行政院國家科學委員會補助專題研究計畫成果報告

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International Portfolio For Property-Liability Insurers

國科會委託研究案

計畫主持人：周國端

計畫編號:88-2416-h-002-022

David G. Jou

Department of Finance

National Taiwan University

Date: 02-14-88

Tel: 02-23660354

Abstract

Globalization is one of the newest trends in the insurance industry. Recently, many insurers, including Prudential, Metropolitan, Aetna, and Cigna, have extended their insurance business to foreign countries. However, relatively few literature studies whether globalization is an efficient strategy for an insurance company.

Markovwitz's portfolio theory provides a decisive framework to evaluate the efficiency of diversification. Portfolio theory examines the optimum selection procedures for an investor's entire portfolio of securities. Many researchers have also extended Markovwitz's portfolio theory to be an international model. They proved that investors can further improve their performance through international investment. This paper intends to use portfolio selection model to evaluate whether insurers can gain excess benefit through international diversification.

This paper first presents a method for the composition of optimal international insurance portfolio. We further use property-liability insurance data of 1975-1986 to simulate the potential gains from international diversification. It shows that globalization can efficiently increase an insurer's expected return while reduce its business risks.

Introduction

Grubel (1968) and Levy and Sarant (1970) demonstrated that portfolio selection model developed by Markowitz (1952) can be applied in the international investments. The models discussed by Grubel and Levy and Sarnat can be used to derive mean-variance efficient frontiers of international investments. However, these models, when applied to international insurance, can't directly work without discussing the interrelationship between underwriting and investment. On the other hand, Kahane and Nye (1975), Kahane (1977), and Cummins and Nye (1981) developed portfolio optimization models for property-liability insurers. These articles have used portfolio theory to demonstrate that diversification in both insurance and investment can create an efficient gain. However, when applied to decision making in international insurance companies, the models of these articles need further modification.

This article presents such a model to analyze international diversification for property-liability insurers. A method is derived to measure the expected return and variance for an international property-liability insurance company, a quadratic programming algorithm is developed to sovle the problem, and empirical results are also given to demonstrate the application of these models derived by this article.

Model

It is important for property-liability insurance companies to recognize the interaction of underwriting and investment as well as the influence of exchange rate when they consider international diversification. For identifying the problem, several assumptions are necessary to be indicated in the beginning. Decision makers only consider one period expected return and risk, which can be estimated by mean and variance respectively. The expected returns and risks are calculated in local currency of parent companies. The political risks, tax, and other barriers of foreign countries are small enough to be omitted. In general, one dollar of written premium creates, on average, one dollar of liabilities in a property-liability insurance company.

For a specific country k, the interrelation between underwriting and investment in a property-liability insurance company can be expressed as:

$$W_M^K = W_U^K + 1, \tag{1}$$

where W_M^K is the asset to surplus ratio and W_U^K is the reserve to surplus ratio.

Thus, the rate of return of a property-liability insurance company R_C^K is:

$$R_C^K = W_M^K R_M^K + W_U^K R_U^K, \tag{2}$$

where R_M^K and R_U^K are the rate of returns on investment and underwriting respectively.

To establish the international portfolio diversification model, the actual rate of return to the parent company R_I^K should be adjusted by the exchange rate and can be defined as:

$$1 + R_I^K = (1 + R_C^K)(1 + R_E^K), \tag{3}$$

where R_E^K is rate of return on exchange rate.

If the parent company diversify its business in N countries, each with weight W_K. Thus, the expected total rate of return and variance of parent company can be obtained as:

$$E[R_I] = \sum_{K=1}^{N} W_K E[R_I^K], \tag{4}$$

$$V[R_I] = \sum_{J=1}^{N}\sum_{K=1}^{N} W_J W_K COV[R_I^J, R_I^K], \tag{5}$$

where E, V, and COV denote expectation, variance, and covariance operator respectively.

Therefore, given a certain level of expected return, the decision maker of an international insurance company chooses not only W_K but also W_M^K and W_U^K for each country to minimize its risk. The model can be expressed as:

$$\begin{aligned} Min \quad & V[R_I] = \sum_{J=1}^{N}\sum_{K=1}^{N} W_J W_K COV[R_I^J, R_I^K], \\ \text{s.t.} \quad & E[R_I] = r, \\ & \sum_{K=1}^{N} W_K = 1, \\ & W_M^K = W_U^K + 1, \quad \forall K. \end{aligned} \tag{6}$$

It is important to recognize that R_I^K depends on W_M^K and W_U^K which are decision variables. Thus, R_I^K can not be directly estimated by historical data without given portfolio weights on underwriting and investment. Equation (6) can not be solved by traditional quadratic programming. To surpass this problem, this paper further assumes that $R_C^K R_E^K$ is small enough to be omitted and therefore,

$$R_I^K \approx R_C^K + R_E^K. \tag{7}$$

Equation (7) is critical to simplify equation (6) into a solvable quadratic programming problem. Recall equations (2) and (3). The actual rate of return to the parent company R_I^K can be redefined as:

$$R_I^K = W_M^K R_M^K + W_U^K R_U^K + R_E^K. \tag{8}$$

Therefore, the expected rate of return and variance of R_I^K as well as covariance of R_I^J and R_I^K can be redefined as:

$$E[R_I^K] = W_M^K E[R_M^K] + W_U^K E[R_U^K] + E[R_E^K]. \tag{9}$$

$$\begin{aligned} V[R_I^K] = {} & (W_M^K)^2 V[R_M^K] + (W_U^K)^2 V[R_U^K] + V[R_E^K] + 2W_M^K W_U^K COV[R_M^K, R_U^K] \\ & + 2W_M^K COV[R_M^K, R_E^K] + 2W_U^K COV[R_U^K, R_E^K]. \end{aligned} \tag{10}$$

$$\begin{aligned} COV[R_I^J, R_I^K] = {} & W_M^J W_M^K COV[R_M^J, R_M^K] + W_M^J W_U^K COV[R_M^J, R_U^K] + W_M^J COV[R_M^J, R_E^K] \\ & + W_U^J W_M^K COV[R_U^J, R_M^K] + W_U^J W_U^K COV[R_M^J, R_U^K] + W_U^J COV[R_M^J, R_E^K] \\ & + W_M^K COV[R_E^J, R_M^K] + W_U^K COV[R_E^J, R_U^K] + COV[R_E^J, R_E^K]. \end{aligned} \tag{11}$$

Thus, the expected total rate of return and variance of parent company can be redefined as:

$$E[R_I] = \sum_{K=1}^{N} W_K \{W_M^K E[R_M^K] + W_U^K E[R_U^K] + E[R_E^K]\}. \tag{12}$$

$$\begin{aligned} V[R_I] = \sum_{J=1}^{N} \sum_{K=1}^{N} W_J W_K \{ & W_M^J W_M^K COV[R_M^J, R_M^K] + W_M^J W_U^K COV[R_M^J, R_U^K] + W_M^J COV[R_M^J, R_E^K] \\ & + W_U^J W_M^K COV[R_U^J, R_M^K] + W_U^J W_U^K COV[R_M^J, R_U^K] + W_U^J COV[R_M^J, R_E^K] \\ & + W_M^K COV[R_E^J, R_M^K] + W_U^K COV[R_E^J, R_U^K] + COV[R_E^J, R_E^K]\}. \end{aligned} \tag{13}$$

Let

$$\begin{aligned} \lambda_M^K &= W_K W_M^K, \\ \lambda_U^K &= W_K W_U^K, \text{ and} \\ \lambda_K &= W_K. \end{aligned} \tag{14}$$

Equation (14) are just variable transformation to make the model solvable through quadratic programming. With equations (12), (13) and (14), equation (6) can be redefined as:

$$
\begin{aligned}
Min \quad & V[R_I] = \sum_{J=1}^{N}\sum_{K=1}^{N} \{\lambda_M^J \lambda_M^K COV[R_M^J, R_M^K] + \lambda_M^J \lambda_U^K COV[R_M^J, R_U^K] \\
& + \lambda_M^J \lambda_K COV[R_M^J, R_E^K] + \lambda_U^J \lambda_M^K COV[R_U^J, R_M^K] + \lambda_U^J \lambda_U^K COV[R_M^J, R_U^K] \\
& + \lambda_U^J \lambda_K COV[R_M^J, R_E^K] + \lambda_J \lambda_M^K COV[R_E^J, R_M^K] + \lambda_J \lambda_U^K COV[R_E^J, R_U^K] \\
& + \lambda_J \lambda_K COV[R_E^J, R_E^K]\}, \\
\text{s.t.} \quad & \sum_{K=1}^{N} \{\lambda_M^K E[R_M^K] + \lambda_U^K E[R_U^K] + \lambda_K E[R_E^K]\} = r, \\
& \sum_{K=1}^{N} \lambda_K = 1, \qquad (15) \\
& \lambda_M^K = \lambda_U^K + \lambda_K \qquad \forall K.
\end{aligned}
$$

Obviously, given historical estimations of expected rate of returns and covariance, equation (15) can be solved by quadratic programming. The optimal λ_M^K and λ_U^K as well as λ_K can be obtained from equation (15). Furthermore, by equation (14), W_M^K and W_U^K as well as W_K can be obtained since $W_M^K = \frac{\lambda_M^K}{\lambda_K}$, $W_U^K = \frac{\lambda_U^K}{\lambda_K}$ and $W_K = \lambda_K$.

Numerical Results

The numerical analysis of this article is intended to demonstrate the application of the above model. Thus, the results are suggestive for the insurance industry as a whole rather than are feasible to a specific insurance company.

Five countries including United States, Japan, Germany, France, and Switzerland, are selected as sample target markets. The total market volume of these five countries represent more than &0 percentage of the world market volume. The expected rate of return and variance are calculate by the time series data for the period 1975-1986, which is published by SIGMA/Swiss Re. These returns are the average of the whole property-liability insurers in each country. All simulation is from the point view of an U.S. based insurance company. International Financial Statistics are used to obtain data of exchange rate in these five selected countries.

As we can see from the figure 1, the numerical analysis results support the implication of traditional financial portfolio and the simulation results are also consistent with the conclusion of our model.

Figure 1 Efficient Frontier (International vs. Local)
(Data: 1975-1986, SIGMA-Swiss Re)