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Fuzzy control on the phase and stroke of a linear compressor of a split-Stirling cryocooler

Yee-Pien Yang and Bin-Juine Huang

Department of Mechanical Engineering, National Taiwan University, Taipei, Taiwan 106, Republic of China

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A new dual fuzzy controller for the linear compressor of a split-Stirling cryocooler is proposed in this paper. In addition to the stroke, the phase of the working fluid, circulating from the compressor to the split regenerator, is an important parameter to the cooling efficiency. The non-linearities and uncertainties of a cryocooler make it impossible to use a conventional controller over a wide range of operation. This fuzzy controller is designed to consist of a fuzzy stroke control and phase control without any explicit system models, but driven in the human thinking mechanism. Computer simulations and experiments showed that the compressor piston in the displacement tracking control failed to catch up with the periodic sinusoidal command. However, the dual fuzzy controller can successfully drive the piston to follow the stroke as well as the phase of the reference command. This new fuzzy control methodology can be employed with split-type cryocoolers, whose stroke and phase both contribute to the refrigeration performance.

Keywords: split-Stirling cryocooler; linear compressor; dual fuzzy controller; phase control

Compressors are fundamental components in miniature split-type Stirling cryocoolers, which are characterized by complete separation of the expansion cylinder from the compressor cylinder, drive motor and crankcase. The working fluid provides gas force by an actuating piston oscillating in the compressor cylinder with a phase-shift relative to the displacer motion in the expansion cylinder. The stroke variation of the displacer indicates the volume change in the expansion space, and its pressure–volume (P–V) diagram illustrates the work done by the gas on the displacer. The net area of the diagram will represent the heat transferred to the expansion space, a positive work, so there is some refrigeration effect, and the tip of the cold finger will become cold. The optimal P–V diagram condition of the cryocooler depends not only on the stroke but also on the phase angle of pressure phasors[1,2].

In addition to the stroke of the compressor that determines the available refrigeration, the phase control of the compressor is also important to improve the cooling performance[3–5]. Stroke controls were present in most of the documentation of the motion control of linear motors, but few have addressed both phase and stroke controls[6,7]. Stolfi and Daniels[8] employed a local feedback loop to control the phase and frequency of the displacer and piston, to which the reference lagged the signal to the displacer, producing the desired piston/displacer phase angle. In this paper, a simultaneous control on the phase and stroke of a linear compressor is developed by the use of fuzzy control theory, in order to deal with system non-linearities, uncertainties and time delay.

Fuzzy theory was initially proposed by Zadeh[9], who induced the principle of incompatibility for complex systems which were beyond precise description by traditional mathematical modelling and analyses. Therefore, the human thinking mechanism was implemented instead of using numbers to describe system behaviours. The refrigeration performance of the split-type Stirling cryocooler depends upon the cooling or ambient atmospheric temperature, working gas pressure, the swept volume in the compression space of the expansion space, and the operating frequency, etc. It is not easy to model or identify the system transfer function of the cryocooler over all the operating range, and it is therefore hard to design a proper and robust controller for the stroke and phase control of the system[10]. Thus, the unobservable system uncertainties can be translated by fuzzy rule bases, and the control signal can be generated by the fuzzy inference engine and defuzzification interface. The most important contribution of this paper is the development of a dual fuzzy controller on the stroke and phase of the linear compressor of a Stirling cryocooler.

Dynamics of linear compressor

The mechanical structure of the linear compressor in this research is shown in *Figure 1*. The armature is a cylinder wound with coils through which the input current is applied, the stator is composed of two separate rings of permanent magnets. A linear variable-differential transformer (LVDT) is placed at the tail of the piston to measure its displacement[11,12]. The electrical equation of the linear compressor[10] can be expressed as

$$V = L\frac{dI}{dt} + RI + B\ell\frac{dX}{dt} \tag{1}$$

and the mechanical equation of motion can be described

$$m\frac{d^2X}{dt^2} + c\frac{dX}{dt} + kX = \ell IB - A\Delta P \tag{2}$$

where L and R are coil inductance and resistance, I and V are driving current and voltage, B is the magnetic flux density in the air gap between the stator and the moving coil, the piston displacement is denoted by X, the effective length of the coil is ℓ, the pressure difference between the upper and lower chambers is ΔP, the cross-section area of the piston is A, and m, k, c denote the equivalent mass, spring constant and viscous damping coefficient, respectively. Taking the Laplace transformation of the above equations yields

$$X(s) = G(s)V(s) + W(s)\Delta P(s) \tag{3}$$

where

$$G(s) = \frac{B\ell}{mLs^3 + (mR + cL)s^2 + [cR + B^2\ell^2 + Lk]s + Rk} \tag{4}$$

and

$$W(s) = \frac{(Ls + R)A}{mLs^3 + (mR + cL)s^2 + [cR + B^2\ell^2 + Lk]s + Rk} \tag{5}$$

are, respectively, the transfer functions of the driving voltage and the pressure difference with respect to the piston displacement of the linear compressor. It is clearly understood that the displacement of the compressor is mainly determined by the driving voltage and pressure difference, and the latter changes with the cooling or ambient temperature, working gas pressure, operating frequency, etc. Therefore, from the control system point of view, the term $W(s)\Delta P(s)$ can be deemed to be bounded uncertainty, to which the controller has to be designed robustly so that the displacement follows the reference command. The transfer function $G(s)$ of the linear compressor at a certain operating point was identified experimentally as[10]

$$G(s) = \frac{X(s)}{V(s)} = \frac{2.4101 \times 10^6}{(s + 1.8148 \times 10^2)(s^2 - 0.6002 \times 10^2 s + 6.7542023 \times 10^4)} e^{-T_d s} \tag{6}$$

in which $T_d = 0.001$ s is the time delay. The designer must take a lot of effort on modelling and experiments for precise system identification of the compressor. However, the coefficients in the transfer function change with various operating conditions, to which a conventional controller may not be robust. *Figure 2* shows the open-loop frequency response of the linear compressor, and its bandwidth is approximately 14 Hz.

Fuzzy controller design

The basic structure of a fuzzy logic controller consists of four components[13]: knowledge base, input fuzzification, inference engine and output defuzzification, as shown in *Figure 3*. The input fuzzy variables are system tracking error

$$E(t) = X(t) - C(t) \tag{7}$$

and its change

$$\Delta E(t) = E(t) - E(t - \Delta t) \tag{8}$$

where the sinusoidal reference command is

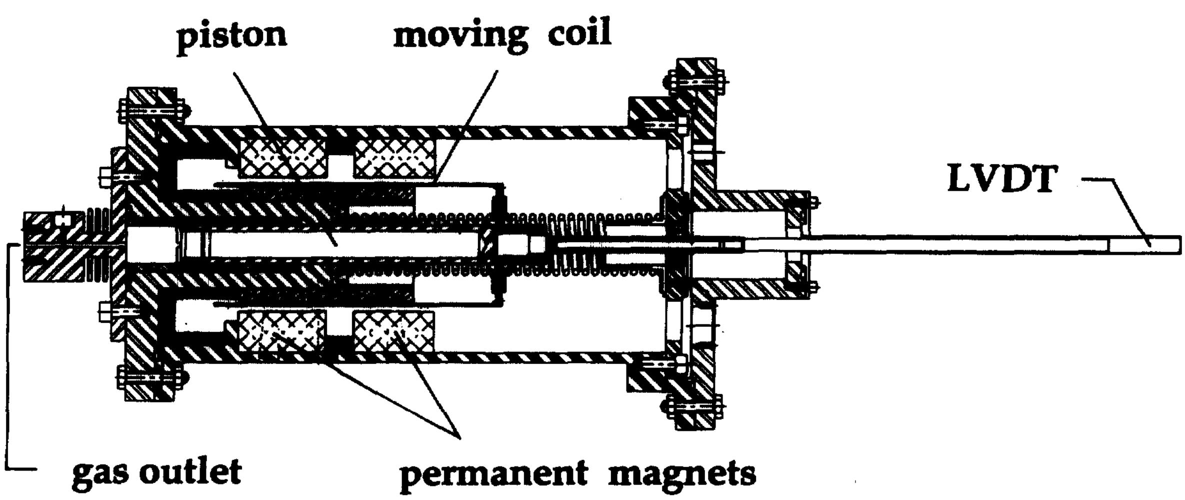

Figure 1 The linear compressor structure.

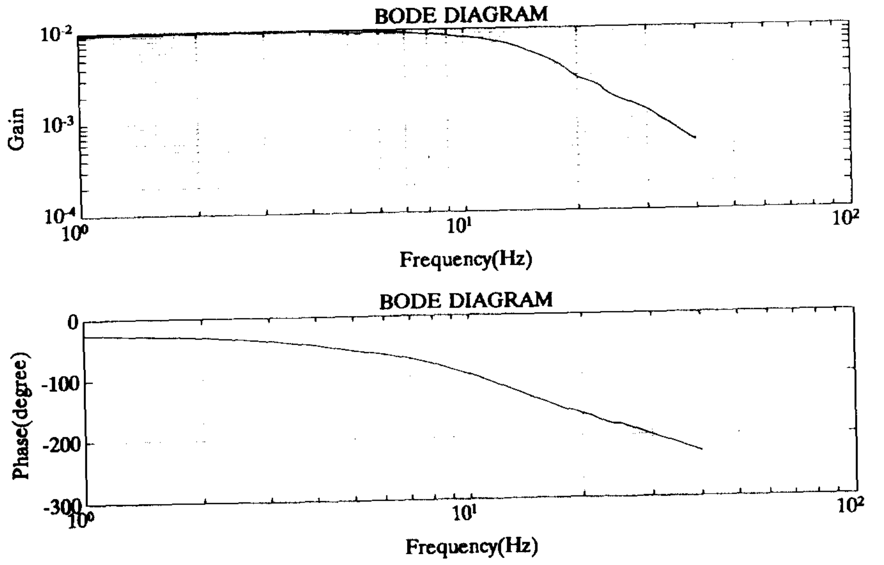

Figure 2 Frequency response of the linear compressor.

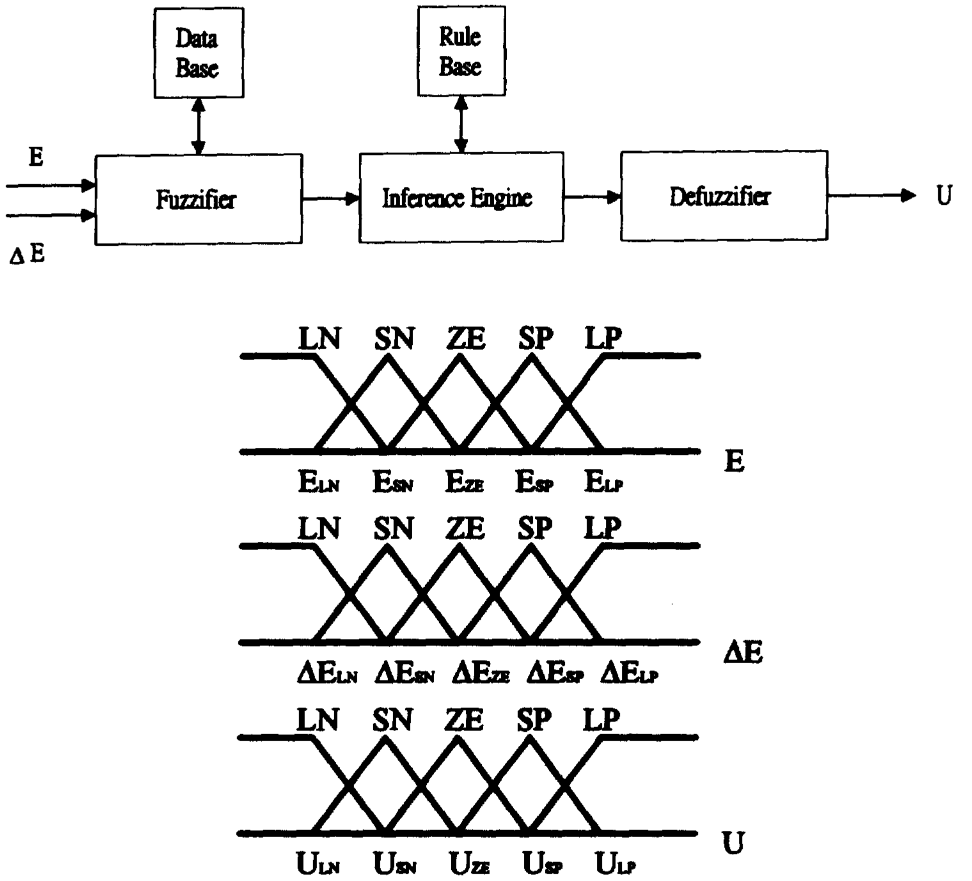

Figure 3 Fuzzy controller and membership functions.

$$C(t) = V_C \sin \omega t \tag{9}$$

and the piston displacement is

$$X(t) = V_X \sin(\omega t - \phi). \tag{10}$$

The variables V_C and V_X are peak voltages of the reference and the displacement, respectively, and ϕ is the phase angle. In the knowledge base, the triangular shape membership functions are defined on the intervals of E, ΔE and V, and five linguistic terms, such as large positive, small positive, zero, small negative and large negative, are given to the fuzzy sets LP, SP, ZE, SN, and LN, also shown in *Figure 3*.

In the fuzzification process, the degrees of fuzzy states of E and ΔE must be interpreted in terms of fuzzy values. For a given set of linguistic values, each weighting function value can be calculated by the use of the membership function. For example, if the system output error value locates between E_{LP} and E_{SP}, the weighting values for each member are

$$W_E(LP) = \frac{E_{LP} - E}{E_{LP} - E_{SP}} \tag{11}$$

$$W_E(SP) = \frac{E - E_{SP}}{E_{LP} - E_{SP}} \quad (12)$$

and

$$W_E(ZE) = W_E(SN) = W_E(LN) = 0 \quad (13)$$

In the inference engine, a rule table presents fuzzy associative memories (FAMs) in a shorthand matrix notation shown in *Table 1*. A linguistic armature voltage rule is fired for each pair of linguistic error variables and linguistic change in error variables. For example, if the error on *SP* has a weighting of $W_E(SP)$ and the change in error on *ZE* has weighting of $W_{\Delta E}(ZE)$ then the armature voltage on *SN* has a weighting of

$$W_{14} = \min(W_E(SP), W_{\Delta E}(ZE)) \quad (14)$$

Then, the defuzzification component performs the inverse operation of fuzzification, and the correction of the controller output is evaluated by the centre of area method

$$u = M_r \frac{\sum_{i=1}^{n} W_i U_i}{\sum_{i=1}^{n} W_i} \quad (15)$$

and total control output to the compressor is

$$V(t) = V(t - \Delta t) + u(t) \quad (16)$$

where W_i is the weighting number of the *i*th control rule, and U_i is the fuzzy value of the *i*th rule, and M_r is the amplitude of the reference command.

Displacement tracking control

Displacement tracking is usually the main objective of the conventional system control. An ideal steady-state displacement of the compressor piston is a sinusoidal wave at a constant frequency, and its stroke is defined as the amplitude of the oscillation. The controller must be designed to respond fast enough to track high frequency commands without time delays. Let E_d be the displacement output error, ΔE_d the error change, and the fuzzy values of each fuzzy set are listed in *Table 2*. The control law is calculated in the same way as Equations (14)–(16).

To verify the inadequacy of the displacement fuzzy control, Equation (6)) is taken as the system model in computer simulations. For a sinusoidal reference command with 20 Hz frequency, without disturbances and loads, the controlled output signal becomes oscillatory, both the stroke and phase do not track the command, as shown in *Figure 4*. The displacement of the compressor seems not to catch up with high frequency commands under single stroke controls owing to the system time delay. In order to keep up with high frequency commands, the frequency response of the controller should be a hundred times faster than the command[14]. In that way, a slight phase delay on the control signal will yield the compressor out of control, and the high-bandwidth controller is also expensive.

Dual fuzzy controller

For an open-loop control system, a given command with a certain magnitude and frequency will result in an output signal with inherent stroke and phase if the system operates in its linear range. It is hard to control the stroke and phase at the same time if only the stroke controller is implemented.

The dual fuzzy controller is designed to deal with the stroke and phase control separately. In addition to the fuzzy stroke controller, the fuzzy phase controller shifts the phase of the command, and the new command to the compressor is rebuilt by a four-quadrant multiplier. The dual fuzzy control system structure is depicted in *Figure 5*. The sampling rate is 4 kHz.

Stroke controller

Instead of tracking a sinusoidal signal, the system follows a constant stroke command, which is the output of a full-wave rectifier. Thus, the stroke controller can be designed with a lower bandwidth, and is therefore less expensive.

Table 2 Membership values for displacement fuzzy sets

Fuzzy set	E_d	ΔE_d	U_i
LP	1.00	0.15	0.5710
SP	0.05	0.01	0.0001
ZE	0	0	0
SN	− 0.05	− 0.01	− 0.0001
LN	− 1.00	− 0.15	− 0.5710

Table 1 Linguistic armature control voltage rule

	ΔE				
	LN	*SN*	*ZE*	*SP*	*LP*
E					
LN	*LP* (rule 1)	*LP* (rule 2)	*LP* (rule 3)	*LP* (rule 4)	*LP* (rule 5)
SN	*LP* (rule 6)	*SP* (rule 7)	*SP* (rule 8)	*ZE* (rule 9)	*ZE* (rule 10)
ZE	*SP* (rule 11)	*SP* (rule 12)	*ZE* (rule 13)	*SN* (rule 14)	*LN* (rule 15)
SP	*ZE* (rule 16)	*ZE* (rule 17)	*SN* (rule 18)	*SN* (rule 19)	*LN* (rule 20)
LP	*LN* (rule 21)	*LN* (rule 22)	*LN* (rule 23)	*LN* (rule 24)	*LN* (rule 25)

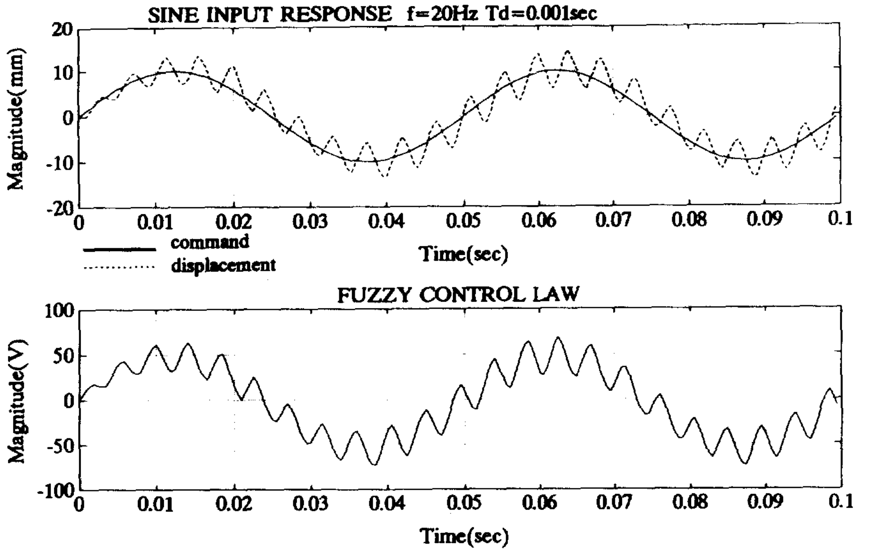

Figure 4 Simulation of fuzzy displacement tracking control.

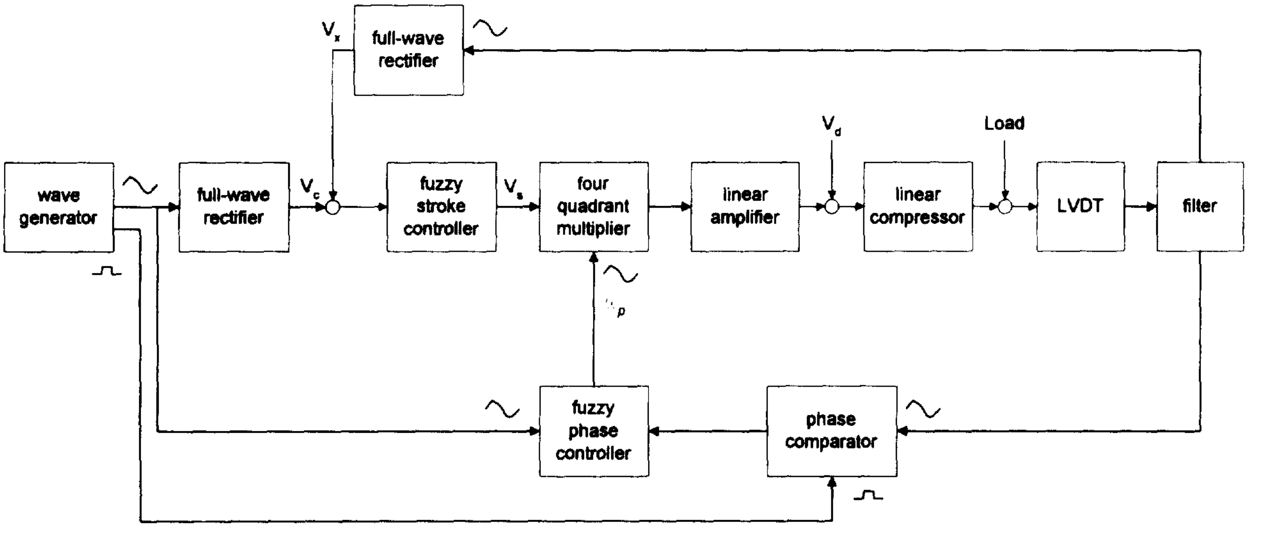

Figure 5 Dual fuzzy control system.

Both the reference and output signals are filtered by a full-wave rectifier to get DC voltages of V_C and V_X, respectively, and are compared with each other to produce the stroke errors

$$E_S = V_X - V_C \tag{17}$$

and their changes

$$\Delta E_S(t) = E_S(t) - E_S(t - \Delta t). \tag{18}$$

Their membership values are assigned via trial and error in *Table 3*. The fuzzy stroke control law V_S is then obtained in the same way as in Equations (14)–(16).

Phase controller

The purpose of the phase controller is to adjust the piston displacement in-phase to the reference command. Leading a phase angle θ is equivalent to lagging a phase $2\pi-\theta$, and the latter is easier to implement with electronic circuits. For phase adjustment, the phase comparator takes the phase difference of the reference square wave command and the piston displacement, and the fuzzy phase controller determines the time-delay number I_P according to the fuzzy rules as shown in *Table 4*, in which E_P and ΔE_P are the phase error and its change.

Thus, the output of the fuzzy phase controller becomes

Table 3 Membership values for stroke fuzzy sets

Fuzzy set	E_S	ΔE_S	U_i
LP	0.50	0.15	0.0150
SP	0.05	0.01	0.0015
ZE	0	0	0
SN	– 0.05	– 0.01	– 0.0015
LN	– 0.50	– 0.15	– 0.0150

Table 4 Membership values for time delay number I_P

Fuzzy set	E_P	ΔE_P	I_P
LP	1.8	0.50	20.0
SP	0.1	0.02	0.01
ZE	0	0	0
SN	– 0.1	– 0.02	– 0.01
LN	– 1.8	– 0.50	– 20.0

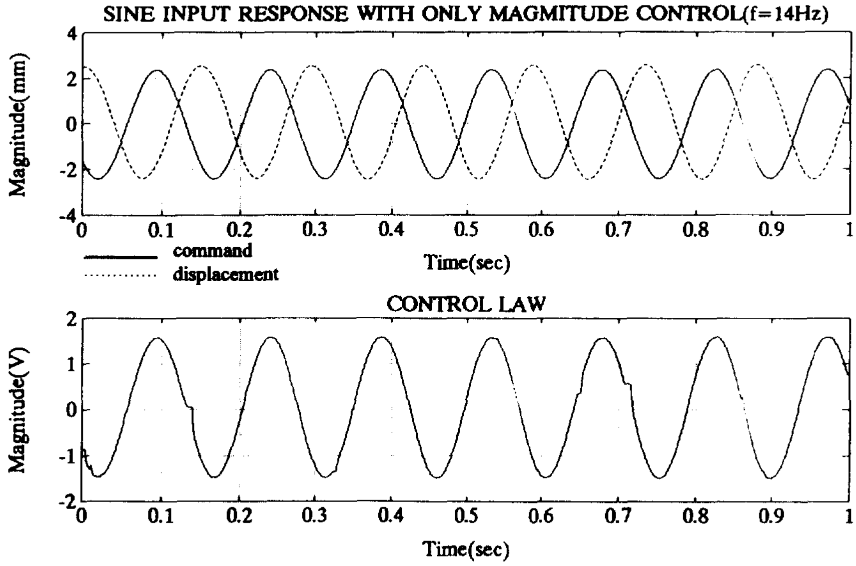

Figure 6 Control response of a single fuzzy stroke control.

$$u_p(t) = C(t - I_p\Delta t) \quad (19)$$

where $C(t)$ is the reference command as defined in Equation (8).

Experimental results

It should be noted that the fuzzy phase controller must be started after the stroke error converges in a certain range, in order not to mislead the fuzzy inference to a correct phase correction. *Figure 6* shows that a single fuzzy stroke controller does not handle the phase delay. The dual fuzzy controller causes the system to follow the reference command successfully as shown in *Figure 7*. Also shown in *Figures 8* and *9*, the proposed dual fuzzy controller is robust to the membership values, input voltage disturbances, and loads, which are produced by turning off the valve of the compressor. Furthermore, the closed-loop system responds up to 20 Hz reference command without stroke attenuation and phase delay, as shown in *Figure 10*.

Summary and conclusions

The proposed dual fuzzy controller successfully performed the stroke as well as phase control of a linear compressor. No derivation of mathematical models is needed and no system identification is required for the implementation of fuzzy controllers, in which the fuzzy control law is robust to the values of membership functions, disturbances and loads. Although the linear compressor inheres a narrow operating range with a bandwidth around 14 Hz, the dual fuzzy controller can even drive the piston to oscillate up to 20 Hz with satisfactory tracking performance in stroke and phase. This research result may be applied to various cryocoolers with periodic motion when its stroke and phase are both essential to the refrigeration performance.

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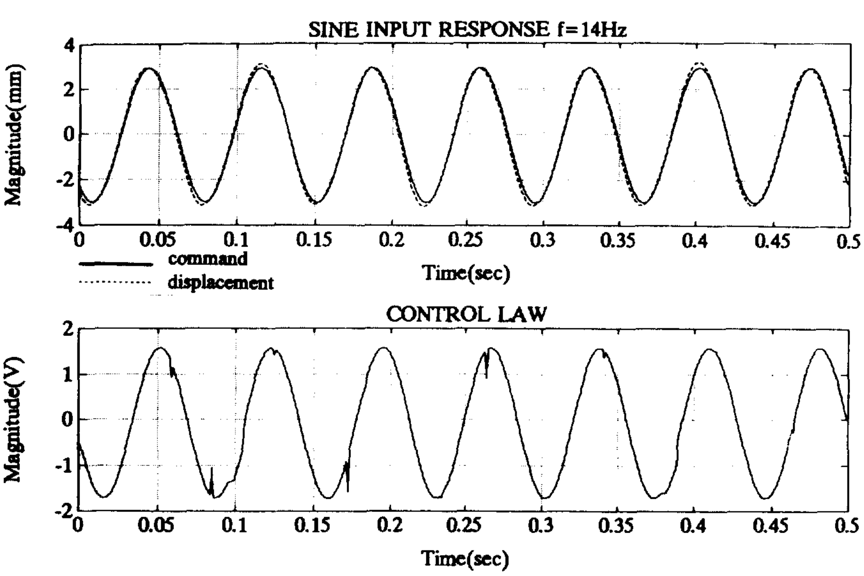

Figure 7 Control performance of dual fuzzy control.

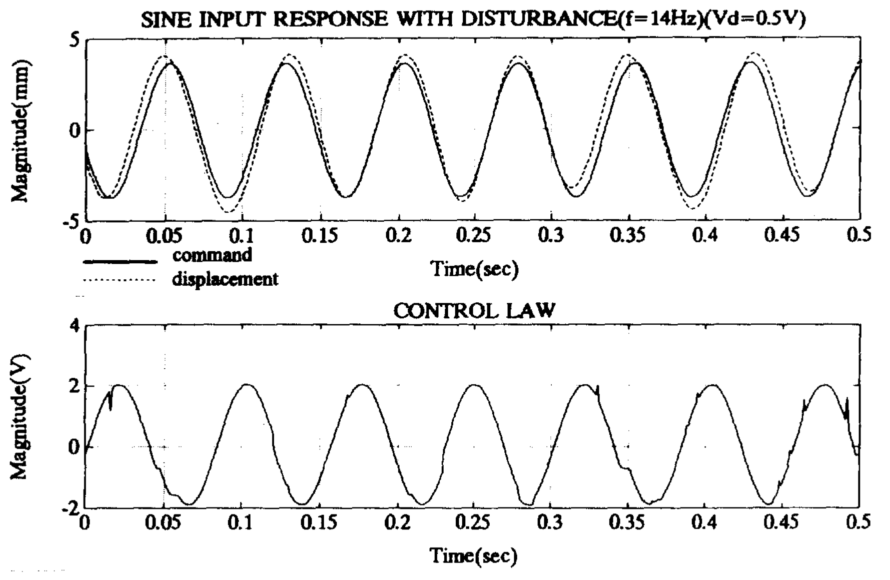

Figure 8 Control p_rformance with input voltage disturbance (0.5 V).

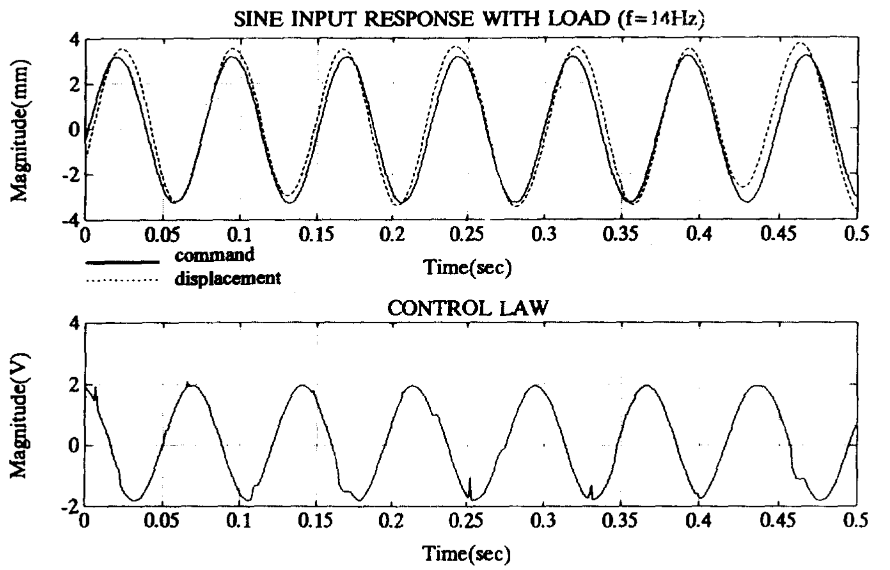

Figure 9 Control performance with different load.

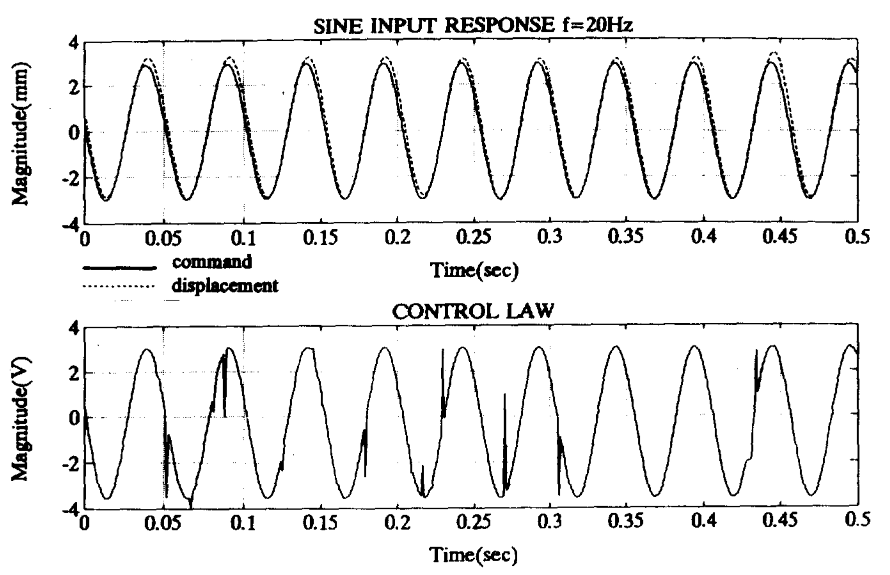

Figure 10 Response to a 20 Hz reference command.

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