DESIGN OF FIR FILTERS AND ALLPASS PHASE EQUALIZERS WITH PRESCRIBED MAGNITUDE AND PHASE RESPONSES BY TWO REAL CHEBYSHEV APPROXIMATIONS

Soo-Chang Pei and Jong-Jy Shyu

Department of Electrical Engineering, National Taiwan University
Taipei, Taiwan, Rep. of China

ABSTRACT

Since the coefficients of a FIR filter with arbitrary complex-valued desired frequency responses are neither symmetric nor antisymmetric, the Remez exchange algorithm cannot be applied directly. The problem can be solved by dividing the original complex approximation into two real ones such that the Remez exchange algorithm can be applied by slightly modifying the Park-McClellan program. This method is much easier than the currently existing methods using linear programming and the performance is satisfied. More importantly, the magnitudes of the resultant complex errors are also equiripple as the direct complex Chebyshev approximation designs. Numerical design examples including lowpass filter and sine-delay FIR allpass phase equalizers are given to show the effectiveness of this approach.

I. INTRODUCTION

Conventionally, we often use the well-known McClellan-Parks program [1] to design the linear phase FIR digital filters. But these filters need large length and long time delay when designed with sharp cutoffs. Hermann and Schuessler proposed the method for designing minimum phase FIR filters [2] which cause less delay but introduce delay distortion due to the delay is not a constant for all frequencies. In order to design filters which have less delay than linear phase filters and have approximately constant group delay in the filter passband, recently Chen and Parks have used a standard linear programming algorithm to solve this complex approximation problem [3], and then Preuss designed them by the complex Remez exchange algorithm [4], which has recently been improved by Schulist [5].

In this paper, we divide the complex Chebyshev approximation problem into two real Chebyshev approximations, and each of them can be solved by using Remez exchange algorithm. This method is fast and easy, also the powerful McClellan-Parks program cna be. applied after slight modification. The overall performance is satisfied and the magnitude of the total complex error is also equiripple in the Chebyshev sense. Moreover, the method can also be applied to design all-pass phase equalizers.

II. PROBLEM FORMULATION FOR FIR DIGITAL FILTER DESIGNS WITH CONSTANT GROUP DELAY IN PASSBAND

The frequency response of a FIR digital filters with real impulse response $h(n)$, $n=0,1,\dots,N-1$ is given by

$$H(w)=\sum_{n=0}^{N-1} h(n)e^{-jnw} \tag{1}$$

For simplicity, we consider only odd-length filter designs, let $N=2L+1$ and

$$h(n)=h_e(n)+h_0(n) \quad n=0,1,\dots,N-1 \tag{2}$$

where $h_e(n)$ and $h_0(n)$ are the even part and odd part of $h(n)$ respectively and are given by

$$h_e(L-n)=h_e(L+n)=\tfrac{1}{2}[h(L-n)+h(L+n)], \quad n=0,1,2,\dots,L \tag{3a}$$

and

$$h_0(L-n)=-h_0(L+n)=\tfrac{1}{2}[h(L-n)-h(L+n)], \quad n=0,1,2,\dots,L \tag{3b}$$

Obviously $h_e(L)=h(L)$ and $h_0(L)=0$; Thus

$$H(w)=\sum_{n=0}^{2L} h_e(n)e^{-jnw}+\sum_{n=0}^{2L} h_0(n)e^{-jnw}$$
$$=e^{-jLw}\Big[\sum_{n=0}^{L}\hat{h}_e(n)\cos nw+j\sum_{n=1}^{L}\hat{h}_0(n)\sin nw\Big] \tag{4}$$

where

$$\hat{h}_e(n)=\begin{cases} h_e(L) & n=0 \\ 2h_e(L-n) & n=1,2,\dots,L \end{cases} \tag{5a}$$

and

$$\hat{h}_0(n)=2h_0(L-n) \quad n=1,2,\dots,L \tag{5b}$$

Then we use Eq.(4) to approximate the desired complex-valued frequency response $D(w)$ given below

$$D(w)=\begin{cases} M(w)w^{jp(w)}=e^{-jLw}\{M(w)\cos[Lw+P(w)]+ jM(w)\sin[Lw+P(w)]\}, & w\in \text{passbands.} \\ 0, & w\in \text{stopbands.} \end{cases} \tag{6}$$

where $M(w)$ and $P(w)$ are the amplitude response

CH 3006-4/91/0000 – 0260 $1.00 © IEEE

and phase response of D(w) respectively. Then the design problem can be separated into two real approximated criteria, which are called by even and odd approximation respectively, i.e.

even approximation:

$$H_e(w)=\sum_{n=0}^{L}\hat{h}_e(n)\cos nw \simeq D_e(w) \quad (7a)$$

$$\text{for } D_e(w)=\begin{cases} M(w)\cos[Lw+P(w)], & w \in \text{passband} \\ 0, & w \in \text{stopband} \end{cases} \quad (7b)$$

and odd approximation:

$$H_0(w)=\sum_{n=1}^{L}\hat{h}_0(n)\sin nw \simeq D_0(w) \quad (7c)$$

$$\text{for } D_0(w)=\begin{cases} M(w)\sin[Lw+P(w)], & w \in \text{passband} \\ 0, & w \in \text{stopband} \end{cases} \quad (7d)$$

The overall filter impulse response h(n) can be obtained by combining the resultant $\hat{h}_e(n)$ and $\hat{h}_0(n)$. Eq.(7a) and (7c) are formulated to find $\hat{h}_e(n)$ and $\hat{h}_0(n)$ such that to minimize the maximum absolute weighted errors defined by

$$||E_e(w)||=\text{Maximum } W_e(w)\cdot|D_e(w)-H_e(w)|, \quad w \in \text{passband/stopband} \quad (8a)$$

and

$$||E_0(w)||=\text{Maximum } W_0(w)\cdot|D_0(w)-H_0(w)|, \quad w \in \text{passband/stopband} \quad (8b)$$

for even and odd approximation respectively, where $W_e(w)$ and $W_0(w)$ are the weighting functions.

The main differences between this problem and the conventional filter approximation problem are in the desired responses for even and odd approximation, the original McClellan-Parks program can be easily modified to fit this problem. Also the weighted errors for two real approximations are each equiripple, if the weighting functions are chosen the same for both even and odd approximations ($W_e(w)=W_0(w)=W(w)>0$), then the magnitudes of the overall complex errors are also equiripple in the complex Chebyshev sense. This separate appromixation approach has the simplicity advantages and easy implementation for practical applications.

Due to the degree of freedom for odd approximation is one less than that for even approximation, the peak error of the former is generally larger than that of the latter. Suppose the peak errors of even and odd approximation are δ_e and δ_0 respectively, i.e.

$$W(w)|D_e(w)-H_e(w)| \le \delta_e \quad (9a)$$

and

$$W(w)|D_0(w)-H_0(w)| \le \delta_0 \quad (9b)$$

then the peak magnitude of the overall complex error is

$$w(w)|D(w)-H(w)| \le \sqrt{\delta_e^2+\delta_0^2} \quad (10)$$

EXAMPLE 1: Design of Low-pass Filters.

A 31 point lowpass filter with L=15, group delay τ=12 [P(w)=-12w], a passband [0,0.06] and a stopband [0.12,0.5] is considered in the design specifications. If the passband weighting is 1 and the stopband weighting is 10 for both even approximation and odd approximation, the frequency magnitude and group delay responses are shown in Fig.1(a) and (b) respectively. Fig.1(c) shows the magnitude of the overall complex error, in which the peak value is 0.04404 and 0.004401 in stopband.

III. DESIGN OF FIR ALLPASS PHASE EQUALIZERS

For a FIR allpass filter, in which its magnitude response is approximately unity with some prescribed phase characteristics, i.e.

$$\begin{aligned} D(w)&=e^{-jLw}\cdot e^{j\Phi(w)}, && 0\le w\le \pi \\ &=e^{-jLw}[\cos\Phi(w)+j\sin\Phi(w)], && 0\le w\le\pi \end{aligned} \quad (11)$$

where $\Phi(w)$ is the phase response and a prescribed function of w; The design problem is similar to that in Section II, i.e. we wish to minimize the maximum absolute error (W(w)=1) defined as

$$||E_e(w)||=\text{maximum}|\cos\Phi(w)-\sum_{n=0}^{L}\hat{h}_e(n)\cos nw|, \quad 0\le w\le\pi \quad (12a)$$

for even approximation, and

$$||E_0(w)||=\text{maximum}|\sin\Phi(w)-\sum_{n=0}^{L}\hat{h}_0(n)\sin nw|, \quad 0\le w\le\pi \quad (12b)$$

for odd approximation.

Due to the phase discontinuity at zero and folding frequencies, relaxation of the band edge specification for the odd approximation is permitted such that better result will be obtained, that is to say that, Eq.(12b) can be reformulated as below to minimize

$$||E_0(w)||=\text{maximum}|\sin\Phi(w)-\sum_{n=0}^{L}\hat{h}_0(n)\sin nw|, \quad w_0\le w\le\pi-w_0 \text{ and } w_0\ll\pi \quad (13)$$

If the phase $\Phi(w)$ is symmetric or antisymmetric about $w=\pi/2$, the FIR allpass filter can be implemented with one-half the usual number of multiplications, in a manner analogous to the linear phase Case [6]. These results are summarized as below:

$$\left.\begin{aligned} h(L-k)&=h(L+k), && k \text{ even} \\ h(L-k)&=-h(L-k), && k \text{ odd} \end{aligned}\right\} \text{for } \Phi(w) \text{ even about } \pi/2 \quad (14)$$

and

$$h(L-k)=h(L+k)=0,\ k \text{ odd}, \quad \text{for } \Phi(w) \text{ odd about } \pi/2 \quad (15)$$

The output coefficients of Eq.(15) are not actually zero in practice, however they are generally very small, we simply set these coefficients to zero for keeping the antisymmetry of the phase $\Phi(w)$.

EXAMPLE 2: Design of Sine-Delay Allpass Phase Equalizers

The desired frequency response is a unit amplitude and a sinusoidial phase characteristic

$$\mathrm{Arg}\, D(w) = -\left(\frac{N-1}{2}\right)w - \beta\cos w \tag{16}$$

where $\Phi(w)=-\beta\cos w$ is antisymmetric about $w=\pi/2$, and the group delay is

$$\tau(w) = -\frac{d}{dw}\mathrm{Arg}\, D(w) = \frac{N-1}{2} - \beta\sin w \tag{17}$$

A 61 point Sine-delay allpass filter is designed with L=30, $\beta=2\pi$ and $w_0=0.04$, Fig.2(a) and (b) show the magnitude response (magnified version) and group delay response respectively, and the peak magnitude error is 0.0005249 which is much smaller than 0.000931 in [6].

IV. CONCLUSION

By separately approximating the real and imaginary parts of FIR filter complex-valued frequency response, McClellan-Parks program can be slightly modified to design the general FIR filters and allpass phase equalizers very effectively. This approach is much easier than the current existing linear programming techniques, and the performance is satisfied, more importantly the overall complex errors are also equiripple in the complex Chebyshev sense. This approach has several practical advantages such as fast design time and easy implementation with comparable accuracy.

REFERENCES

[1] J.H. McClellan, T.W. Parks and L.R. Rabiner, "A Computer program for designing optimum FIR linear phase digital filters", IEEE Trans. Audio Electroacoust. vol.AU-21, pp.506-526, Dec. 1973.

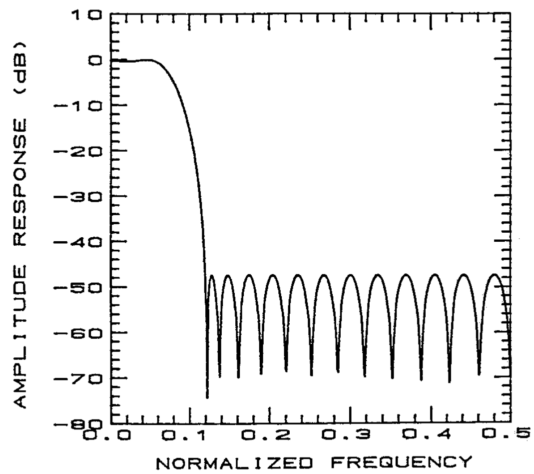
[2] O. Herrmann and H.W. Schuessler, "Design of nonrecursive digital filters with minimum phase", Electron. Lett., vol. 6, no. 11, pp.329-330, May 1970.

[3] X. Chen and T.W. Parks, "Design of FIR filters in the complex domain," IEEE Trans. Acoust., Speech, Signal Processing, vol.ASSP-35, pp.144-153, Feb. 1987.

[4] K. Preuss, "On the design of FIR filters by complex Chebyshev approximation," IEEE Trans. Acoust., Speech, Signal Processing, vol.ASSP-37, pp.702-712, May 1989.

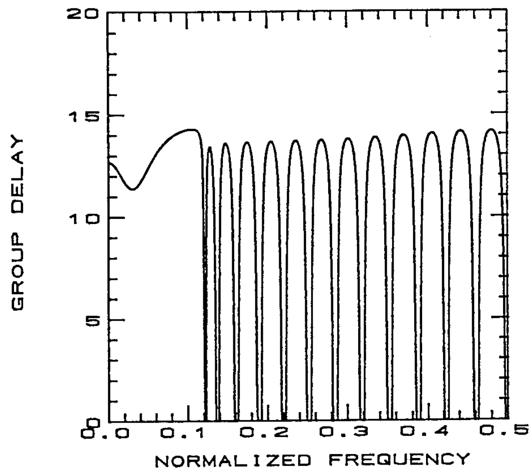
[5] M. Schulist, "Improvements of a complex FIR filter design algorithm," Signal Processing, vol. 20, pp.81-90, May 1990.

[6] K. Steiglitz, "Design of FIR digital phase networks," IEEE Trans. Acoust., Speech, Signal Processing, vol.ASSP-29, pp. 171-176, Apr. 1981.

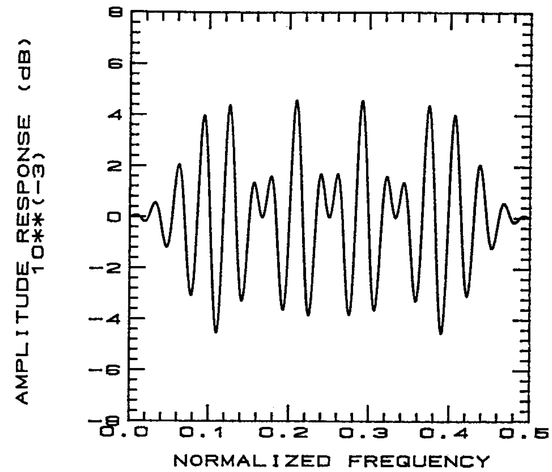


(a)

Fig.1 Example 1: A 31 point lowpass filter with $f_p=0.06$, $f_s=0.12$ and $\tau=12$. (a) magnitude response, (b) group delay response. (c) equiripple magnitude response of complex error. (d) equiripple error for even approximation. (e) equiripple error for odd approximation. (f) trace of complex error in the passband [-0.06,0.06] (dotted line: error radius of [5]). (g) trace of complex error in the stopband [0.12,0.88] (dotted line: error radius of [5]).



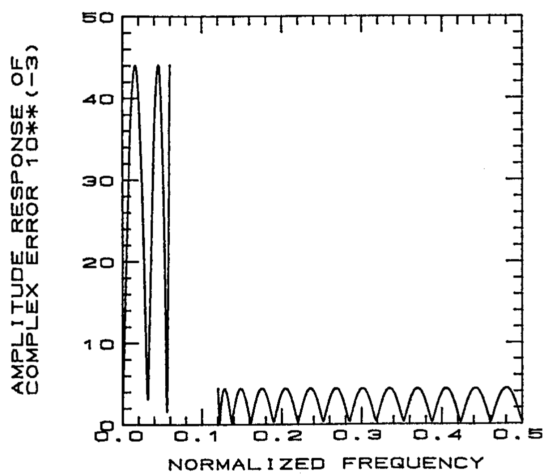
(b)



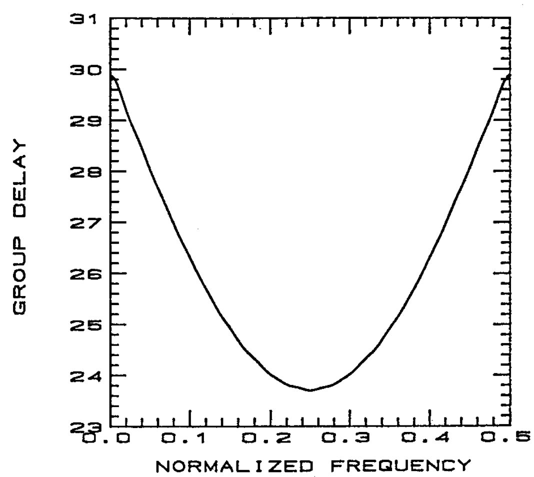
(a)

Fig.1 Example 1: A 31 point lowpass filter with f_p=0.06, f_s=0.12 and τ=12. (a) magnitude response, (b) group delay response. (c) equiripple magnitude response of complex error. (d) equiripple error for even approximation. (e) equiripple error for odd approximation. (f) trace of complex error in the passband [-0.06,0.06] (dotted line: error radius of [5]). (g) trace of complex error in the stopband [0.12,0.88] (dotted line: error radius of [5]).

Fig. 2 Example 5: A 61 point Sine-delay allpass phase equalizer for $0.02 \le f \le 0.48$. (a) magnified magnitude response, (b) group delay response.



(c)



(b)

Fig.1 Example 1: A 31 point lowpass filter with f_p=0.06, f_s=0.12 and τ=12. (a) magnitude response, (b) group delay response. (c) equiripple magnitude response of complex error. (d) equiripple error for even approximation. (e) equiripple error for odd approximation. (f) trace of complex error in the passband [-0.06,0.06] (dotted line: error radius of [5]). (g) trace of complex error in the stopband [0.12,0.88] (dotted line: error radius of [5]).

Fig. 2 Example 5: A 61 point Sine-delay allpass phase equalizer for $0.02 \le f \le 0.48$. (a) magnified magnitude response, (b) group delay response.