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特殊拉格拉奇流形(2/3)

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Abstract

During this year, I wrote a paper titled "A Stability Criterion for Nonparametric Minimal Submanifolds" with Prof. Mu-Tao Wang from Columbia University. In this paper, we use calibrating forms to obtain new stability criterion for minimal graphs of higher codimension. This paper has been submitted.

I have a weekly seminar since last September with Jih-Hsin Cheng from Academia Sinica and I-Hsun Tsai from Taiwan University to study a conjecture of Yau and Thomas. The conjecture is about mean curvature flow of graded Lagrangian. If the conjecture is true, one will obtain the existence of Special Lagrangian submanifolds.

Key words: stability, minimal submanifold, mean curvature flow, special Lagrangian.

中文摘要

在本年度的計畫中，完成論文一篇，已在投稿中，論文的題目是「極小子流形穩定性的判別條件」。這是和哥倫比亞大學王慕道教授合作的工作，我們是利用 Calibrating forms 來得到額外訊息，進而證明這個關於高餘維極小子流形穩定性的判別方法。

從去年九月開始，我和中研院鄭日新教授以及台大的蔡宜洵教授，每星期有固定的討論會。我們主要是研究 Yau 和 Thomas 關於均曲率流的一個猜想。如果這個猜想成立，則我們將得到特殊拉格拉奇子流形的存在性定理。

關鍵詞： 極小子流形、穩定性、均曲率流、特殊拉格拉奇子流形。

Midreport for NSC 91-2115-M-002-004
Special Lagrangians in Calabi-Yau Manifolds

Yng-Ing Lee

May 31, 2003

In the second year of this project, I wrote a paper titled "A Stability Criterion for Nonparametric Minimal Submanifolds" [2] with Prof. Mu-Tao Wang from Columbia University. In the paper, we use calibrating forms to obtain new stability criterion for minimal graphs of higher codimension. This paper has been submitted.

The notion of calibration not only characterizes nice examples such as Special Lagrangian submanfolds and complex submanifolds, but also gives an additional tool in studying general submanifolds. With this observation, M.T. Wang makes a significant progress in the mean curvature flow of higher codimension (see [5], [6]) and obtains a series of results in related problems. This direction becomes very active and attracts a lot of mathematicians' attentions. I had a lot of discussions with M.T. Wang last summer and studied most of his work. I think the main difficulties in this field are not overcome yet. The problem I am most interested in is using mean curvature flow to obtain the existence of Special Lagrangian submanifolds. This a conjecture of Yau and Thomas [4]. I have a group seminar with Jih-Hsin Cheng from

Academia Sinica and I-Hsun Tsai from Taiwan University to study the conjecture.

We study the problem first from Geometric Measure Theory's point of view and come to the step of understanding mean curvature flow of singular spaces which is based on Tom Ilmanen's work. Ilmanen visited Taiwan last summer and had very nice interactions with me. I wish we can invite him to Taiwan again in the near future, so we can learn his method in more details. I think the notion of weak solutions for a general flow will play a very important role in future's research. Although our group is still in a very primitive stage in proving Yau and Thomas's conjecture, we do learn many important knowledge and experience in the weekly meeting.

In the coming year, I wish to finish those problems studied in the first and second year of this project. One is to generalize the result on stability criterion for minimal submanifolds, which is a joint program with M.T. Wang. Another is to study Special Lagrangian loop and use perturbation to construct new Special Lagrangian submanifolds. This a joint program with Conan Leung. The last one is the program with J.H. Cheng and I H. Tsai, which studies Yau and Thomas's conjecture.

References

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