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Supplement to the Iris-loaded Wave Guide as a Boundary Value Problem III

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The problem, we treated in our last paper can be considered also as one for the magnetic field intensity H_φ. For an exact solution, the two results must coincide. For an approximation we can obtain information about the degree of approximation by comparing the two results and finding out for which values of h/R and in which regions of kb the two results show a good agreement.

IN our last paper[1] we formulated the problem of the E-wave in an iris-loaded wave guide with infinitely thin irises as a variational principle for the longitudinal electric field intensity and obtained with the simple trial function

$$\left.\frac{\partial E_z}{\partial z}\right|_{z=0} = A I_0(\sqrt{h^2-k^2}\, r)$$

the transcendental equation for the determination of the propagation-constant h,

$$\text{(A-a)} \quad (A-\beta)-\delta(A^2-\gamma^2) = 0 \qquad (1)$$

with

$$A = \frac{I_0(c_0 a)}{c_0 a I_0'(c_0 a)}, \qquad \alpha = \frac{J_0(ka)N_0(kb)-N_0(ka)J_0(kb)}{ka[J_0'(ka)N_0(kb)-N_0'(ka)J_0(kb)]}; \qquad \beta = \frac{J_0(kb)}{kaJ_0'(ka)}$$

$$\delta = \frac{2a(hl)^4}{\pi l(1-\cos hl)} \sum_{n=1}^{\infty} \frac{1-(-)^n\cos hl}{[(hl)^2-n^2\pi^2]^2}\,\frac{1}{\sqrt{n^2\pi^2-(kl)^2}} - \frac{1}{(ka)^2 J_0'(ka)\left[J_0'(ka)\frac{N_0(kb)}{J_0(kb)}-N_0'(ka)\right]};$$

$$\gamma^2 = \frac{l^2}{a^2}\,\frac{(hl)^2\sum\frac{1-(-)^n\cos hl}{[(hl)^2-n^2\pi^2]^2}\,\frac{1}{\sqrt{n^2\pi^2-(kl)^2}}}{\sum\frac{1-(-)^n\cos hl}{[(hl)^2-n^2\pi^2]^2}\sqrt{n^2\pi^2-(kl)^2}}\,.$$

Now it is remarkable that the problem can also be treated as one for the magnetic field intensity H_φ. One has then to find a solution of

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial H_\varphi}{\partial r}\right)+\frac{\partial^2 H_\varphi}{\partial z^2}+\left(k^2-\frac{1}{r^2}\right)H_\varphi = 0$$

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(1) W. Kroll, Chin. J. Phys. 3, 98 (1965).

with the boundary conditions

$$\frac{\partial}{\partial r}(rH_\varphi)=0\ :\ r=b; \qquad \frac{\partial H_\varphi}{\partial z}=0\ :\ z=0.\ z=l;\ r>a$$

This problem can be solved in a way quite similar to that of the electric problem. By the introduction of the Green function G_{H}

$$G_{\mathrm{H}}(r,z;r',z')=\frac{1}{l}\sum(2)f_n(r,r')\cos\frac{n\pi z}{l}\cos\frac{n\pi z'}{l}$$

with

$$f_0(r,r')=\frac{\pi}{2J_0(kb)}\begin{cases}J_1(kr)[J_1(kr')N_0(kb)-N_1(kr')J_0(kb)]; & r<r'\\ J_1(kr')[J_1(kr)N_0(kb)-N_1(kr)J_0(kb)]; & r>r'\end{cases}$$

$$f_n(r,r')=\frac{1}{I_0(k_nb)}\begin{cases}I_1(k_nr)[I_1(k_nr')K_0(k_nb)\ +K_1(k_nr')I_0(k_nb)]; & r<r'\\ I_1(k_nr')[I_1(k_nr)K_0(k_nb)\ +K_1(k_nr)I_0(k_nb)]; & r>r'\end{cases}$$

we obtain for the determination of the propagation constant h of the guide the variational principle

$$\delta M=0$$

with

$$M=\frac{\int_0^a r\frac{\partial H_\varphi}{\partial z}dr\int_0^a r'\frac{\partial H_\varphi}{\partial z}G_{\mathrm{H}}(r,\ 0;\ r',0)\,dr'}{\int_0^a r\frac{\partial H_\varphi}{\partial z}dr\int_0^a r'\frac{\partial H_\varphi}{\partial z}G_{\mathrm{H}}(r,0;r',l\,)\,dr'}$$

where $\frac{\partial H_z}{\partial z}$ is the normal derivative of H_φ for $z=0$. When we proceed in the same way as in the electric problem we obtain with the simple trial function

$$\frac{\partial H_\varphi}{\partial z}=BJ_1(\sqrt{h^2-k^2}\,r)$$

an equation of the form

$$(A'-a')\ (A'-\beta')-\delta'(A'^2-\gamma^2)=0 \qquad (2)$$

with

$$A'=\frac{I_1\ (c_0a)}{c_0aI_0(c_0a)};\quad \alpha'=\frac{J_1(ka)N_0(kb)-N_1(ka)J_0(kb)}{ka[J_0(ka)N_0(kb)\ -N_0(ka)J_0(kb)]};\quad \beta'=\frac{J_1(ka)}{kaJ_0(ka)};$$

$$\delta'=\frac{2a(hl)^4}{\pi l(1-\cos hl)}\sum_{n=1}^{\infty}\frac{1-(-)^n\cos hl}{[(hl)^2-n^2\pi^2]^2}\sqrt{n^2\pi^2-(kl)^2}\quad \frac{1}{(ka)^2J_0(ka)\left[J_0\ (ka)\frac{N_0(kb)}{J_0(kb)}-N_0\ (ka)\right]}$$

When the two trial functions represented the exact solution the two equations should coincide or at least should lead to identical results. We expect then that, the better the approximation, the better will be the agreement of the two results obtained from the electic problem and from the magnetic problem, We[1] have solved the equation for the electric problem for small values of *hl*, that is, the equation

$$\frac{\sqrt{h^2-k^2}\,aI_0'(\sqrt{h^2-k^2}\,a)}{I_0(\sqrt{h^2-k^2}\,a)}=ka\frac{J_0'(ka)N_0(kb)-N_0'(ka)J_0(kb)}{J_0(ka)N_0(kb)-N_0(ka)J_0(kb)}$$

The equation for the magnetic problem for small values of *hl*, that is, the equation

$$\frac{I_1(\sqrt{h^2-k^2}\,a)}{\sqrt{h^2-k^2}\,aI_0(\sqrt{h^2-k^2}\,a)}=\frac{J_1(ka)N_0(kb)-N_1(ka)J_0(kb)}{ka[J_0(ka)N_0(kb)-N_0(ka)J_0(kb)]}$$

has been solved by Chu and Hansen[2]. A comparison of the two results shows, that we obtain a near agreement for small values of h/k, as. it should be. The solutions of the two equations (1) and (2) we expect to agree in a larger region of h/k values. In any case by comparison of the resalt obtained from the electric problem with that obtained fror the magnetic problem we can get information about the degree of approximation of the result for the propagation constant h.

(2) Chu and Hansen, J. Appl. Phys. 18, 996 (1947).