

Frequency-Domain Interpolation-Based Channel Estimation in Pilot-Aided OFDM Systems

Pei-Yun Tsai and Tzi-Dar Chiueh

Graduate Institute of Electronics Engineering
and Department of Electrical Engineering
National Taiwan University, Taipei, Taiwan, 10617.

Abstract—In this paper, we propose a novel frequency-domain interpolation algorithm for channel estimation in comb-type pilot-aided orthogonal frequency-division multiplexing (OFDM) systems. There exist two major types of pilot-aided OFDM channel estimation methods: time-domain and frequency-domain. We show that these two estimation methods have mathematical equivalence. The performance of these channel estimators mainly depends on how the channel impulse response (CIR) is reconstructed from frequency-domain channel response sample at the pilot sub-carriers. By closely examining time-domain CIR characteristics, we propose a new frequency-domain interpolation-based algorithm that can overcome the limitation of conventional frequency-domain algorithms. Meanwhile, this new algorithm has the advantages of less latency and computation complexity when compared to time-domain approaches. Simulation results show that the proposed algorithm outperforms frequency-domain interpolation-based and time-domain algorithms in most cases.

I. INTRODUCTION

OFDM has attracted considerable attention since last decade, mainly because its substantial advantages in high-rate transmission over frequency-selective fading channels. By dividing a wide-band frequency-selective-fading channel into a large number of narrow-band flat-fading sub-channels, OFDM systems can easily compensate adverse channel effects by a simple one-tap frequency-domain equalizer. Moreover, with a cyclic prefix, inter-symbol interference can be mitigated and the mutually-overlapped spectra improve the spectrum efficiency. These features facilitate the adoption of OFDM in communications standards, such as digital audio broadcasting (DAB), digital video broadcasting-terrestrial (DVB-T), and IEEE 802.11a/g wireless LAN.

In OFDM technology, high-rate transmission is achieved by using higher-order constellations. Robust coherent detection of such OFDM signals calls for accurate channel estimation. To this end, pilot sub-carriers are often interlaced with data sub-carriers. The comb-type pilot insertion has been shown to be suitable for channel estimation in fast fading channels [1]. Various comb-type pilot-aided channel estimation schemes for OFDM systems have been proposed. Among them, there are two major types: time-domain windowing and frequency-domain interpolation.

In the time-domain windowing algorithms, a time-domain CIR is obtained by first inverse Fourier transforming the

frequency-domain channel response at the pilot sub-carriers. In this case, the number of pilot sub-carriers M must be greater than the normalized maximum excess delay, i.e., $M > \tau_{max}/T_s$ [2], where τ_{max} and T_s represent the maximum excess delay and the sample time. Thereafter, different windowing techniques are applied to the contaminated time-domain CIR in order to reduce noise and aliasing effect. In [3], the CIR is directly cut off below a threshold. Similarly, Minn [4] and Fukuhara [5] keep only the more significant samples in the CIR. Garacia [6] gathers M samples in the CIR. Yang [2] utilizes all the CIR samples and applies *minimum mean squared error* (MMSE) weighting.

In the frequency-domain interpolation algorithms, linear interpolation of the responses at the pilot sub-carriers has been proposed to estimate the frequency-domain channel response for all sub-carriers [7]. In [8], a second-order interpolation technique has been shown to outperform linear interpolation. Coleri [1] uses a low pass filter and spline cubic interpolation. Usually, in frequency-domain interpolation techniques, pilot sub-carriers over-sample the channel frequency response in the frequency domain by at least a factor of two, i.e., $M \geq 2\tau_{max}/T_s$ [9].

We can see that these two types of channel estimation algorithms developed along different directions. In the time-domain windowing algorithms, researchers have tried to increase estimation accuracy by weighting the time-domain CIR samples. In the frequency-domain interpolation algorithms, higher-order polynomials are adopted to approximate the ideal sinc interpolation regardless of the CIR characteristics. Nevertheless, these two types of algorithms have their own limitations. In the time-domain windowing algorithms, although *discrete Fourier transform* (DFT) can be implemented by *fast Fourier transform* (FFT) algorithms, buffers are needed for temporary data storage. Moreover, FFT and IFFT operations amount to overhead in latency as well as complexity, which is non-existent in the frequency-domain interpolation algorithms. On the other hand, in order to get acceptable performance, the conventional frequency-domain interpolation algorithms need more pilot sub-carriers, which reduce transmission efficiency.

In this paper, we first show correspondence between these two types of channel estimation algorithms. Then, we propose

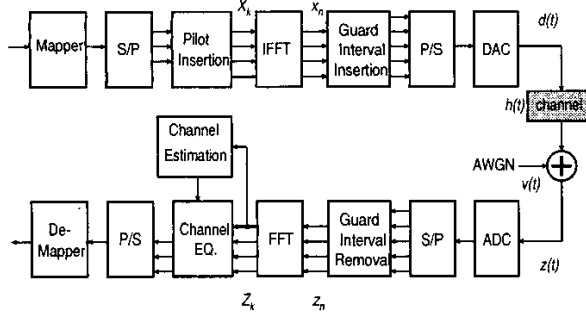


Fig. 1. Baseband block diagram of a typical pilot-aided OFDM system.

a novel frequency-domain interpolation-based channel estimation algorithm that can efficiently extract pilot sub-carrier information as the time-domain methods. Furthermore, the proposed technique strikes a balance between noise suppression, aliasing removal, and CIR preservation by effectively shifting the time window and using a new interpolation function.

The paper is organized as follows. In Section II, the description of the comb-type pilot-aided OFDM system is given. Section III illustrates the correspondence between time-domain windowing and frequency-domain interpolation. The proposed frequency-domain interpolation function is presented in Section IV. Simulation results and comparisons are given in Section V. Section VI concludes this paper.

II. SYSTEM DESCRIPTION

Fig. 1 shows a typical block diagram of an OFDM system based on pilot-aided channel estimation. The IFFT block transforms frequency-domain data, X_k , on the k -th sub-carrier into time-domain samples x_n as

$$x_n = \frac{1}{N} \sum_{k=-N/2+1}^{N/2} X_k \cdot e^{j2\pi nk/N}, n = -N_g, \dots, N-1, \quad (1)$$

where N is the number of total sub-carriers. In order to deal with *inter-symbol interference* (ISI), a cyclic prefix of N_g samples is inserted at the beginning of every symbol. The comb-type pilot allocation is shown in Fig. 2. Assume that M pilot sub-carriers are uniformly distributed in a total of N sub-carriers. The pilot sub-carrier spacing is $D = N/M$, where D is an integer. Note that in addition to N_u transmission sub-carriers, there are reserved sub-carriers used as guard bands on both ends of the spectrum. Among the N_u transmission sub-carriers, there are M_p pilot sub-carriers for channel estimation.

At the receiver, the baseband signal is first sampled to obtain z_n . With the cyclic prefix removed, z_n is sent to the

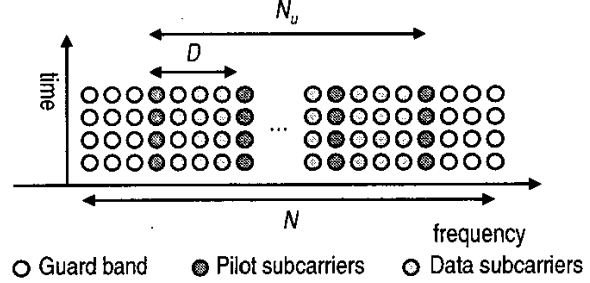


Fig. 2. Pilot sub-carriers allocation.

FFT block for transformation to the frequency domain:

$$Z_k = \sum_{n=0}^{N-1} z_n \cdot e^{-j2\pi nk/N}, k = -N/2+1, \dots, N/2. \quad (2)$$

Assume that the duration of the cyclic prefix is long enough so that there is no ISI and that the down-conversion is accurate enough so that there is no inter-carrier interference (ICI). Then the frequency-domain received baseband data Z_k is given by

$$Z_k = X_k H_k + V_k, \quad (3)$$

where H_k is the channel response on the k -th sub-carrier and V_k is the noise term. The frequency-domain channel response is given by

$$H_k = \sum_r h_r e^{-j2\pi \frac{\tau_r k}{T_s N}}, \quad (4)$$

where h_r and τ_r denote the gain and delay of the r -th path and the CIR has the form of $h(t) = \sum_r h_r(t) \cdot \delta(t - \tau_r(t))$.

From the received frequency-domain data on the pilot sub-carriers, the channel response on those sub-carriers can be computed as $\hat{H}_{mD} = Z_{mD}/X_{mD}$, $m = -M_p/2+1, \dots, M_p/2$. Henceforth, the channel response on all data sub-carriers can be estimated, and the received data are equalized by the channel estimation, $\hat{H}_k$

$$\hat{X}_k = \frac{Z_k}{\hat{H}_k}, k = -N_u/2+1, \dots, 0, \dots, N_u/2. \quad (5)$$

III. TIME-DOMAIN WINDOWING AND FREQUENCY-DOMAIN INTERPOLATION

In time-domain channel estimation algorithms, M -point inverse Fourier transform is applied to the M pilot sub-carrier channel responses to reconstruct the time-domain CIR, $\tilde{h}_n$:

$$\tilde{h}_n = \frac{1}{M} \sum_{m=-M_p/2+1}^{M_p/2} \tilde{H}_{mD} e^{j2\pi mn/M}, n = 0, \dots, M-1. \quad (6)$$

Due to the sub-sampling in the frequency domain, $\tilde{h}_n$ is a folded version of the original CIR with a period of M samples. We can usually avoid aliasing by setting $M > N_g$ since CIR energy mainly appears in the first N_g taps.

Usually, a time-domain window is applied on the periodic M -sample CIR. Let us denote this window by $w = [w_{-b} \ w_{-b+1} \ \dots \ w_{L-b-1}]^T$, where L is the window width and b controls its starting position. After windowing, the estimates of the frequency-domain channel response are obtained by Fourier transforming the weighted and zero-padded CIR as

$$\begin{aligned} \hat{H}_k &= \sum_{n=-b}^{L-b-1} \tilde{h}_{\langle n \rangle} w_n e^{-j2\pi nk/N} \\ &= \frac{1}{M} \sum_{m=-M_p/2+1}^{M_p/2} \tilde{H}_{mD} \sum_{n=-b}^{L-b-1} w_n e^{-j2\pi n(k-mD)/N}, \end{aligned} \quad (7)$$

where $\langle \cdot \rangle$ denotes modulo M . The equation above can be interpreted as interpolation in the frequency domain using $\tilde{H}_{mD}$ as base points and interpolation coefficients W_l where

$$W_l = \frac{1}{M} \sum_{n=-b}^{L-b-1} w_n e^{-j2\pi nl/N}. \quad (8)$$

Similarly, in a frequency-domain interpolation algorithm, a corresponding time-domain window can be derived. For a set of J -tap interpolation coefficients, W_k , the windowing vector is given by

$$w_n = \frac{1}{D} \sum_{k=-JD/2+1}^{JD/2} W_k e^{j2\pi nk/N}. \quad (9)$$

The fact that these two types of operations are closely related offers a possibility of mapping between these two types of algorithms. Therefore, one can design a channel estimation algorithm that has advantages from both the frequency-domain algorithms and the time-domain algorithms.

IV. NEW FREQUENCY-DOMAIN INTERPOLATION FOR CHANNEL ESTIMATION

The inverse Fourier transform of M pilot sub-carrier frequency-domain responses generates a periodic time-domain signal with a period of MT_s , as shown in Fig. 3. If the OFDM symbol boundary acquisition is accurate, the first pulse of the CIR will be at the origin and the remaining impulse response appears in the guard interval $[0, N_g T_s]$. However, energy leakage occurs in the uniformly T_s -spaced CIR due to the non- T_s -spaced channel delay τ_r [10]. Therefore there exist pre-cursor as well as post-cursor in the reconstructed CIR. A time-domain window is used to preserve the major portion of the CIR and at the same time reject the aliased components. It is clear from Fig. 3 that the window must be shifted to the right instead of centering at the origin as the conventional polynomial interpolators in Eq. (9). Note that shifting the window in the time domain is equivalent to rotating the phase of the interpolation coefficients in the frequency domain.

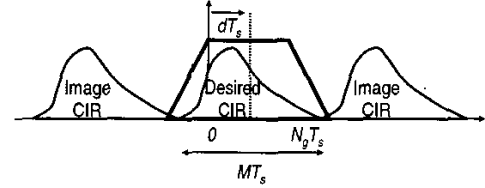


Fig. 3. Periodic CIR in the time domain.

Usually, the magnitude of the time-domain CIR decays gradually to the right. The time-domain CIR reconstructed by the samples of the frequency-domain channel response is corrupted by noise and aliasing effect. The weighting window must be flat over the duration where CIR is strong so that it is not distorted [11]. On the two edges of the window, the weighting should be smaller in order to suppress noise and aliasing effect. Moreover, smooth weighting in the time domain entails fast-decaying magnitude in the frequency-domain interpolation coefficients, and thus fewer of them are needed.

In light of the above considerations, we choose the raised-cosine function as the frequency-domain interpolation coefficients and set

$$W_{l,RC} = \frac{\sin(\pi Ml/N)}{\pi Ml/N} \cdot \frac{\cos(\pi \beta Ml/N)}{1 - 4\beta^2 (Ml/N)^2} \cdot e^{-j2\pi dl/N}, \quad (10)$$

where β is the roll-off factor; d is the time shift and it is decided by the worst-case channel delay spread. Note that in a short delay-spread channel, the estimation error is insensitive to d since the major portion of the CIR will be covered by the window with a wide range of d .

V. SIMULATION RESULTS AND COMPARISONS

In order to show the effectiveness of the proposed raised-cosine-based frequency-domain channel estimation algorithm, we conduct simulation on some practical scenarios. In the simulation, we use the typical urban channel model given in [12], which has 20 paths and a maximum excess delay of 24.28 samples at a sampling rate of 11.52 MHz. There are 1024 sub-carriers with 29 pilot sub-carriers evenly inserted for channel estimation. The guard interval has a length of 26 samples.

Fig. 4 demonstrates the improvement in channel estimation accuracy if we consider the time shift effect by incorporating a phase rotation term in several frequency-domain interpolation-based algorithms. Since the time-domain weighting window derived from the polynomial interpolators' coefficients has wide and non-flat mainlobe [13], aliasing is unavoidable. So the CIR will be distorted, yielding larger estimation error. We can also see the proposed raised-cosine interpolator is by far the best frequency-domain interpolation-based channel estimator in terms of estimation accuracy. In addition, note that for each algorithm, the error

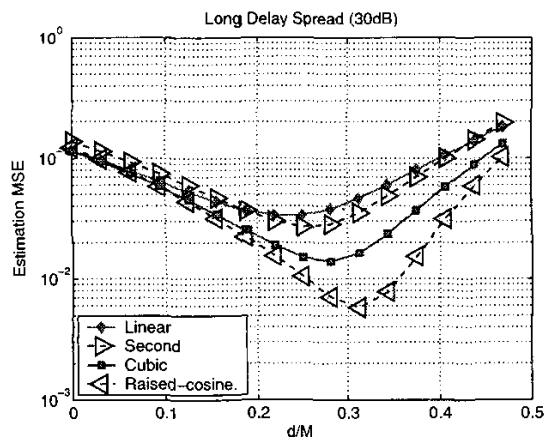


Fig. 4. Improvement in estimation accuracy if the time shift is considered in frequency-domain interpolation-based channel estimation.

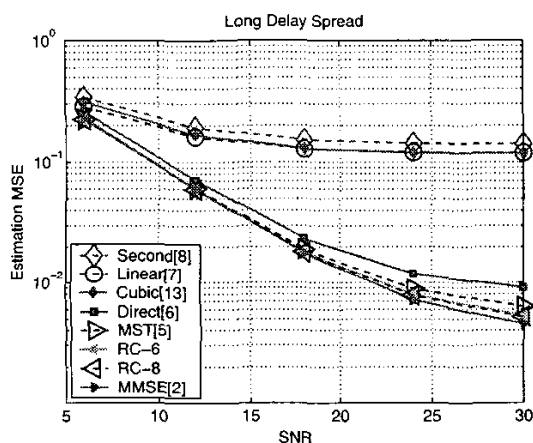


Fig. 5. Estimation error of different channel estimators in a channel with long delay spread.

decreases as the time shift increases, and after reaching a minimum, the error starts to rise again.

Fig. 5 illustrates the performance of several channel estimators under different levels of SNR. RC-6 and RC-8 denote the proposed frequency-domain interpolation channel estimation algorithm using 6-tap and 8-tap raised-cosine coefficients, respectively. We then apply such estimated channel response to a frequency-domain equalizer in a 16-QAM sub-carrier data receiver. The resulting bit error rates versus SNR are depicted in Fig. 6. From these two figures, we see that in the case of channels with long delay spread, the proposed algorithm is significantly better than all other frequency-domain algorithms and most of the time-domain algorithms.

Computational complexity and memory requirement are also important issues in the choice of channel estimators.

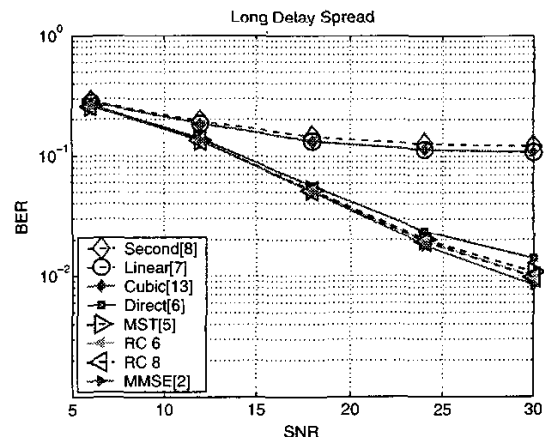


Fig. 6. Bit error rate performance using different channel estimators in a channel with long delay spread.

The time-domain windowing estimators require $M \log_4 M + N \log_4 N$ complex multiplications and $M + N - 2$ complex buffers for radix-4 FFT/IFFT and $2M$ real multiplications for windowing by w in the time domain. The entries in w are pre-computed and stored thus occupy M buffers. Due to the latency of $2N - 1$ samples caused by IFFT and FFT, $2N_u$ complex data must be also buffered. For the proposed frequency-domain J -tap interpolation channel estimator (shown in Fig. 7), we need $2N_u J + 4N_u$ real multiplications and $2J$ real data buffers and $(D - 1)J$ real buffers for the real coefficients. Since the latency of the frequency-domain interpolation is much reduced to $JD/2 - 1$, only $J(D - 1)/2$ complex data buffers are necessary.

As the FFT size increases, the overhead in the buffers and multiplications increases rapidly if a time-domain windowing channel estimator is implemented. The numbers of multiplications and of buffers needed by the time-domain channel estimators and the frequency-domain estimators for different FFT size are plotted in Fig. 8. Note that a 6-tap frequency-domain interpolation estimator is used for comparison. For the proposed estimator, the reduction in computational complexity is evident, especially when the number of sub-carriers is large.

VI. CONCLUSION

In this paper, we examined the time-domain and frequency-domain channel estimation algorithms in the comb-type pilot-aided OFDM systems. We showed that there exists correspondence between these two types of channel estimators. We then proposed to include a phase rotation term in the frequency-domain interpolation to account for the equivalent time shift for better CIR window location. In addition, the proposed raised-cosine interpolation coefficients not only have good attenuation on noise and aliasing but also preserve most CIR information and provide very good estimation accuracy. The

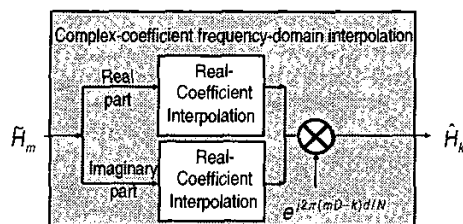


Fig. 7. Block diagram of the proposed frequency-domain interpolation channel estimator.

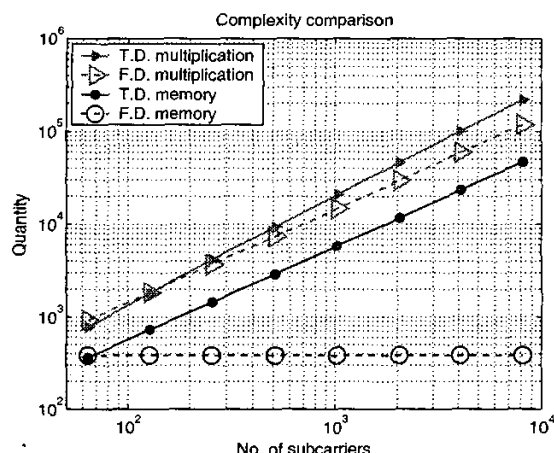


Fig. 8. Channel estimator complexity versus FFT size. T.D.: time-domain estimators; F.D.: frequency-domain estimators.

complexity of the proposed frequency-domain interpolation is shown to be less than all time-domain channel estimators in almost all FFT sizes. With its low hardware complexity and accurate estimation performance, the raised-cosine frequency-domain interpolation channel estimator will find many applications in OFDM communication systems.

REFERENCES

- [1] S. Coleri, M. Ergen, A. Puri and A. Bahai, "Channel estimation techniques based on pilot arrangement in OFDM systems," *IEEE transaction on Broadcasting*, vol. 48, no.3, pp. 223-229, Sep. 2002.
- [2] B. Yang, Z. Cao and K. B. Letaief, "Analysis of low-complexity windowed DFT-based MMSE channel estimator for OFDM systems," *IEEE transaction on Communications*, Vol. 49, No. 11, pp.1977-1987, Nov. 2001.
- [3] Y. Zhao and A. Huang, "A novel channel estimation method for OFDM mobile communication systems based on pilot signals and transform domain processing," in *Proc. of 1997 IEEE 47th Vehicular Technology Conference*, Vol. 3, May 1997, pp. 2089-2093.
- [4] H. Minn, V. K. Bhargava, "An investigation into time-domain approach for OFDM channel estimation," *IEEE Transactions on Broadcasting*, Vol. 46, Issue 4, pp. 240-248, Dec. 2000.
- [5] T. Fukuhara, H. Yuan, Y. Takeuchi and H. Kobayashi, "A novel channel estimation method for OFDM transmission technique under fast time-variant fading channel," in *Proc. of 2003 IEEE 57th Vehicular Technology Conference*, 2003.
- [6] M. J. Fernandez-Getino Garcia, J. M. Paez-Borrillo and S. Zazo, "DFT-based channel estimation in 2D-pilot-symbol-aided OFDM wireless systems," in *Proc. of 2001 IEEE 53th Vehicular Technology Conference*, Vol. 2, 2001, pp. 810-814.
- [7] J. Rinne and M. Renfors, "Pilot spacing in orthogonal frequency division multiplexing systems on practical channels," *IEEE Transaction on Consumer Electroics*, vol. 42, no. 4, pp. 959-962, Nov. 1996.
- [8] M. H. Hsieh and C. H. Wei, "Channel estimation for OFDM systems based on comb-type pilot arrangement in frequency selective fading channels," *IEEE Transaction on Consumer Electroics*, vol. 44, no. 1, pp. 217-225, Feb. 1998.
- [9] P. Hoeher, S. Kaiser and P. Robertsson, "Two dimensional pilot-symbol-aided channel estimation by Wiener filtering," in *IEEE International Conference on Acoustics, Speech, and Signal Processing*, vol. 3, 1997, pp. 1845-1848.
- [10] J. -J van de Beek, O. Edfors, M. Sandell, S. K. Wilson and P. O. Borjesson, "On channel estimation in OFDM systems," in *Proc. of 1995 IEEE 45th Vehicular Technology Conference*, Vol. 2, Jul. 1995, pp. 815-819.
- [11] H. Tang, K. Y. Lau and R. W. Brodersen, "Interpolation-based maximum likelihood channel estimation using OFDM pilot symbols," in *IEEE Global Telecommunications Conference*, vol. 2, 2002, pp. 1860-1864.
- [12] 3GPP Technique Specification Group (TSG) RAN WG4: Deployment Aspects, TR 25.943 v2.1.0 (2001-06).
- [13] L. Erup, F. M. Gardner and R. A. Harris, "Interpolation in Digital Modems-Part II: implementation and performance," in *IEEE Transaction on communications*, vol. 41, no. 6, pp. 998-1008, Jun. 1993.