure 8. Product-mix proportions of different semiconductor technologies.

Table 2. MSE of different product-mix estimates for demands of different technologies.

Methods	PMSE	MSE
1. Mean-proportion	0.0245890253	193 707 314
2. Least-SSEp	0.0198083407	926 967 800
3. Least-SSEd	0.0259923295	186 223 492
4. Dynamic wt. mean-proportion	0.0198953295	198 057 131
5. Dynamic wt. least-SSEp	0.0186601176	1 093 635 664
6. Dynamic wt. least-SSEd	0.0216717924	177 668 486

As shown in table 2, the least-SSEp method has the best performance in PMSE and the least-SSEd has the best performance in MSE as expected. With dynamic weights applied to the methods, the performances are further improved by more than 5%. The limited improvement by the dynamic weighting scheme is due to the incomplete product life cycle from which the demand data are sampled. The mean-proportional method has average performances in both PMSE and MSE. Figures 9 and 10 show the estimated product-mix proportions obtained respectively by the dynamic weighted least-SSEp and weighted least-SSEd, versus the actual demand proportions. It can be seen that the proportion estimates by least-SSEp are more responsive to the product-mix changes whereas the least-SSEd method results in

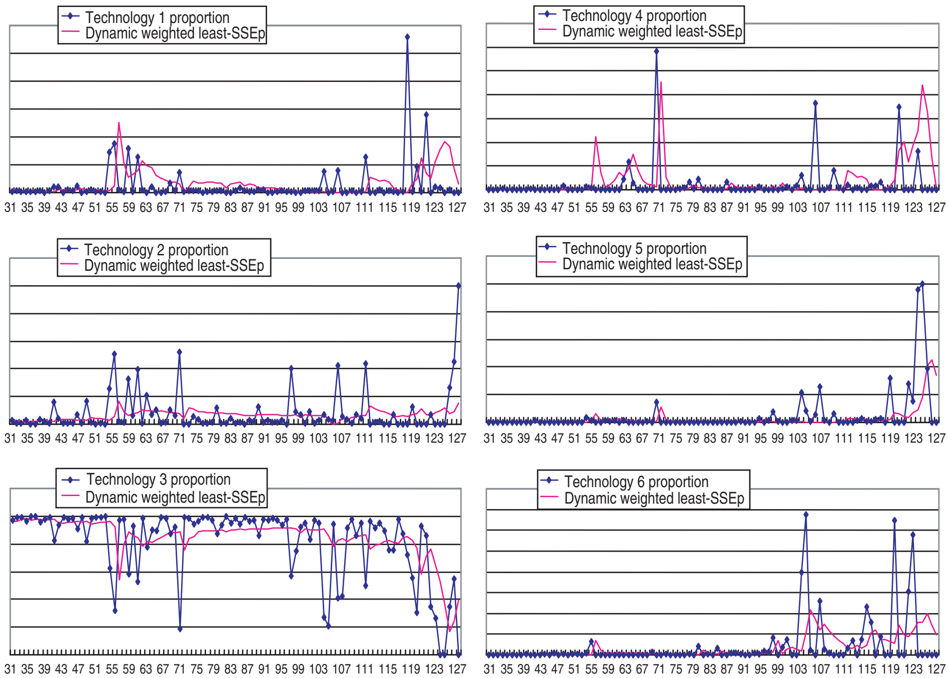

Figure 9. Proportion estimates with dynamic weighted least-SSEp method.

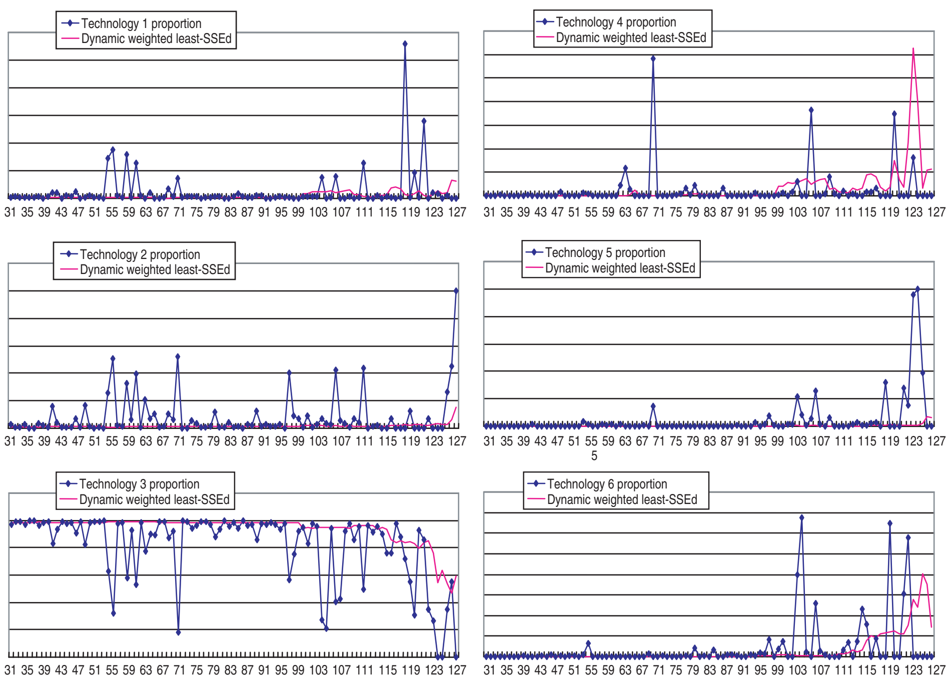

Figure 10. Proportion estimate with dynamic weighted least-SSEd method.

more robust product-mix estimates. This explains why the least-SSEp method performs best in estimating the product-mix proportions while the least-SSEd methods has the best performance in predicting the individual demands by disaggregating the total demand based on its robust estimates of the product-mix proportions.

6. Concluding remarks

The contribution of this research is three-fold. First, we have developed two least-square product-mix estimates. The least-SSEd estimate is the first in the literature and appears to outperform conventional estimates in minimizing the demand estimate errors. The least-SSEp estimate turns out to be the conventional proportion-mean estimate but is first established with theoretical bases by this research. Second, we have extended the least-square estimates to weighted estimates to allow varied emphases on the historical demands with different ages. Third, we have developed a dynamic weighting scheme to capture the PLC effect based on an effective PLC leading indicator. The dynamic weighting scheme is able to automatically adjust the weight distribution over the historical demands and make the weighted least-square estimate adapt to the PLC transition. In this research, the focus is on developing the weighted least-square estimates of the demand product-mix which is subject to the mix constraint and differs from the conventional demand forecast techniques. Only the single exponential weighting is used in the weighted scheme. Adoption of the double exponential weighting scheme, as commonly used in the demand trend prediction, is possible to further improve the forecast accuracy. Such an adoption requires extension of (14), (15) and (18) and should be an interesting future research topic. The three developed methodologies have all been tested and validated by the simulated and actual semiconductor demand data sets. The least-SSEd estimate is shown to perform best in estimating the individual product demands while the least-SSEp is best in estimating the product-mix proportions. The conventional mean-proportion estimate appears to fall in-between with a relatively stable performance in both demand and proportion estimations. As expected, estimates devised with the dynamic weighting scheme outperform the equally-weighted estimates, though implementation of the weighted scheme is somehow complicated. With the readily available computing power of modern personal computers, we believe that the superior performance and the adaptive nature of the dynamic weighting scheme are worth the efforts. Even though only applications to semiconductor demands are included in this paper, the methodologies presented are developed with general theoretical bases and can be readily applied to other industry sectors.

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