A Novel Iterative Approach for Doppler Spread Estimation in LOS Environment

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Abstract—**In this paper, a novel iterative algorithm for maximum Doppler frequency estimation is proposed. In line-of-sight (LOS) environments, it is known that the performance of the conventional maximum Doppler frequency estimator obtained by the Rayleigh fading assumption will degrade severely. Thus we propose an estimator that estimates the Rician K factor first, then iteratively estimate the Doppler spread and the angle of arrival (AOA) of the LOS component through the autocorrelation function. In addition, the Cramér-Rao bounds (CRB) for the K factor and maximum Doppler frequency estimations are also derived for comparison. Simulation results indicate that the proposed estimator is not only superior to the conventional estimator but also robust to the channel parameters variation.**

I. Introduction

In mobile communication systems, signals usually experience fading and Doppler spread due to multipath effect. Doppler spread would distort the transmitted signal and make the channel estimation or synchronization more difficult. The maximum Doppler frequency is an important parameter of the channel. On one hand, it is related to the mobile's speed which is an important factor in adaptive handoff techniques [1]; on the other hand, the maximum Doppler frequency is also related to the required bandwidth when designing an adaptive channel estimation filter [2]. Once the Doppler spread is estimated, the adaptation rate in adaptive transmission techniques, such as AMC (Adaptive Modulation and Coding) [3], can be optimized. All of above illustrate the necessity of Doppler spread estimation.

Some researches on maximum Doppler frequency estimation methods based on the statistics of fading signals have been developed. In noncoherent communications, level-crossing rate (LCR) [4] or autocovariance of the envelope [1] can be used to estimate the maximum Doppler frequency. When the signal is detected coherently, the channel estimates can be acquired at the receiver. [5] and [6] have proposed the estimators that utilize autocorrelation or phase difference of these channel estimates as the sufficient statistics for estimation. All of them assume the signal experiences Rayleigh fading. However, when the mobile is in an environment with a line-of-sight (LOS) component, where the channel is Rician distributed instead, the statistic parameters above—like LCR, autocorrelation etc.,—will depend on the Rician K factor and the direction of the LOS component. Hence the performance will be seriously degraded. In this paper, we propose an estimator that iteratively estimates the maximum Doppler frequency in the LOS environment. Before the estimation, the K factor should be estimated first. Thus we also investigate the K factor estimators that can be utilized in our proposed algorithm. We will show that our proposed estimator provides a tremendous improvement over the conventional autocorrelation-based estimator [5] that is derived under the assumption of Rayleigh fading.

The rest of this paper is organized as follows. The Rician fading signal model is described in section II. In section III, the K factor estimators are investigated; and a novel iterative Doppler spread estimator will also be proposed. We will derive the Cramér-Rao bounds (CRB) of the channel parameters' estimates under Rician fading channels in section IV for comparison. In section V, the simulation results are presented and compared, followed by a conclusion given in section VI.

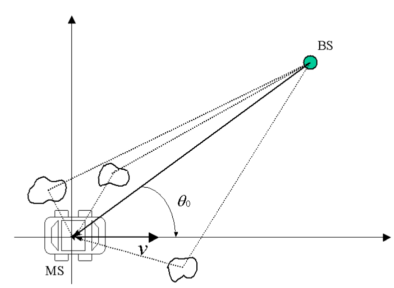

Fig. 1 Illustration of the environment with a line-of-sight (LOS) component between mobile and base station.

II. Signal Modeling

In the scattering environment with a LOS component, as shown in Fig. 1, the received signal composed of a deterministic function and a narrowband Gaussian process, can be expressed as

$$x(t)=\sqrt{\frac{K\Omega}{K+1}}e^{j(2\pi f_m\cos(\theta_0)t+\phi_0)}+\sqrt{\frac{\Omega}{K+1}}h(t), \tag{1}$$

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where K is the Rician factor defined as the ratio of the power of the LOS component to that of the diffused component. As the mobile traveling with a velocity v, the induced maximum Doppler frequency is $f_m = v \cdot f_c / c$, here f_c is the carrier frequency, c is the speed of light. θ_0 denotes the angle between the LOS and the moving direction (or the angle of arrival (AOA) in this paper), and ϕ_0 is the random phase of the LOS component. $h(t)$ is the narrowband signal composed of scattering components. When the number of scatterers is large, the central limit theorem can be invoked and $h(t)$ is treated as a Gaussian random process with unit variance [4]. The total power of the received signal is $\Omega = E[|x(t)|^2]$.

Consider the normalized autocorrelation of the received signal

$$\rho_x(\tau) = E[x(t+\tau)x^*(t)] \Big/ E\left[|x(t)|^2\right], \tag{2}$$

it can be represented as the combination of $\rho_{LOS}(\tau)$ and $\rho_h(\tau)$ as

$$\rho_x(\tau) = \frac{K\rho_{LOS}(\tau) + \rho_h(\tau)}{K+1}, \tag{3}$$

where $\rho_{LOS}(\tau)$ and $\rho_h(\tau)$ are the normalized autocorrelation of the LOS component and the diffuse component respectively. When we apply the well-known Clark-Jakes model to the diffuse component, (3) can be rewritten as

$$\rho_x(\tau) = \frac{K\exp(j2\pi f_m \cos\theta_0 \tau) + J_0(2\pi f_m \tau)}{K+1}. \tag{4}$$

where $J_0(x)$ is the 0^{th} order Bessel function of the first kind. From (4), it can be shown that in Rician fading channels, the autocorrelation of the fading signal can be described by the following three parameters: K, the Rician factor; f_m, the maximum Doppler frequency, and θ_0, the AOA of the LOS component. Our goal is to estimate the maximum Doppler frequency, f_m. However, we have to estimate K and θ_0 first, then invert (4) to estimate the maximum Doppler frequency. In the following section, we propose an algorithm which estimates the Rician factor first, and use it to iteratively estimate the maximum Doppler frequency and the AOA of the LOS component.

III. The Proposed Estimation Algorithm

In our proposed estimator, the autocorrelation property in (4) is utilized. We first estimate the Rician K factor and f_0—the Doppler frequency of the LOS part—then guess an initial maximum Doppler frequency. With these two values, the AOA of the LOS component can be estimated by the relationship $f_0 = f_m \cos\theta_0$. Next, the estimated AOA is substituted in (4), together with the estimated K factor, the maximum Doppler frequency can then be estimated by inverting the autocorrelation function. An iterative estimation procedure will be illustrated step-by-step in subsection B.

A. The Rician K Factor Estimator

The K factor estimated by the moment method has been derived in [7], which uses the fact that the ratio between the m^{th} power of the n^{th} moment and the n^{th} power of the m^{th} moment is a function of K only; i.e.

$$f_{n,m}(K) = \frac{\mu_n^m}{\mu_m^n} \tag{5}$$

where μ_n and μ_m are the n^{th} and m^{th} moment of $|x(t)|$ respectively. The Rician K factor can then be estimated by inverting the function of (5). This expression is useful in noncoherent communication system, where the channel phase is not available. We can use the statistics of the received signal's envelope to make an inference on the K factor.

Tepelenliğlu and Abdi [8] proposed an I/Q-based estimator in 2002. Assume the channel estimates are perfect and made per T_s second. Denote the sequence of channel estimates as $x[n] = x(nT_s)$. Let

$$\hat{\omega}_0 = \arg\max_{\omega} \left| \frac{1}{N}\sum_{n=0}^{N-1} x[n] e^{-j\omega n} \right|. \tag{6}$$

Define

$$X_1 = \frac{1}{N}\sum_{n=0}^{N-1} x[n] e^{-j\hat{\omega}_0 n}, \text{ and } X_2 = \frac{1}{N}\sum_{n=0}^{N-1} |x[n]|^2; \tag{7}$$

then the Rician K factor is estimated by

$$\hat{K}_{IQ} = \frac{|X_1|^2}{X_2 - |X_1|^2}. \tag{8}$$

In this estimator, I/Q components of channel responses are both used. The Doppler frequency of the LOS component is found by picking the maximum value in the Doppler spectrum. The K factor is estimated by the ratio of the power of the LOS component ($|X_1|^2$) to the scattered component ($X_2 - |X_1|^2$).

A comparison between the envelope-based estimator and I/Q component-based estimator has been discussed in [9]. It has been shown that the Cramér-Rao bound (CRB) of any envelope-based estimator is larger than that of any I/Q component-based estimator. Because estimation theory tells us that using more information to make an inference would yield better estimation. In our proposed approach, we have to obtain the Doppler frequency of the LOS component before the K factor is estimated. Therefore, we prefer the I/Q component-based K factor estimator in the following proposed algorithm.

B. The Proposed Iterative Estimation Procedure

In this subsection, we give a step-by-step procedure of the proposed algorithm.

Step1. Estimate the Doppler frequency of the LOS component, $\hat{f}_0 = \hat{\omega}_0 / 2\pi$ and the K factor, $\hat{K}$, by the I/Q-based estimator described above.

Step2. Guess a maximum Doppler frequency, say $\hat{f}_m^{(0)}$. The initial value can be the value obtained by the conventional estimation method, which estimates f_m

by inverting the Bessel function. i.e.

$$\hat{f}_m^{(0)} = \frac{1}{2\pi m T_s} J_0^{-1}(\hat{\rho}), \tag{9}$$

where T_s is the time duration between two successive channel samples, and $\hat{\rho}$ is the measured correlation coefficient defined as

$$\hat{\rho} = \frac{\mathrm{Re}\left\{\frac{1}{N}\sum_{n=0}^{N-1} x[n]x^*[n+m]\right\}}{\mathrm{Re}\left\{\frac{1}{N}\sum_{n=0}^{N-1} |x[n]|^2\right\}}. \tag{10}$$

Step3. Iteratively estimate the maximum Doppler frequency and the AOA of the LOS component. Let M be the number of iterations. For $i = 1,2,\ldots,M$, let

$$\hat{\theta}_0^{(i)} = \cos^{-1}\left(\hat{f}_0 \big/ \hat{f}_m^{(i-1)}\right). \tag{11}$$

Construct the function

$$F^{(i)}(x) = \frac{\hat{K}\cos\left(x\cos\hat{\theta}_0^{(i)}\right) + J_0(x)}{\hat{K}+1}, \tag{12}$$

then

$$\hat{f}_m^{(i)} = \frac{1}{2\pi m T_s} F^{(i)^{-1}}(\hat{\rho}). \tag{13}$$

In each iteration, the previous estimated maximum Doppler frequency is used to estimate the $\theta_0^{(i)}$ by equation (11). $F^{(i)}(x)$ is formed by taking the real part of eq.(4) since we use the autocorrelation coefficient of quadrature component as the statistic for estimation. The variable x and the maximum Doppler frequency f_m can be related as $x/2\pi = f_m\tau$. By inverting it, the maximum Doppler frequency estimate of the i[th] iteration can be acquired. Although it is not easy to find a closed form of the inverse function $F^{(i)^{-1}}(x)$, it can be computed by look-up-table. The table is updated once a new $\hat{\theta}_0^{(i)}$ is estimated.

IV. CRB for Channel Parameters

In statistical inference, Cramér-Rao bound (CRB) is known as the lower bound of the variance of any unbiased estimator. When multiple parameters are unknown and are jointly estimated, the CRBs of these parameters are equal to the diagonal terms of the inverse of the Fisher information matrix (FIM). Thus all we have to do is to find the FIM $\mathbf{J}$. Consider the channel estimates corrupted by additive white Gaussian noise $w(t)$ with zero mean and unit variance. Then (1) can be rewritten as

$$x(t) = \sqrt{\frac{K\Omega}{K+1}} e^{j(2\pi f_m \cos\theta_0 t + \phi_0)} + \sqrt{\frac{\Omega}{K+1}} h(t) + \sqrt{\frac{\Omega}{\gamma}} w(t), \tag{14}$$

where γ is the signal-to-noise ratio (SNR). Define $x[n] = x(nT_s)$, the signal sequence is

$$x[n] = \sqrt{\frac{K\Omega}{K+1}} e^{j(2\pi f_m \cos\theta_0 T_s n + \phi_0)} + \sqrt{\frac{\Omega}{K+1}} h[n] + \sqrt{\frac{\Omega}{\gamma}} w[n], \tag{15}$$

Let N be the number of data points collected for estimation. Define the parameter vector $\varsigma = [K \;\; f_m \;\; \theta_0 \;\; \phi_0 \;\; \Omega \;\; \gamma]^T$. Assume all of the parameters are unknown to the receiver, and we are interested in the CRB of the first two parameters, K, and f_m. With these parameters, a 6×6 FIM can be formed.

Since $h(t)$ can be viewed as a zero-mean Gaussian process in a scatterer-rich environment. Therefore, (15) is a complex Gaussian random vector with mean vector

$$\mathbf{m}(\varsigma) = \sqrt{\frac{K\Omega}{K+1}} e^{j\phi_0}\left[e^{j2\pi f_m\cos\theta_0 T_s\cdot 0}, \ldots, e^{j2\pi f_m \cos\theta_0 T_s\cdot(N-1)}\right]^T \tag{16}$$

and covariance matrix

$$\mathbf{\Gamma}(\varsigma) = \frac{\Omega}{K+1}\mathbf{R}(f_m) + \frac{\Omega}{\gamma}\mathbf{I}_\mathbf{N}. \tag{17}$$

Apply Clark-Jakes model, the element of matrix $\mathbf{R}(f_m)$ is

$$[\mathbf{R}(f_m)]_{k,l} = J_0(2\pi f_m T_s(k-l)). \tag{18}$$

Since $\mathbf{x}[n]$ is a complex Gaussian random vector, the element of FIM can be computed from [10]

$$\begin{aligned}[\mathbf{J}(\varsigma)]_{k,l} = 2\,\mathrm{Re}&\left[\left(\frac{\partial \mathbf{m}(\varsigma)}{\partial \varsigma_k}\right)^H \mathbf{\Gamma}^{-1}(\varsigma)\left(\frac{\partial \mathbf{m}(\varsigma)}{\partial \varsigma_l}\right)\right]. \\ &+ tr\left[\mathbf{\Gamma}^{-1}(\varsigma)\frac{\partial \mathbf{\Gamma}(\varsigma)}{\partial \varsigma_k}\mathbf{\Gamma}^{-1}(\varsigma)\frac{\partial \mathbf{\Gamma}(\varsigma)}{\partial \varsigma_l}\right]\end{aligned} \tag{19}$$

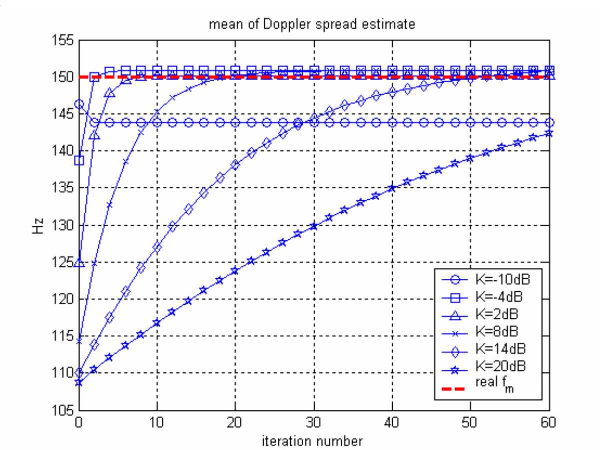
Thus the CRB of K and f_m are the first and second diagonal element of inverse FIM respectively.

In the next section, the estimation error of our proposed estimator will be compared to the square root of CRB. Here we define the estimation error as the square root of the sum of variance and the square of the bias, denoted as RMSE (Root Mean Square Error).

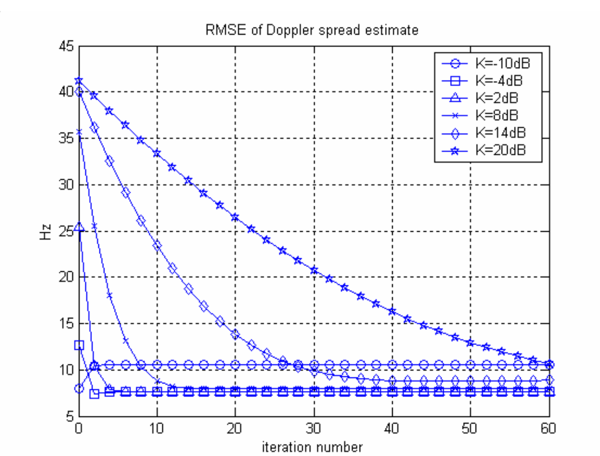
V. Simulations

In this section, we will compare our proposed Doppler spread estimator with the conventional one, which estimates the maximum Doppler frequency by inverting the Bessel function described in the previous sections. All of the following results are simulated 500 times to get the mean value or the corresponding RMSE. The scattering parts of the fading signal are generated by Jakes-like generator [11]. In most of these experiments, the maximum Doppler frequency and the AOA of the LOS component are set as f_m=150Hz, $\theta_0 = \pi/3$ respectively and the channel sampling rate is f_s=1.5kHz. A total of 256 data points are collected in each experiment, which approximately equals to a time period of 170.67 ms. We can reasonably assume that the mobile remains the same velocity and the scattering points around it do not change positions during this period. In addition, the correlation lag and the SNR are set as $\tau = 3T_s$ and γ=40 dB respectively.

The convergence rates of our iterative algorithm under different K factors are shown in Fig. 2 (a) and (b), where we estimate the K factor by the I/Q-based estimator. We observe that the number of iteration needed for convergence increases as K becomes larger. This is because we use the maximum Doppler frequency estimated by the conventional method as the initial estimate, or the 0th iteration shown in the figure. In the conventional method, Rayleigh fading channel is assumed. However, as K increases, the channel deviates more from the Rayleigh assumption. Thus our algorithm needs more iterations to obtain an unbiased estimate. There is another phenomenon worth noting. When K is smaller than 0 dB, the estimates are biased no matter how many iterations are made, and the RMSE is slightly larger than those with moderate K factor. For small K, the Doppler frequency of the LOS component estimate is never accurate, because the peak corresponding to LOS component in spectral domain is difficult to detect accurately.



(a)



(b)

Fig. 2 Mean and RMSE of Doppler spread estimates versus iteration number under different K factors. (a) Mean of Doppler spread estimates; (b) RMSE of Doppler spread estimates.

To demonstrate the superiority of the novel estimator over the conventional one, we compare the two approaches under different environments. In Fig. 3, we vary the K factor and compare the performance of our proposed estimator and the conventional one. The two K factor estimators described in the previous sections are both utilized in our algorithm. The estimation result when the K factor and the AOA of the LOS are perfectly estimated is also indicated in this figure for comparison, which can be viewed as the lower bound of our estimator. All of the RMSE values for our approach are computed after 60 iterations to guarantee the convergence. The CRB is also plotted for comparison. The results show that if K and θ_0 can be estimated perfectly, our estimator can approach CRB as K factor gets larger. For low K factor, the one utilizing the I/Q-based K factor estimator performs better than the other one using the moment-based K factor estimator due to smaller RMSE of the K factor estimation. The conventional approach has smaller RMSE than that obtained by this new algorithm because the channel is more Rayleigh-like. When K factor is increased, both estimators perform similarly and have tremendous improvement over the traditional one. In later simulations, we will set K =3dB and use the I/Q-based method to estimate K factor in our proposed algorithm because it provides smaller RMSE.

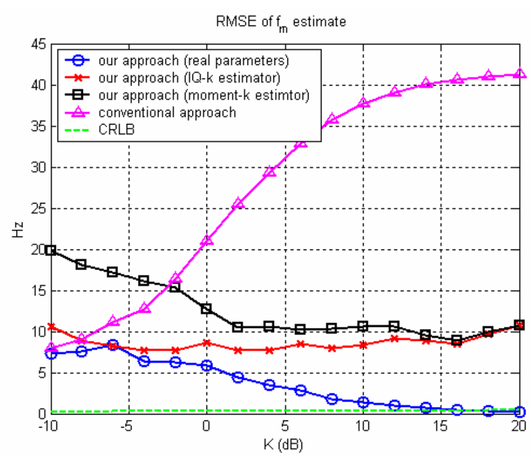


Fig. 3 RMSE of Doppler spread estimates versus K factor.

In Fig. 4, we change the angle of arrival of the LOS component to examine the performance variation. Results show that the RMSE of the Doppler spread estimated by our approach is not sensitive to θ_0. On the other hand, the RMSE of the conventional method forms a V-shape. The error has a minimum when θ_0 is around 0.8 radian. Under high SNR, the minimum point occurs when the AOA of the LOS component happens to satisfy the equation

$$J_0(2\pi f_m \tau) \approx \frac{K\cos(2\pi f_m \cos\theta_0 \tau) + J_0(2\pi f_m \tau)}{K+1}. \quad (20)$$

That is, under Rician fading channels, when the channel parameters happen to have the autocorrelation value that equals to the Bessel function, the conventional Doppler spread estimator will have small estimation error. Substituting equation (20) by K =3dB, f_m =150Hz, and $\tau = 3T_s$; the AOA can be solved as $\theta_0 \approx 0.827$ radian. This agrees with the numerical result in Fig. 4.

The performance of the proposed estimator under different maximum Doppler frequency is also investigated. In Fig. 5, we find that the proposed estimator is unbiased after convergence is achieved. On the contrary, the conventional estimator biases more from the true maximum Doppler frequency as f_m increases. The RMSE of the estimates obtained by the proposed algorithm increases slightly when f_m is increased. The RMSE increases at low f_m region because of the inaccurate estimation of the K factor due to the short data length.

VI. Conclusions

In this paper, we proposed a novel Doppler spread estimator which utilizes the K factor estimates to iteratively estimate the maximum Doppler frequency in the environment with a LOS component. We also derived the Cramér-Rao bounds of the channel parameters for Rician fading signal corrupted by additive noise. It has been shown by simulations that the proposed estimator can outperform the conventional autocorrelation-based Doppler spread estimator in the sense of RMSE. With moderate SNR, the iterative approach is more robust to channel parameters variation. Even when the channel is reduced to Rayleigh fading, the proposed one can still have good performance. When the K factor is not too large, a small number of iterations is enough to make the RMSE converge.

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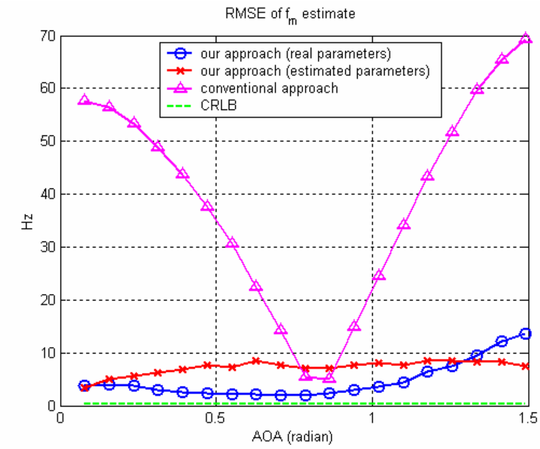
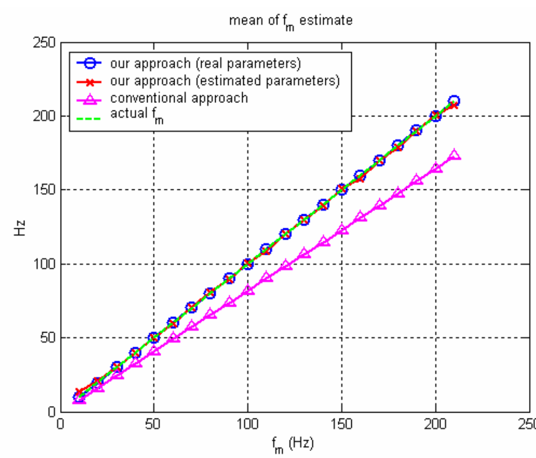
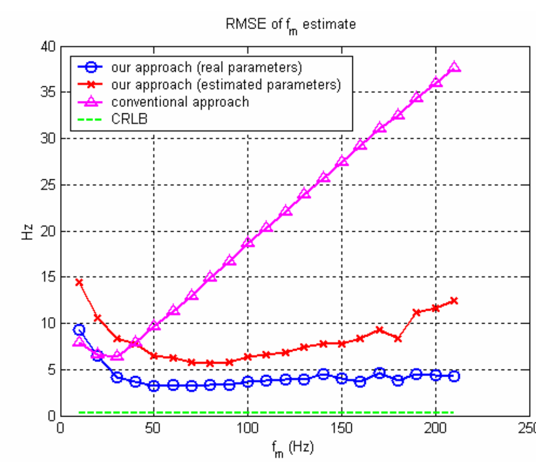


Fig. 4 RMSE of Doppler spread versus AOA of LOS component.



(a)



(b)

Fig. 5 Mean and RMSE of Doppler spread estimates versus maximum Doppler frequency. (a) Mean of Doppler spread estimates; (b) RMSE of Doppler spread estimates.