SRC Project 879
Progress report in liaison meeting

Task 879.2: Integration of Demand Planning and Manufacturing Planning

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The Problem of Product Grouping

- **Tool #6 in not a sensitive tool.**
- **Tool #7 is a sensitive tool across product families.**
- **Tool #3 is sensitive to both inter- and intra-family variation.**

Family / Tool ID	B				F	C					J	
	2	4	5	10	16	3	6	11	14	15	25	28
1	3.20	3.20	3.20	2.80	2.60	3.20	3.60	3.60	3.60	3.60	3.00	3.00
3	6.36	6.36	6.36	8.44	6.36	6.36	8.24	7.76	7.76	7.76	8.12	8.12
5	5.52	5.52	5.52	5.68	5.84	5.52	7.44	6.32	6.32	6.32	6.48	6.48
6	0.92	0.92	0.92	0.92	0.92	0.92	0.92	0.92	0.92	0.92	0.92	0.92
7	5.92	5.92	5.92	5.92	4.00	5.92	5.92	5.92	5.92	5.92	5.92	5.92
8	22.35	22.35	22.35	23.37	23.26	19.55	32.79	27.82	27.82	27.82	26.75	26.75
9	0.22	0.22	0.22	0.34	0.34	0.22	0.34	0.34	0.34	0.34	0.22	0.22

Example: 4 products and 3 tool sets

ocessing Time	Product			
	1	2	3	4
Tool 1	6.33	6.23	5.16	5.15
Tool 3	20.93	20.99	16.17	16.18
Tool 4	15.6	15.61	7.93	7.94

	All products in 1 Group		
	Upper (Ug,k)	Lower (Lg,k)	Range (Rg,k)
Tool 1	6.33	5.15	1.18
Tool 3	20.99	16.17	4.82
Tool 4	15.61	7.93	7.68

Notation

Product group: g

Tool ID \ Family	B				C				
	2	4	5	10	3	6	11	14	15
1	3.20	3.20	3.20	2.80	3.20	3.60	3.60	3.60	3.60
3	6.36	6.36	6.36	8.44	6.36	8.24	7.76	7.76	7.76
7	5.92	5.92	5.92	5.92	5.92	5.92	5.92	5.92	5.92
9	0.22	0.22	0.22	0.34	0.22	0.34	0.34	0.34	0.34

Product:

Tool: k

Range of processing times: $R_{g,k}$

$$R_{g,k} = U_{g,k} - L_{g,k}$$

Decision variable: $X_{i,g}$

(Erroneous) $$Min \quad \sum_g \sum_k R_{g,k}$$

Subject to:

$$X_{i,g} \cdot t_{i,k} \leq U_{g,k} \qquad \forall i,g,k$$

$$X_{i,g} \cdot t_{i,k} \geq L_{g,k} \qquad \forall i,g,k$$ Erroneo

$$U_{g,k} - L_{g,k} = R_{g,k} \qquad \forall g,k$$

$$\sum_g X_{i,g} = 1 \qquad \forall i$$

empty groups) $$\sum_i X_{i,g} \geq 1 \qquad \forall g$$ Not necessa

Minimum Processing Time $L_{g,k}$

$$_{i,k} \cdot X_{i,g} \geq L_{g,k}$$ (erroneous)

hould be replaced by disjuctive constraints

$$t_{i,k} \cdot X_{i,g} \geq L_{g,k} \quad if \;\; X_{i,g} = 1 \qquad ...(1)$$

$$or$$

$$t_{i,k} \cdot X_{i,g} \geq 0 \quad if \;\; X_{i,g} = 0 \qquad ...(2$$

Disjunctive Constraints

$$Y_{i,g} \in \{0,1\}$$

$$\text{H : a large number}$$

$$\Rightarrow$$

$$X_{i,g} \cdot t_{i,k} \geq L_{g,k} - H(1 - Y_{i,g})$$

$$X_{i,g} \cdot t_{i,k} \geq -H \cdot Y_{i,g}$$

$$X_{i,g} = Y_{i,g}$$

A Formulation for Product Grouping

(Formulation Q) Min $\sum_k \sum_g R_{g,k}$

$$
\begin{aligned}
X_{i,g} \cdot t_{i,k} &\leq U_{g,k} && \forall i,g,k \\
X_{i,g} \cdot (t_{i,k} - H) + H &\geq L_{g,k} && \forall i,g,k \\
X_{i,g} \cdot (t_{i,k} + H) &\geq 0 && \forall i,g,k \\
U_{g,k} - L_{g,k} &= R_{g,k} && \forall g,k \\
\sum_{g} X_{i,g} &= 1 && \forall i
\end{aligned}
$$

The Constraint for Empty Groups

Family / Tool ID	B				C					J	
	2	4	5	10	3	6	11	14	15		
1	3.20	3.20	3.20	2.80	3.20	3.60	3.60	3.60	3.60		
3	6.36	6.36	6.36	8.44	6.36	8.24	7.76	7.76	7.76		
5	5.52	5.52	5.52	5.68	5.52	7.44	6.32	6.32	6.32		

- Group J is empty.
- Moving any product from Groups B or C to Group J will result in a grouping that is at least as good.
- Therefore, the no-empty-group constraint is not necessary.

Work in Progress

Tool requirements $= D_i \cdot W_{i,k}$

where D_i is the demand of product i

$W_{i,k}$ is the processing time of product i on tool k

- The above analysis has focused on $W_{i,k}$. Need to include the effect of D_i.
- The tool requirements of multiple product groups need to be combined.
  - Between sensitive tools
  - Between sensitive tools and non-sensitive tools
  - Between non-sensitive tools

Formal Proof for the Empty Group Constraint

- Definition: Given a constrained minimization problem P, a problem Q is a quasi-relaxation of P if for every feasible solution of P with objective function value v, there is a feasible solution of Q having an objective function values less than or equal to v.
- The formulation with the no-empty group constraint will be called P. This constraint states that each group must contain at least 1 product. Removing this constraint has the effect of legitimating solutions with one or more empty groups. It is straightforward to say that Q is a quasi-relaxation of P, because Q is less constrained. We will prove that the reverse is also true, that is, P is also a quasi-relaxation of Q.

P is a Quasi-relaxation of Q

- To prove it, we must show that, for every feasible solution of Q, there is a corresponding feasible solution of P having a better or equal objective function value.
- The feasible solutions of Q belong to either of two cases.

1. In the first case, the solutions contain no empty groups. Therefore, those solutions are also solutions of P.
2. In the second case, the solutions contain one or more empty groups. Suppose that s is a feasible solution of Q. There exists a non-empty group g1 and an empty group g2. By moving a product from g1 to g2, the resultant solution is a solution in P. Because g2 contains only one product, its contribution to the objective function is 0. Because g1 now has one less product, its contribution to the objective function will be less or the same. Therefore, the resultant solution has a function value at least as good as that of solution s.