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Wavenumber formulation for $V(z)$ curves of line-focus acoustic microscopy

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Abstract

A stiffness matrix formulation is proposed to incorporate electro-mechanical relations for simulating the $V(z)$ curves of a line-focus acoustic microscope for layered media. Examples for the layer/substrate configuration of polycrystalline ZnO and silicon, which are assumed to be isotropic, are well discussed. Leaky Rayleigh, Sezawa and/or pseudo-Sezawa waves with respect to various layer thicknesses, can be easily modeled and interpreted from their $V(z)$ curves. Results obtained from different electro-mechanical relations are also compared.

Keywords: Acoustic microscopy; $V(z)$ curves; SAW velocity

1. Introduction

A line-focus acoustic microscope is consisted of a piezoelectric transducer, a buffer rod, a thin matching layer and the lens. The transducer emits an incident wavefield into the buffer rod. The downward propagating waves penetrate through the thin matching layer of the lens and the coupling fluid. Waves then reach the solid specimen under evaluation and are scattered. The incident and the scattered wavefields together determine the voltage output of the transducer. The record of this voltage output of the transducer is often called the $V(z)$ curve of the acoustic microscope, since it is a function of the relative elevation z between the specimen and the geometrical focal point of the lens. The pattern of peaks and dips in a $V(z)$ curve is closely related to the mechanical properties of the specimen. Hence $V(z)$ curves are also referred as acoustic material signatures (AMSs) of the specimens and can be used to characterize the specimens.

A complete modeling of $V(z)$ curves contains three ingredients: (1) the model of incident waves insonifying the specimen; (2) the model of backscattered waves scattered from the specimen; (3) the model for converting the wavefields into the voltage output of the transducer. There have been various models concerning both the incident and the scattered waves: the Fourier spectral decomposition approach [1]; the geometrical ray-optics approach [2]; and pure numerical methods, such as the boundary element method (BEM) [3] and the hybrid finite/boundary element method [4], while the voltage output is usually related to the wavefields through an electro-mechanical reciprocal relation originated by Auld [5], or through a simple reciprocity argument between the voltage and wavefields proposed by Atalar [1].

2. Methods of solution

2.1. Incident wavefield

In this paper, the emitted wavefield from the piezoelectric transducer is assumed to be as a Gaussian beam. The emitted wave tracks through the buffer rod, the anti-reflection coating film of the lens, and enters the coupling fluid. Firstly, the BEM procedure proposed by Achenbach et al. [3] is used to calculate the wavefield in the coupling fluid. This wavefield then acts as an incident wave to insonify the specimen in the latter analysis. Both

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the paraxial approximation and the ambiguities of the pupil function of the lens in the ray-optics approach are thus eluded from this full-wave analysis. Moreover, due to the toned voltage burst across the transducer and the signal-gating electronic circuits inside the microscope, the effects of multiple reflection of waves between the lens and the specimen are usually gated out. Thus, this incident wavefield is independent on the specimen being evaluated, and can then be computed once and for all in advance, checked in full detail and saved for later use. It can be treated as a characteristic of the particular acoustic microscope operated on a particular frequency. Here, the incident wavefield in the horizontal wavenumber domain, rather than the physical domain, is saved to complement the proposed wavenumber formulation.

2.2. Scattered wavefield

To construct the scattered wave field, a stiffness matrix formulation in the wavenumber domain is employed. First of all, the Fourier transform over x with kernel $\exp(+ikx)$ will be employed to construct the solution in the wavenumber domain. The transformed displacement fields within a genetic layer are expressed in terms of the layer-face displacements, as

$$\{\hat{u}_n, \mathrm{i}\hat{v}_n\}^{\mathrm{T}} = [\mathbf{N}_n(z)]\hat{\mathbf{w}}_n \tag{1}$$

where $\hat{\mathbf{w}}_n = \{\hat{U}_{n-1}, \mathrm{i}\hat{V}_{n-1}, \hat{U}_n, \mathrm{i}\hat{V}_n\}^{\mathrm{T}}$, the superimposed hat indicates the Fourier transformed quantity, and $\hat{U}_n$ and $\hat{V}_n$ are the layer-face displacement at the nth interface, respectively; while the superscript T indicated the transpose of a matrix or a vector. The 'shape function' matrix $[\mathbf{N}_n(z)]$ is of size 2×4. By using of the generalized Hooke's law and (1), the tractions along the upper and lower faces of the nth layer can be expressed as

$$\{\mathbf{\Sigma}_n\} = [\mathbf{K}_n]\{\mathbf{U}_n\} \tag{2}$$

where $\{\mathbf{\Sigma}_n\} = \{-(\mathbf{\Sigma}_{xz})_n^u, -\mathrm{i}(\Sigma_{zz})_n^u, (\Sigma_{xz})_n^l, \mathrm{i}(\Sigma_{zz})_n^l\}^{\mathrm{T}}$. The matrix $[\mathbf{K}_n]$ is called the 'local stiffness matrix' of the size of 4×4 for the nth layer.

Conditions of traction continuity and/or traction jump across interfaces are now ready to conclude a system of $2N-2$ linear equations for U_n and V_n $(n = 1, 2, \ldots, N-1)$. This is equivalent to the 'assembly' process in the finite element analysis. It leads to a symmetric $(2N-2) \times (2N-2)$ global stiffness matrix with bandwidth 7.

2.3. Electro-mechanical relations

Finally, the voltage output of the transducer is expressed as a wavenumber integral of incident and scattered wavefields within the coupling fluid, by Auld's electro-mechanical reciprocal relation. The expression can be cast in the form of

$$V(z) \propto \int_0^{\infty} \frac{1}{L} \cdot \hat{p}^{\mathrm{in}}(k) \cdot \hat{p}^{\mathrm{sc}}(k; z)\, \mathrm{d}k, \tag{3}$$

where $L(k) = -\rho_{\mathrm{w}}\omega^2/(k^2 - \omega^2/c_{\mathrm{w}}^2)^{1/2}$. The wavenumber integral must then be evaluated numerically.

Atalar [1] proposed a slightly different electro-mechanical relation, which can be cast in the similar form as

$$V(z) \propto \int_{-\infty}^{\infty} \hat{u}_z^{\mathrm{in}}(k) \cdot \hat{u}_z^{\mathrm{sc}}(k; z)\, \mathrm{d}k. \tag{4}$$

It is worthwhile noting that if the same argument of Eq. (4) is employed in the voltage and the emitted/received pressures, instead of z-components of velocities, the output voltage will then be, instead of (4),

$$V(z) \propto \int_{-\infty}^{\infty} \hat{p}_0^{\mathrm{in}}(k) \cdot \hat{p}_0^{\mathrm{sc}}(k; z)\, \mathrm{d}k. \tag{5}$$

3. Numerical results

Examples for the layer/substrate configuration of polycrystalline ZnO and silicon are discussed. The $V(z)$ curves for these cases are compared with the ones computed by the boundary element method, and satisfactory agreements are obtained. The calculated $V(z)$ curves for the layer thickness of 1.4, 4.6, and 12.8 μm are also presented. Finally, leaky Rayleigh waves, leaky Sezawa waves, or leaky pseudo-Sezawa waves are interpreted from these simulated $V(z)$ curves. The differences in SAW speeds resulting from three electro-mechanical relations Eqs. (3)–(5), are presented in Table 1.

4. Conclusions

The incident wavefield in the coupling fluid is pre-calculated by the BEM approach to elude the paraxial approximation and the ambiguities of the pupil function of the lens. The scattered wavefield is then computed by the stiffness formulation in the wavenumber domain. A

Table 1
SAW velocities obtained from different electro-mechanical relations

h (μm)	Theoretical	A	B	C
1.4	4184	4187 (+0.07)	4192 (+0.19)	4187 (+0.07)
4.6	3130	3130 (——)	3132 (+0.06)	3128 (−0.06)
	5155	5129 (+1.24)	5200 (+0.87)	5239 (+1.63)
12.8	2661	2660 (−0.04)	2657 (−0.15)	2662 (+0.04)
	3796	3788 (−0.21)	3760 (−0.95)	3802 (+0.16)
	5145	5074 (−1.40)	5014 (−2.55)	5164 (+0.37)

A: Auld's relation, B: Atalar's relation, C: modified Atalar's relation.

typical $V(z)$ curve calculation by this full wave analysis takes less than two minutes on a 486DX2 PC versus two hours in a Cray YMP/EL for the pure boundary element method. Fast computation of $V(z)$ curves makes the inversion for material properties feasible through a line-focus acoustical microscope. The SAW velocities predicted from $V(z)$ curves have relative error up to 2.55% in the particular example.

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