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效用與遺贈--高齡老人在健康動態下的資產管理

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Utility and Bequest:
A life-Cycle Model of Asset Management and Health Dynamics
Among the Oldest Old

Final Report

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Abstract

In our work we investigate asset management by the elderly, focusing on the links between health and health changes and asset management, and between consumption and asset use by the household and bequests.

Our main focus is on a structural dynamic model with two choice variables each period: (1) saving/spend-down, (2) stock buy/sell. The essence of the model is that the household tries to maximize a combination of lifetime expected utility and the bequest function, and that as the end-of-life nears the bequest function becomes more important, and if the shape of the bequest function is different than the utility function, investment strategies may change as a result. This is of significant policy importance because the elderly control such a large fraction of the asset in the country. So if they tend to shift investment behavior late in life it is important to know.

In the model a household has both a utility function over consumption each period and a bequest function, with the utility function having two sub-forms, one for good health and one for poor health. The curvature embodied in each function (risk aversion equivalent) can differ, and indeed we expect them to. For example, elderly may well be more risk averse when in poor health than in good health, because of feelings of vulnerability as well as the possibility of medical outlays; in addition, the curvature of the bequest function may well be different -- thus elderly may be more risk-neutral or even risk-seeking when managing assets intended as a bequest.

We build a structural model of household asset management decision-making and estimate the model using panel data from the first 3 waves (1992/1995/1998) of the Asset and Health Dynamics among the Oldest Old (AHEAD) database. We focus especially on household decisions about whether to enter or exit the stock market; we also investigate house asset changes (housing mobility) and shifts in ownership of other assets, notably cash and cash-like.

We build a separate model of mortality and morbidity. Then, to estimate the structural model, for each household, given the age of head (and spouse), we use this model to build in the probabilities for each path going forward of health and mortality -- what is the probability of remaining in good health next period, falling into poor health, or death of head or spouse. Using these probability values, we compute the utility for the household for each asset decision, building in transaction costs and costs due to health. We estimate the model using maximum likelihood.

摘要

本文提出一項整合效用與遺贈的生命循環理論架構，對於高齡老人在面臨健康狀況的不確定性下，如何達成最適資產管理進行分析。此一分析的重點在於健康動態與投資策略的風險平衡，以及生前消費與死後遺贈的效用關聯。

本文理論基礎構築在一項包括兩項選擇變數的動態結構模型。其基本精神在於高齡老人的家庭透過在生命循環末段中，每個時期的儲蓄消費及股票買賣的兩項選擇，而使終生預期效用及死後遺贈價值的總合極大化。對於高齡老人而言，當生命盡頭明顯接近時，遺贈價值相對生前效用而言，將變得更為重要; 而兩者在應對風險態度上的相對差異將會進而改變整體的投資策略。

本文模型設計假定高齡老人家庭在生前消費與死後遺贈有不同的效用價值，並與健康好壞而有明顯區別。這些效用函數也因此在其曲度上，也就是代表因應風險的態度上，有其顯著差異。基本而言，健康情況惡劣要比健康情況良好在資產管理上較為消極趨避風險(由於心理發生恐慌或實際醫療支出的負擔)。相反地，在資產管理上作為遺贈要比作為消費較為積極地應對風險(由於生前自己享受與死後繼承人享受的效用不同)。

本文並將設計一項高齡老人健康風險的計量模型，並將用由1992到1998共分三期的美國高齡老人資產與健康動態(AHEAD)的個案追蹤資料(Panel Data)，來估算在生命循環模型中的健康死亡風險 – 亦即健康良好，健康惡劣，及死亡三種狀況下的相對機率。

本文將根據上述的AHEAD資料及模型估算的健康風險機率，對本文的動態生命循環模型中所設定的各項理論要素，包括高齡老人在生前效用與死後遺贈上運用投資策略的避險態度，進行實證評估。

Utility and Bequest:
A life-Cycle Model of Asset Management and Health Dynamics
Among the Oldest Old

1. Introduction

Our theoretical insight is to develop a multi-period structural model with two choice variables in each period under a dynamic framework: (1) saving/consumption and (2) stock buy/sell. The essence of the model is that an elderly households would try to maximize a combination of lifetime expected utility and bequest, and as the end-of-life nears the bequest becomes more important, and the household's investment strategies may change as a result. This is of significant public policy implication because these households are in control of a significant fraction of national assets.

The structural model we built is a life-cycle expected utility maximization model of consumption and bequest. It includes two equations: an asset utilization (saving/consumption) decision; and an asset management (stock /cash) decision; with uncertain investment return (portfolio) risk. Such portfolio risks can be controlled by individual households' investment choices, and then will be internalized in the structural model. The mortality risk can be observed and accounted for, yet is beyond one's control. However, an individual household member's reactions to such mortality risk are reflected in his or her risk aversion associated with consumption and bequest.

In this report, we would like to investigate the asset utilization and asset management by the elderly by introducing these considerations. First, we develop a theoretical model in the life cycle model framework with bequest motive. By including both risky asset (stock) and non-risky asset (cash), we would like to study the representative household's asset allocations (portfolio) under the influence of mortality risk, and the potential difference in risk aversion between utility and bequest functions. Intuitively, the bequest function could be more risk neutral in comparison to the utility function. The main reason may come from the notion that utility from bequest actually transfers utility from the offspring who would be more willing to take risks for future life as a younger generation. From this perspective, as the representative agents are getting older, the mortality risks become higher, and the utility from bequest plays an increasingly important role in their expected life time utility. As a result, they will tend to shift the asset to more risky one - stock holding, particularly when they approach near the end of life.

In the second step, we develop a simulation model to solve the theoretical model. Since the model is a dynamic programming setting across six periods, and involves two state variables and three decision variables, it is quite complex to deal with. We solve it backward by dynamic programming. That is, first the dynamic problem is transformed into static problem, period by period. We solve the value function in the end period, say period T, given all possible initial conditions can be made in period T-1. By using these computing results, we can go back to period T-1 and solve the expected value function, given all possible initial conditions, which can be induced in period T-2. Applying these procedures until the beginning period, we may investigate the agent's asset managemen decisions across periods.

We conduct two sets of simulations to discuss three issues with regard to asset utilization and management for the elderly. Our baseline set is the agent (representative household) with a risk averse bequest function. An alternative set is the agent with a risk neutral bequest function. Both sets include different initial wealth endowments.

First, we would like to know the correlation between saving rates and wealth. In literature, this issue has been quite controversial during 1950s and 1960s. In their research, Dynan, Skinner, and Zeldes (2004) demonstrated an empirical finding of positive relationship between saving rate and lifetime income. The simulation results in our model are consistent with that finding.

Second, we would like to know how risk aversion pertaining to bequest would affect the investment decision. Our simulation results show that when the bequest function remains risk neutral, the asset allocation would shift considerably toward more risky assets – such as from cash to stock holding, just as expected.

Finally, we would like to know how the asset allocations changes as agents are getting older. The simulation results suggests that agents allocate more on risky assets when they are getting older, which holds for both risk neutral and risk averse situations as well.

All these simulation results presented in this report are based on a reduced-scale model (a prototype so to speak) of our further research. We will incorporate health dynamics as alternative factors in asset allocation decisions, and the number of life periods will also be expended. A more complete model in the near future would offer us more insights and knowledge about asset management and utilization decisions made by the elderly.

2. The Model

This model is based on a structural dynamic life cycle model. In six periods of life, the household tries to maximize a combination of lifetime expected utility and the bequest function.

At the beginning of each period, the household confronts an environment defined by the following state variables: Income, I_t, Cash, A_t, and stock, K_t, on hand. In each period the household decides the amount of cash holding, X_t, and the amount of stock holding, S_t. Both S_t and X_t can be positive or negative, but we assume that household cannot sell over the amount of stock in hand or spend over the money it has.

$$X_t + A_t \geq 0 \qquad (1)$$
$$S_t + K_t \geq 0 \qquad (2)$$

Household make decisions in each period, in order to maximize their current and future expected utility, and the expected utility from bequest. The utility function is given by,

$$U(C_t) = 1 - \frac{1}{C_t^{\alpha U}} \qquad (3)$$

consumption is determined as,

$$C_t = I_t - S_t - X_t \qquad (4)$$

In the end of each period, the household could be dead or remain alive to the next period, which is decided by the mortality probability, P_{death}. If household go forward to the next period, it would decide how to allocate assets again; If it is dead, the asset would become bequest to offspring, and the weighted utility from bequest would be part of household's utility.

The value of bequest in next period will be decided by the market interest r_t and the return on stock, r_{kt}. The utility from bequest and the value of bequest in the next period are shown in following equations:

$$B(W_t) = 1 - \frac{1}{W_t^{\alpha B}} \qquad (6)$$

$$\text{where} \quad W_t = A_{t+1} + K_{t+1} \qquad (7)$$

$$A_{t+1} = (A_t + S_t) \times r \qquad (8)$$

$$K_{t+1} = (K_t + X_t) \times r_{kt} \qquad (9)$$

Market interest rate r_t is a certainty; while stock return r_{kt} follows a log-normal distribution:

$$f(x) = \frac{1}{2\pi\sigma x} \exp\left(\frac{x-\mu}{2\sigma^2}\right) \qquad (10)$$

In summary, the household's utility maximization in each period can be characterized as the following: given initial endowments A_t, K_t, I_t, market rate r_t, and the probability distribution of investment returns on stock, r_{kt}; a agent (representative) household would like to maximize (11), subject to (1), (2), and (4).

$$V_t(A_t, K_t, I_t) = U(C_t) + \frac{P_{death} \times \omega}{r_t} \int_{a1}^{a2} B(W_t) f(r_{kt}) dr_{kt} + (1 - P_{death}) \int_{a1}^{a2} V_{t+1}(A_{t+1}, K_{t+1}, I_{t+1}) dr_{kt}$$

$$\text{for} \quad t = 1,2,3,\ldots,6 \qquad (11)$$

Note that, we restrict P_{death} to be 0 in the period 1; and to be 1 on the period 6.

3. Solving the Model

The model is solved by working backward, in the spirit of dynamic programming. More precisely, we solve the end period V_6^* first, and use the V_6^* to solve V_5^*, V_4^*, $\ldots V_1^*$, recursively:

$$V_t(A_t, K_t, I_t) = U(C_t) + \frac{P_{death} \times W}{r_t} \int_{a1}^{a2} B(W_t)\, f(r_{kt}) dr_{kt} + (1 - P_{death}) \int_{a1}^{a2} V_{t+1}^*(A_{t+1}, K_{t+1}, I_{t+1}) dr_{kt}$$

for $t = 1,2,3,...,6$, (12)

$V_{t+1}^*(A_{t+1}, K_{t+1}, I_{t+1})$ are given

In periods prior to the sixth period, given arbitrary initial wealth endowment set (A_t, K_t, I_t), the state variables of V_{t+1}^*, $(A_{t+1}, K_{t+1}, I_{t+1})$, are determined by budget constraints and current control variables S_t and X_t. As a result, we may need to approximate V_{t+1}^*, thus requiring the knowledge of the value of V_{t+1}^* in all possible initial endowment conditions. We set up a grid of state variables, and solve all these contingencies. For example, given the upper bound and lower bound pertaining to $(A_{t+1}, K_{t+1}, I_{t+1})$, we set up N points within each interval respectively, resulting in N^2 initial conditions in total. Upon solving all these possible contingencies, and V_{t+1}^* requested by (12) can be solved by interpolation.

3.1 *The Sixth Period Solution*

Since the household in the last period (the seventh period) has to be dead for certain, the utility maximization problem of the sixth period is,

$$V_t(A_t, K_t, I_t) = U(C_t) + \frac{W}{r_t} \int_{a1}^{a2} B(W_t)\, f(r_{kt}) dr_{kt} \tag{13}$$

Compared to all previous periods (from second to fifth period) when the household has face the uncertainty of death, the utility maximization in the sixth period is straightforward and quite simple: how to distribute their cash and stock - either consume in this period or leave it to their children after death?

Since it is virtually impossible to have a **closed form solution**, we are looking for a **numerical solution**. The solutions S_t and X_t lie in the region formed by constraints (1), (2), (4), and (5), a triangle region.

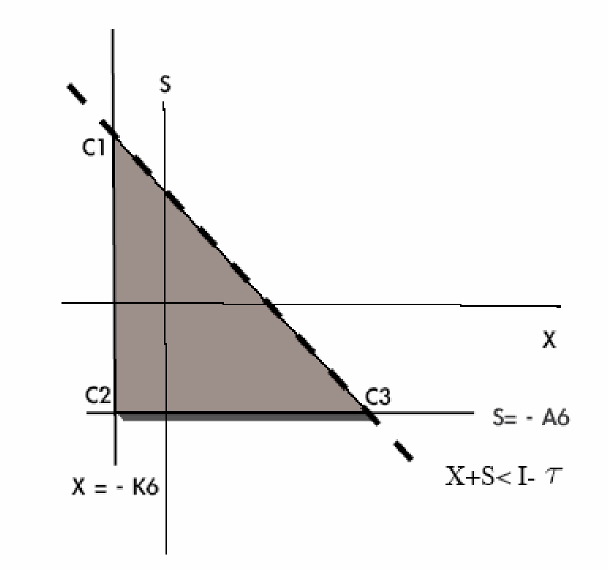

Figure 1

Figure 1 shows the constraint region, the shadow area. Our work is to find the maximum of this region. By applying grid search first, we find that the shape of the value over the domain is quite monotone. There exists a unique optimum. We build an algorithm based on gradient search to find the optimum. This algorithm is divided into three steps. First, we simplify our problem to one dimension. Using bisection method to bracket the optima in the bottom edge, b^* and left edge, l^* of the constraint region;

compare these two optima, and pick the bigger one. Secondly, starting from this optimum, we try to search the optimum inside the domain by using the Broyden-Fletcher-Goldfarb-Shanno (BFGS) algorithm.

In our first step, we assume there are four cases – solving the problem case by case. Take the bottom edge for example, from C_2 to C_3 in Figure 1, V could be (i) monotone increasing, where both $\frac{dV(C_2)}{dx}$, $\frac{dV(C_3)}{dx}$ are greater than 0, (ii) monotone decreasing, $\frac{dV(C_2)}{dx}$, $\frac{dV(C_3)}{dx}$ are less than 0[1], (iii) first decreasing, meeting the minimum, and then increasing, where $\frac{dV(C_2)}{dx} < 0$, $\frac{dV(C_2)}{dx} > 0$, and (iv) increasing, meeting the optimum, and then decreasing, where $\frac{dV(C_2)}{dx} > 0$, $\frac{dV(C_3)}{dx} < 0$. It's easy to find the maximum for cases 1, 2, and 3: just pick the bigger one of $V(C_2)$ and $V(C_3)$. However, for case 4, we apply the bisection search to bracket the maximum. That is, the maximum is supposed to have been bracketed in interval (C_2, C_3). We then evaluates the function at an intermediate point x and obtain a new, yet smaller bracketing interval, either (C_2, x) or (x, C_3). This process continues until the bracketing interval is acceptably small, the maximum obtained. When each case has been considered, we can get the maximum, b^*, in the bottom side. We apply the above algorithm again to find the optima of left side, l^*; compare b^* and l^*; and pick the maximum of these two. From this point, we could do the next step: internal region search.

[1] In case 1 and 2, We also check five points between C_2 and C_3 in order to make sure if V is monotone along the interval; while in case 3, we also check five middle points to make sure that there only exist one minima, V.

The goal of the BFGS algorithm we use to search the internal solution is to accumulate information from successive line maximizations so that such line maximizations lead to the exact optima. In the process of searching, the algorithm will generate a matrix and update it iteratively to help us find the direction of maximum.

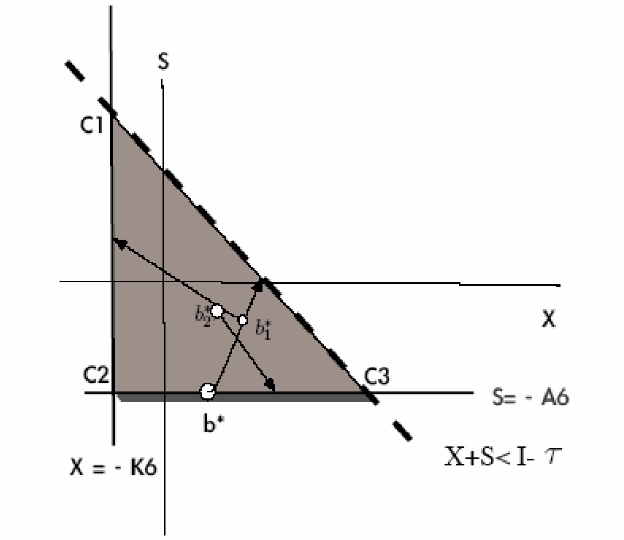

Figure 2

Figure 2 illustrates how this search works. Suppose that the starting point is the optimum in the bottom, b^*. The algorithm will first search the internal solution through the direction, $(\frac{dV(b^*)}{dx}, \frac{dV(b^*)}{dy})$, and then find the optimum, b_1^* of this direction. Starting from b_1^*, the algorithm updates the search direction to find another optimum, b_2^*, again along the new direction. We repeat this procedure until the estimated bias between the true and estimated value remains under the threshold.

3.2 ***The Previous (Second to Fifth) Period Solution***

Once all the given cases are solved, we know the maximum of the all points in the grid - all the possible initial conditions, which will be used to solve the fifth period problem. Basically, the fifth period problem (14) will be solved by the same algorithm as period 6 - searching the optima in the bottom edge and left edge first, and then go to the internal search. We divide the problem into two parts (see (14)): part 1 actually is almost the same as 6th period problem; while part 2 is the complex one, requiring to apply new algorithms. In the process to find the maximum, we have to compute V_5 and its derivatives at any point in the domain, so the value of $V_6(A_6, K_6, I_6)$, $\frac{dV_6}{dA_6}$, and $\frac{dV_6}{dK_6}$ in part 2 have to be estimated. Since A_6 and K_6 are the function of X_5 , S_5 , r_5 , r_{k5} , and r_{k5} is a random variable, we cannot obtain $V_6(A_6, K_6, I_6)$ directly for any given X_5 and S_5. That is the reason we have to build the grid of a large number of points--- we need it to interpolate V_5 at any (A_6, K_6) we want. It is illustrated in figure 3. .

$$V_t(A_t, K_t, I_t) = U(C_t) + \underbrace{\frac{P_{death} \times W}{r_t} \int_{a1}^{a2} B(W_t) f(r_{kt}) dr_{kt}}_{part1}$$

$$+ \underbrace{(1 - P_{death}) \int_{a1}^{a2} V_6(A_6, K_6, I_6) dr_{k5}}_{part2} \qquad (14)$$

When estimating the value of $V_6(A_6^*, K_6^*, I_6)$, we proceed the interpolation from one dimension A_6 and extend to K_6. We use the quadric form in estimating V_6. Doing the second order of interpolation, the first step is to find the three points, (A_6^1, A_6^2, A_6^3), (K_6^1, K_6^2, K_6^3) , nearest to the point to be estimated, (A_6, K_6) in each dimension . Therefore, there are nine points of combinations in total.

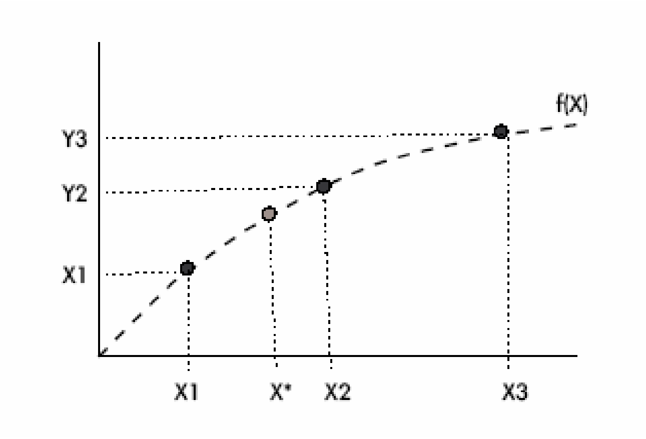

Figure 4

To interpolate on one dimension, A_6 , we fix each of the 3 points in another dimension, K_6 . That is, we first compute $\hat{V}_6(A_6^*, K_6^j)$ (for j=1 ... 3) by $V_6(A_6^1, K_6^j)$, $V_6(A_6^2, K_6^j)$, and $V_6(A_6^3, K_6^j)$. As Figure 4 shows, $\hat{V}_6(A_6^*, Kj)$ will be estimated by the functional form $f(x) = ax^2 + bx + c$. The parameters of $f(x)$, *a, b, and c,* can be estimated by the points $(x, y) =$ $(A_6^1, V_6(A_6^1, K_j))$, $(A_6^2, V_6(A_6^2, K_j))$, $(A_6^3, V_6(A_6^3, K_j))$. Once $\hat{V}_6(A_6^*, Kj)$ has been computed, we get other three pairs of data, $(K_6^1, V_6(A_6^*, K_6^1))$, $(K_6^2, V_6(A_6^*, K_6^2))$, $(K_6^3, V_6(A_6^*, K_6^3))$ and then proceed to the next dimension. Using the same procedure, both $V_6(A_6^*, K_6^*, I_6)$ and its derivatives can be estimated.

Given the estimated value of the sixth period, $V_6(A_6^*, K_6^*, I_6)$, we apply the same algorithm to solve fifth period maximization problem. And the same algorithm would be applied backward to solve forth to the first period problem. In each period, we also set up a grid over the possible rang of the state variables and solve it. These solutions would be used for interpolation in the previous period problem. As a result, the dynamic problem would be solved.

4. Simulation Results

We conduct two sets of simulations with different parameters to discuss the following two issues: (1) how does the wealth affect asset allocation? , (2) how does the risk aversion of the bequest function affect asset allocation?, and (3) how does age affect asset allocation? Table 1 presents our simulations parameters estimated from the Asset and Health Dynamics for the Oldest Old (AHEAD) data.

In order to understand the validity of our dynamic model, we undertake a small scale simulation in a four-period setting – with Period 4 for certain death. As a result, in this reduced-scale simulation, Period 3 is equivalent to the Sixth period (Period 2 is equivalent to the fifth/fourth/third/second period) in the full scale 7-period model.

In the beginning period, the wealth holding is set up less than 120. After three periods, the maximum investment returns cannot exceed 5,000.[2] Based on this setting, we set up 40,000 grids in the second period and 250,000 grids in the final period. The number of grid we set up is based on the range of attainable return. Therefore, the range in the final period is wider than the previous period, so is the number of grid.

[2] The upper bound of next period is computed by ($(A_t + K_t + I_t) \times \max(r_t, r_{kt})$). We compute the upper bound of each period asset based on this equation and parameters we set up.

Table 1
Simulation Model Parameters
for Baseline Households (with Risk Averse Utility Functions)

Parameter	Description	Value
Parameters of Initial Endowment		
N_t	Number of points in the grid in each dimension	$N_3 = 500$, $N_2 = 200$ $N_1 = 6$
N_t^2	Total number of case in each period	$N_3^2 = 250000$, $N_2^2 = 40000$ $N_1 = 6$
ε	Precision level	1e-6
$(\underline{A}_3, \overline{A}_3)$	Bound of cash in third period	(0,3900)
$(\underline{K}_3, \overline{K}_3)$	Bound of stock in third period	(0,3900)
$(\underline{A}_2, \overline{A}_2)$	Bound of cash in second period	(0,450)
$(\underline{A}_2, \overline{A}_2)$	Bound of stock in second period	(0,450)
Parameters of Stock Returns		
μ		0.040336679
σ^2		0.19885094
A	Upper bound of integral	1e-5
B	Lower bound of integral	3.8
P_t	Truncate probability	0.9999
Parameters of Utility Maximization		
I_t	Income in each period	20.00
ω	Weight of bequest function	5.00
r_t	Interest rate	1.04
P_{death}	Mortality Rate in each period	0.20

Table 2A
Simulation Results
Initial Endowment Position and Current Allocation
for Baseline Households with Risk Averse Bequest Functions

Case No	Initial Cash Holding	Initial Stock Holding	Stock Investment	Cash Holding	Consumption
1	5	5	4.14042	11.730643159483	14.12893
2	15	15	0.43294	29.215933630411	20.35113
3	25	25	0.00000	45.132769342986	24.86723
4	35	35	0.06137	58.782600983786	31.15603
5	45	45	0.19982	73.494154055210	36.30602
6	55	55	0.00000	88.681715742633	41.31828

Table 2B
Simulation Results
Initial Endowment Position and Current Allocation
for Baseline Households with Risk Neutral Bequest Functions

Case No.	Initial Cash Holding	Initial Stock Holding	Stock Investment	Cash Holding	Consumption
1	5	5	28.77741	0.000000000000	1.22259
2	15	15	48.77717	0.000000000000	1.22283
3	25	25	68.77707	0.000000000000	1.22293
4	35	35	88.77702	0.000000000000	1.22298
5	45	45	108.77701	0.000000000000	1.22299
6	55	55	128.76535	0.012236648807	1.22241

Table 3A
Simulation Results
Asset Utilization (consumption/saving portion)
and Asset Management (stock/cash holding)
for Baseline Households with Risk Averse Bequest FunctionsE

Consumption	Stock Investment	Cash Holding	Saving (Stock + Cash)
0.47096	0.13801	0.39102	0.52904
0.40702	0.00866	0.58432	0.59298
0.35525	0.00000	0.64475	0.64475
0.34618	0.00068	0.65314	0.65382
0.33005	0.00182	0.66813	0.66995
0.31783	0.00000	0.68217	0.68217

Table 3B
Simulation Results
Asset Utilization (consumption/saving portion)
and Asset Management (stock/cash holding)
for Baseline Households with Risk Neutral Bequest Functions

Consumption	Stock Investment	Cash Holding	Saving (Stock + Cash)
0.04075	0.95925	0.00000	0.95925
0.02446	0.97554	0.00000	0.97554
0.01747	0.98253	0.00000	0.98253
0.01359	0.98641	0.00000	0.98641
0.01112	0.98888	0.00000	0.98888
0.00940	0.99050	0.00009	0.99060

Tables 2A and 2B present simulation results on initial endowment and asset allocation/management positions for risk averse (2A) and risk neutral (2B) households, respectively. Tables 3A and 3B present simulation results on consumption/saving ratio and stock/cash holding portion for risk averse (3A) and risk neutral (3B) households, respectively. We believe that the bequest function would be less risk averse than the utility function, since the bequest is left to offspring, who has more room to take risk. We conduct the set in which the Arrow–Pratt risk measure, ρ, of bequest function equals 2, the same as the utility function as our baseline (2A/3A); while in another set (2B/3B) , the bequest function is risk neutral. The simulation results are shown in Tables 2A /3A, and 2B/3B, respectively.

As the initial stock and cash holdings increase, both consumption and saving of the agent increase. More precisely, we show the consumption and asset allocation proportion of the whole wealth in Table 2A and Table 2B. As wealth increases, the proportion of saving increases as well. This finding is consistent with the results demonstrated by Dynan, Skinner, and Zeldes (2004) that the rich save more proportionally.

Let's further consider the effect from risk aversion of bequest function. In the same wealth level, there is a prevailing effect that agents shift their asset holding from cash to stock. Based on the parameters we set up, we know the stock yield higher return, but with risk. Under the influence of bequest motives with risk neutrality, risky assets become more attractive.

We will still need to verify this notion in the empirical data in the future. However the current simulation results shed us some insight on this issue.

By examining the raw data in each period for both sets, we can investigate another issue: given the same wealth endowment, how asset allocations differ across multiple periods. We find that as the agent approaches to end of life, his total saving increases, yet his asset portfolio tends to move from stock to cash. There could be many driven forces with intersection behind this observation. We offer a potential explanation for this: in the second period, there is still some room for agent to take risk, so that they invest more in risky assets, compared with the end period. While in the end period, in comparison with the previous periods, the utility from bequest becomes more important. Based on these results, the increasing saving rate reflects that bequest rather than self finance is the driven force behind saving for the elderly.

In conclusion, we have discussed many interesting findings from this simplified simulation. We would like in our subsequent work to get more insights from full scale simulations for the entire six periods, expanding from current reduced scale of three periods. In doing so, we may require the incorporation of health dynamics (mortality /morbidity rates) into expected utility maximization across multiple periods.

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