行政院國家科學委員會專題研究計畫　成果報告

貝氏統計在結構系統模態與動態識別之應用(2/2)

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1. 中文摘要

本計畫之主旨在發展以貝氏統計為基礎之結構識別方法。首先，蒐集結構資料，包括幾何性質及起始的桿件性質分佈，並據以建立結構之有限元素模型。接著，量測結構之模態或動態反應，再根據結構反應，以貝氏統計法修正桿件性質分佈。此識別法可用以修正結構之數學模型，也可用於偵測結構之受損桿件，及評估結構之可靠度。

關鍵詞：非破壞檢測、貝氏統計、損傷評估、模態測試、動態測試

2. Abstract

The objective of this project is to develop a method to detect the damage of structures. Dynamic tests are performed on the structure to obtain either the modal data or dynamic response of the structure. Then, the Bayesian approach is adopted to construct the posterior distribution of element properties based on the modal data or dynamic response. Taylor's expansion is used in the formulation of conditional probability that is required in the Bayesian modification. The posterior means obtained by the Bayesian approach are used as an indicator of element damages. Numerical examples are presented to illustrate the proposed method.

Keywords：NDT, Bayesian statistics, damage detection, modal test, dynamic test.

3. Introduction

The system identification of structures plays an important role in many areas, for example, damage detection, material evaluation, dynamic control, and guidance of structures. System identification can be considered as an inverse problem. In a forward problem, the parameters of a system are given, and the response of the system is predicted. In an inverse problem, on the other hand, the response of the system is measured, while the parameters of the system are determined.

Depending on the type of response that is adopted as the input of the identification process, system identification can be classified into static identifications, modal identifications, and dynamic identifications. Banan et al. [1,2] used the static response to identify the parameters of structures by recursive quadratic programming algorithm. Liu and Chian [3] and Liu [4] developed a direct inverse method to identify the element properties of a truss and frame, respectively, using the static strains of the structures. Yun and Shinozuka [5], Hoshiya and Saito [6], and Hoshiya and Maruyama [7] used the extended Kalman filter to identify linear and nonlinear systems. Fritzen and Zhu [8], Lin and Ewins [9], and Imregun et al. [10,11] employed frequency response function to update the finite element models. Agbabian et al. [12] used the least-squares method to detect the structural changes. Masri et al. [13, 14] applied a similar approach to identify the residual and restoring forces of nonlinear structures. Yao and Wang [15] estimated the structural physical parameters by the least-squares method. Banks et al. [16] used least-squares method with cubic splines approximation for non-destructive detection and location of damage.

Most of the aforementioned approaches have applied the 'best of fit' concept to determine the parameters of the target system, and iterations are usually required to solve the inverse problem.

The Bayesian approach proposed in this paper is based on a totally different concept. The Bayesian statistics was developed to update the distribution parameters of a random variable based on new observations. Its applications in engineering can be found in the literature. For example, Tang [17] used the Bayesian method to update the probability for cracks to develop in material. Bartlett and Sexsmith [18] applied the Bayesian statistics to detect the fraction of various materials in a concrete bridge.

Liu and Chen [19] applied the Bayesian method in the re-evaluation of structural safety. In their approach, an inverse problem is solved first to identify the element properties of the structure. Then, the identification results are used as observation data in the Bayesian approach to modify the distributions of the properties. Liu and Yiu [20] applied the Bayesian method to detect the damage of structures based on the static response. Different from the method proposed in [19], no identification process is carried out in advance in their approach.

This study adopts a similar approach as in Liu and Yiu [20] to update the distributions of the structural properties. However, the modal data or the dynamic response of the structure is used as the observation in the modification. The details of the proposed approach are as follows.

4. Bayesian Statistics

The basic idea of Bayesian statistics is to combine the prior knowledge about a random variable before the data are observed, i.e., the prior probability distribution, with the information coming from the observation to obtain a posterior distribution of the variable.

For continuous random variables, the Bayesian modification is as follows [19]:

$$f''_{\mathbf{t}|\mathbf{V}}\left(\mathbf{V}=\mathbf{v}\middle|\mathbf{t}\right)=\frac{f_{\mathbf{t}|\mathbf{V}}\left(\mathbf{t}\middle|\mathbf{V}=\mathbf{v}\right)f'_{\mathbf{V}}\left(\mathbf{V}=\mathbf{v}\right)}{\int_{-\infty}^{\infty} f_{\mathbf{t}|\mathbf{V}}\left(\mathbf{t}\middle|\mathbf{V}=\mathbf{v}\right)f'_{\mathbf{V}}\left(\mathbf{V}=\mathbf{v}\right)d\mathbf{v}} \tag{1}$$

where $f'_{\mathbf{V}}\left(\mathbf{V}=\mathbf{v}\right)$ is the prior probability distribution of the random variables $\mathbf{V}$, $f_{\mathbf{t}|\mathbf{V}}\left(\mathbf{t}\middle|\mathbf{V}=\mathbf{v}\right)$ is the conditional probability of observing the effect $\mathbf{t}$ as $\mathbf{V}=\mathbf{v}$, $f''_{\mathbf{V}}\left(\mathbf{V}=\mathbf{v}\middle|\mathbf{t}\right)$ is the posterior or modified probability distribution of $\mathbf{V}$, and $\left(\int_{-\infty}^{\infty} f_{\mathbf{t}|\mathbf{V}}\left(\mathbf{t}\middle|\mathbf{V}=\mathbf{v}\right)f'_{\mathbf{V}}\left(\mathbf{V}=\mathbf{v}\right)d\mathbf{v}\right)^{-1}$ is a normalizing constant.

In the application of damage detection, let $\mathbf{V}$ denote the element properties of the target structure. Usually, the statistics of the structural properties are available at the construction stage. Such data may be obtained by experiments, theoretical derivations, and/or engineering judgment. Therefore, some prior information is known about the structural properties. Let $f_{\mathbf{v}}'\left(\mathbf{v}\right)$ denote the prior distribution of $\mathbf{V}$.

Suppose structural test is carried out on the target structure, and $\mathbf{t}$ is the modal or dynamic data obtained in the test. Apparently, such information should be incorporated to give a new estimate of the structural properties.

According to the Bayesian formula, the updated probability density of $\mathbf{V}$ can be obtained if the conditional probability density $f_{\mathbf{t}|\mathbf{v}}\left(\mathbf{t}\mid\mathbf{v}\right)$ of observing $\mathbf{t}$ as $\mathbf{V}=\mathbf{v}$ is available. The derivation of $f_{\mathbf{t}|\mathbf{v}}\left(\mathbf{t}\mid\mathbf{v}\right)$ and $f''_{\mathbf{V}}\left(\mathbf{V}=\mathbf{v}\middle|\mathbf{t}\right)$ is discussed in the following section.

5. Modification Using Modal Data

Firstly, consider $\mathbf{t}=\mathbf{\Lambda}=\left[\omega_1^2 \;\; \omega_2^2 \;\; \Lambda \;\; \omega_m^2\right]$ = squares of measured natural frequencies. For a given set of element properties, assume $\mathbf{\Lambda}=N\left(\overline{\mathbf{\Lambda}},\sigma_\lambda^2\mathbf{I}\right)$, where $\overline{\mathbf{\Lambda}}$, the mean vector of $\mathbf{\Lambda}$, is obtained by solving $\mathbf{K}\boldsymbol{\phi}=\lambda\mathbf{M}\boldsymbol{\phi}$., where $\mathbf{K}$, $\mathbf{M}$, and $\boldsymbol{\phi}$ are the stiffness matrix, mass matrix, and modal shape of the structure, respectively. Then,

$$f_{\mathbf{\Lambda}|\mathbf{V}}\left(\mathbf{\Lambda}\mid\mathbf{v}\right)=\mathrm{N}\left(\overline{\mathbf{\Lambda}},\sigma_\lambda^2\mathbf{I}\right)$$
$$=\frac{1}{\left(2\pi\right)^{m/2}\sigma_\lambda^m}\exp\left[-\frac{1}{2\sigma_\lambda^2}\left(\mathbf{\Lambda}-\overline{\mathbf{\Lambda}}\right)^T\left(\mathbf{\Lambda}-\overline{\mathbf{\Lambda}}\right)\right] \tag{2}$$

In the above expression, $\overline{\mathbf{\Lambda}}$ is an implicit function of $\mathbf{V}$. Therefore, it is difficult to obtain a simple expression for the posterior distribution of $\mathbf{V}$. To overcome this problem,

$\overline{\mathbf{\Lambda}}$ is expanded into a Taylor's series:

$$\overline{\mathbf{\Lambda}}(\mathbf{v}) = \mathbf{\Lambda}_0 + \frac{\partial \mathbf{\Lambda}}{\partial \mathbf{v}}|_{v_0}\ \Delta\mathbf{v} + \Lambda \qquad (3)$$

where $\mathbf{\Lambda}_0 = \mathbf{\Lambda}(\mathbf{v}_0)$, $\mathbf{v}_0$ is the vector of the original properties, and $\Delta\mathbf{v} = \mathbf{v} - \mathbf{v}_0$. Then, keep only the first two terms in the above expression and neglect all the higher order terms of $\Delta\mathbf{v}$.

Substitute the linear approximation of $\overline{\mathbf{\Lambda}}$ into the conditional probability, one gets

$$f_{\Lambda|\mathrm{V}}(\mathbf{\Lambda} \mid \mathbf{v}) = \mathrm{N}(\boldsymbol{\mu}_\lambda, \mathbf{C}_\lambda) \qquad (4)$$

in which $\boldsymbol{\mu}_\lambda = \sigma_\lambda^{-2}\mathbf{C}_\lambda\mathbf{H}^T(\mathbf{H}\mathbf{v}_0 + \mathbf{\Lambda}_0 - \mathbf{\Lambda})$, and $\mathbf{C}_\lambda = \sigma_\lambda^2(\mathbf{H}^T\mathbf{H})^{-1}$, where $\mathbf{H} = -\left.\frac{\partial \Lambda}{\partial \mathbf{v}}\right|_{\mathrm{v}_0}$.

Suppose the prior distribution of $\mathbf{V}$ is normal with prior mean $\boldsymbol{\mu}'_{\mathrm{V}}$ and prior covariance $\mathbf{C}'_{\mathrm{V}}$. Then, the posterior distribution of $\mathbf{V}$ is

$$f''_{\mathrm{V}}(\mathbf{v}) = \mathrm{N}(\boldsymbol{\mu}''_{\mathrm{V}}, \mathbf{C}''_{\mathrm{V}}) \qquad (5)$$

where

$$\boldsymbol{\mu}''_{\mathrm{V}} = \left(\mathbf{C}'^{-1}_{\mathrm{V}} + \mathbf{C}^{-1}_\lambda\right)^{-1}\left(\mathbf{C}'^{-1}_{\mathrm{V}}\boldsymbol{\mu}'_{\mathrm{V}} + \mathbf{C}^{-1}_\lambda\boldsymbol{\mu}_\lambda\right) \quad (6)$$

$$\mathbf{C}''_{\mathrm{V}} = \left(\mathbf{C}'^{-1}_{\mathrm{V}} + \mathbf{C}^{-1}_\lambda\right)^{-1} \qquad (7)$$

are the posterior mean vector and covariance matrix of $\mathbf{V}$, respectively.

Next, consider $\mathbf{t} = \lfloor \boldsymbol{\varphi}_1\ \boldsymbol{\varphi}_2\ \Lambda\ \boldsymbol{\varphi}_q \rfloor$ = measured modal shapes. For a given set of element properties, suppose $\boldsymbol{\phi}_j = N(\overline{\boldsymbol{\phi}}_j, \mathbf{\Sigma}_{\phi_j})$, where $\overline{\boldsymbol{\phi}}_j$, the mean vector of $\boldsymbol{\phi}_j$, is obtained by solving $\mathbf{K}\boldsymbol{\phi} = \lambda\mathbf{M}\boldsymbol{\phi}$. Then,

$$f_{\phi|\mathrm{V}}(\boldsymbol{\phi}_1, \Lambda, \boldsymbol{\phi}_q \mid \mathbf{v}) = \prod_{j=1}^{q}(2\pi)^{-\frac{N}{2}}\sqrt{|\mathbf{\Sigma}_{\phi_j}|}$$

$$\exp\left[-\frac{1}{2\sigma^2_{\phi_j}}\left(\boldsymbol{\phi}_j - \overline{\boldsymbol{\phi}}_j\right)^T\mathbf{\Sigma}^{-1}_{\phi_j}\left(\boldsymbol{\phi}_j - \overline{\boldsymbol{\phi}}_j\right)\right] \qquad (8)$$

Again, expand $\overline{\boldsymbol{\phi}}_j$ into a Taylor's series:

$$\overline{\boldsymbol{\phi}}_j(v) \approx \boldsymbol{\phi}_{j0} + \frac{\partial \boldsymbol{\phi}_j}{\partial \mathbf{v}}|_{v_0}\ \Delta\mathbf{v} \qquad (9)$$

where $\boldsymbol{\phi}_{j0} = \boldsymbol{\phi}_j(\mathbf{v}_0)$.

Substitute the linear approximation of $\overline{\boldsymbol{\phi}}_j$ into the conditional probability, one obtaines

$$f_{\phi|\mathrm{V}}(\mathbf{t} \mid \mathbf{v}) = \mathrm{N}(\boldsymbol{\mu}_\phi, \mathbf{C}_\phi) \qquad (10)$$

in which

$$\boldsymbol{\mu}_\phi = \mathbf{C}_\phi[\mathbf{R}^T_{\phi_1}\mathbf{\Sigma}^{-1}_{\phi_1}(\boldsymbol{\varphi}_0 + \mathbf{R}_{\phi_1}\mathbf{v}_0 - \boldsymbol{\varphi}_1) + \Lambda + \mathbf{R}^T_{\phi_q}\mathbf{\Sigma}^{-1}_{\phi_q}(\boldsymbol{\varphi}_0 + \mathbf{R}_{\phi_q}\mathbf{v}_0 - \boldsymbol{\varphi}_q)] \qquad (11)$$

$$\mathbf{C}_\phi = [(\mathbf{R}^T_{\phi_1}\mathbf{\Sigma}^{-1}_{\phi_1}\mathbf{R}_{\phi_1}) + \Lambda + (\mathbf{R}^T_{\phi_q}\mathbf{\Sigma}^{-1}_{\phi_q}\mathbf{R}_{\phi_q})]^{-1} \qquad (12)$$

where $\mathbf{R}_{\phi_j} \equiv -\frac{\partial \boldsymbol{\phi}_j}{\partial \mathbf{v}}|_{\mathrm{v}_0}$.

Finally, applying the Bayesian formula, the posterior mean and covariance can be obtained.

$$\boldsymbol{\mu}''_v = \left(\mathbf{C}'^{-1}_v + \mathbf{C}^{-1}_\phi\right)^{-1}\left(\mathbf{C}'_v\boldsymbol{\mu}'_v + \mathbf{C}_\phi\boldsymbol{\mu}_\phi\right) \qquad (13)$$

$$\mathbf{C}''_v = \left(\mathbf{C}'^{-1}_v + \mathbf{C}^{-1}_\phi\right)^{-1} \qquad (14)$$

Once the posterior distribution is known, one may use the posterior mean $\boldsymbol{\mu}''_{\mathrm{V}}$ as a point estimator of $\mathbf{V}$. Thus, $\boldsymbol{\mu}''_{\mathrm{V}}$ can also be interpreted as the identified properties based on modal data.

6. Update Using Dynamic Data

Now, consider the case that $\mathbf{t}$ is the dynamic response of the target structure. For a given set of element properties, assume $\mathbf{t}$ = $N(\bar{\mathbf{t}}, \sigma_t^2\mathbf{I})$, where $\bar{\mathbf{t}}$, the mean vector of $\mathbf{t}$, is obtained by solving the following equation of motion:

$$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{C}\dot{\mathbf{u}} + \mathbf{K}\mathbf{u} = \mathbf{f} \qquad (15)$$

where $\mathbf{C}$ is the damping matrix of the structure, $\ddot{\mathbf{u}}$, $\dot{\mathbf{u}}$, and $\mathbf{u}$ respectively are the acceleration, velocity, and displacement vectors, and $\mathbf{f}$ is the external force vector. The response $\mathbf{t}$ could be the acceleration, velocity, and/or displacement of the structure.

Hence,

$$f_{\mathbf{t}|\mathbf{V}}(\mathbf{t}\mid\mathbf{v}) = \mathrm{N}(\bar{\mathbf{t}},\sigma_t^2\mathbf{I}) = \frac{1}{(2\pi)^{n/2}\sigma_t^n}\exp\left[-\frac{1}{2\sigma_t^2}(\mathbf{t}-\bar{\mathbf{t}})^T(\mathbf{t}-\bar{\mathbf{t}})\right] \tag{16}$$

where n is the dimension of $\mathbf{t}$.

Similar to the modal date, $\bar{\mathbf{t}}$ is an implicit function of $\mathbf{V}$. Hence, $\bar{\mathbf{t}}$ is expanded into a Taylor's series:

$$\bar{\mathbf{t}}(\mathbf{v}) = \mathbf{t}_0 + \left.\frac{\partial\mathbf{t}}{\partial\mathbf{v}}\right|_{\mathbf{v}_0}\Delta\mathbf{v} + \Lambda \tag{17}$$

where $\mathbf{t}_0 = \mathbf{t}(\mathbf{v}_0)$. Substitute the linear approximation of $\bar{\mathbf{t}}$ into the conditional probability, one gets

$$f_{\mathbf{t}|\mathbf{V}}(\mathbf{t}\mid\mathbf{v}) = \mathrm{N}(\boldsymbol{\mu}_t,\mathbf{C}_t) \tag{18}$$

where

$$\boldsymbol{\mu}_t = \sigma_t^{-2}\mathbf{C}_t\mathbf{S}^T(\mathbf{S}\mathbf{v}_0 + \mathbf{t}_0 - \mathbf{t}) \tag{19}$$

$$\mathbf{C}_t = \sigma_t^2(\mathbf{S}^T\mathbf{S})^{-1} \tag{20}$$

and

$$\mathbf{S} = -\left.\frac{\partial\mathbf{t}}{\partial\mathbf{v}}\right|_{\mathbf{v}_0} \tag{21}$$

Following the same procedure as in the modal data update, one obtains the following posterior mean vector and covariance matrix of **V:**

$$\boldsymbol{\mu}''_{\mathbf{V}} = (\mathbf{C}'^{-1}_{\mathbf{V}} + \mathbf{C}_t^{-1})^{-1}(\mathbf{C}'^{-1}_{\mathbf{V}}\boldsymbol{\mu}'_{\mathbf{V}} + \mathbf{C}_t^{-1}\boldsymbol{\mu}_t) \tag{22}$$

$$\mathbf{C}''_{\mathbf{V}} = (\mathbf{C}'^{-1}_{\mathbf{V}} + \mathbf{C}_t^{-1})^{-1} \tag{23}$$

7. Computation of Response Sensitivity

To attain an Bayesian estimate of the current conditions of the structure, it is required to compute the sensitivity of structural response with respect to the material properties.

The sensitivity of natural frequencies with respect to the material properties can be obtained by taking derivative of the eigen-equation $\mathbf{K}\boldsymbol{\phi} = \lambda\mathbf{M}\boldsymbol{\phi}$ with respect to $\mathbf{v}$. Suppose that the eigenvectors are normalized such that $\boldsymbol{\Phi}^T\mathbf{M}\boldsymbol{\Phi} = \mathbf{I}$, and $\boldsymbol{\Phi}^T\mathbf{K}\boldsymbol{\Phi} = \boldsymbol{\Lambda}$. One can show that

$$\frac{\partial\boldsymbol{\Lambda}}{\partial\mathbf{v}} = \boldsymbol{\Phi}^T\frac{\partial\mathbf{K}}{\partial\mathbf{v}}\boldsymbol{\Phi} \tag{24}$$

For linear structures, $\mathbf{K}$ is a linear function of $\mathbf{v}$. Therefore, the derivative of $\mathbf{K}$ with respect to $\mathbf{v}$ can be easily derived.

The sensitivity of modal shape with respect to the material properties has been derived by Wang [21]. First, take derivative of the eigen-equation $\mathbf{K}\boldsymbol{\phi} = \lambda\mathbf{M}\boldsymbol{\phi}$ with respect to $\mathbf{v}$. Assume that the properties relating to $\mathbf{M}$ do not change as the structure is damaged, and only $\mathbf{K}$ is a function of $\mathbf{v}$. Then,

$$(\mathbf{K} - \lambda_j\mathbf{M})\frac{\partial\boldsymbol{\phi}_j}{\partial\mathbf{v}} + \left(\frac{\partial\mathbf{K}}{\partial\mathbf{v}} - \frac{\partial\lambda_j}{\partial\mathbf{v}}\mathbf{M}\right)\boldsymbol{\phi}_j = \mathbf{0} \tag{25}$$

Pre-multiply the above equation by the eigen-matrix, and expand the sensitivity by the eigenvectors, i.e., $\frac{\partial\boldsymbol{\phi}_j}{\partial\mathbf{v}} = \boldsymbol{\Phi}\mathbf{c}_j$ [22], one gets

$$\begin{aligned}\boldsymbol{\Phi}^T(\mathbf{K} - \lambda_j\mathbf{M})\boldsymbol{\Phi}\mathbf{c}_j &= -\boldsymbol{\Phi}^T\frac{\partial\mathbf{K}}{\partial\mathbf{v}}\boldsymbol{\phi}_j + \boldsymbol{\Phi}^T\frac{\partial\lambda_j}{\partial v}\mathbf{M}\boldsymbol{\phi}_j \\ &= (\boldsymbol{\Lambda} - \lambda_j\mathbf{I})\mathbf{c}_j\end{aligned} \tag{26}$$

Hence,

$$[\mathbf{c}_j]_i = \frac{1}{\lambda_j - \lambda_i}\boldsymbol{\phi}_i^T\frac{\partial\mathbf{K}}{\partial\mathbf{v}}\boldsymbol{\phi}_i \quad i \neq j \tag{27}$$

Since $\boldsymbol{\phi}_j^T\mathbf{M}\boldsymbol{\phi}_j = 1$,

$$\frac{\partial\boldsymbol{\phi}_j^T}{\partial\mathbf{v}}\mathbf{M}\boldsymbol{\phi}_j = 0 = \mathbf{c}_j\boldsymbol{\Phi}\mathbf{M}\boldsymbol{\phi}_j = [\mathbf{c}_j]_j \tag{28}$$

Finally, the sensitivity of modal shapes can be expressed as

$$\frac{\partial\boldsymbol{\phi}_j}{\partial\mathbf{v}} = \sum_{i\neq j}\frac{1}{\lambda_j - \lambda_i}\boldsymbol{\phi}_i^T\frac{\partial\mathbf{K}}{\partial\mathbf{v}}\boldsymbol{\phi}_j\boldsymbol{\phi}_i \tag{29}$$

Similar to the sensitivity of natural frequencies and modal shapes, the sensitivity of dynamic response can be obtained by taking derivative of the equilibrium equation with respect to the material properties.

$$\mathbf{M}\frac{\partial\ddot{\mathbf{u}}}{\partial\mathbf{v}}+\mathbf{C}\frac{\partial\dot{\mathbf{u}}}{\partial\mathbf{v}}+\mathbf{K}\frac{\partial\mathbf{u}}{\partial\mathbf{v}}+\frac{\partial\mathbf{K}}{\partial\mathbf{v}}\mathbf{u}=0 \qquad (30)$$

Change the order of differentiation, one gets

$$\mathbf{M}\frac{\partial^2}{\partial t^2}\left(\frac{\partial\mathbf{u}}{\partial\mathbf{v}}\right)+\mathbf{C}\frac{\partial}{\partial t}\left(\frac{\partial\mathbf{u}}{\partial\mathbf{v}}\right)+\mathbf{K}\left(\frac{\partial\mathbf{u}}{\partial\mathbf{v}}\right)=-\frac{\partial\mathbf{K}}{\partial\mathbf{v}}\mathbf{u} \qquad (31)$$

Therefore, the sensitivity of **u** with respect to **v** can be obtained by solving the above differential equation. Notice that this equation is the same as the original dynamic equation except that **u** and **f** are replaced by $\frac{\partial\mathbf{u}}{\partial\mathbf{v}}$ and $-\frac{\partial\mathbf{K}}{\partial\mathbf{v}}\mathbf{u}$, respectively. Hence, one can use the same finite element code and the same model to solve for $\frac{\partial\mathbf{u}}{\partial\mathbf{v}}$. No coding or modeling is required.

8. Numerical Examples

Consider a simple truss as shown in Fig. 1. The truss has 4 elements and 4 nodes. The cross-sectional area of the elements are $1\times10^{-3}\,\mathrm{m}^2$, and the mass density is $\rho=2700$ kg/m^3. Suppose the prior distributions of the axial rigidities of the elements are independent normal with means μ'_{EA} =70MN and standard deviations σ'_{EA} = 7MN.

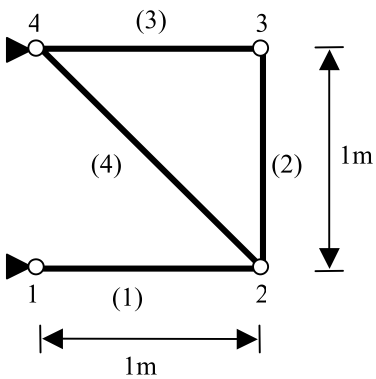

Figure 1. Example Truss

Assume that one element is damaged, and its axial rigidity is reduced to 63MN. Let $\sigma_\lambda=100\mathrm{rad}^2/\mathrm{sec}^2$. Table 1 shows the results of Bayesian modification.

Table 1. Posterior means based on 4 natural frequencies

damaged element	1	2	3	4
EA_1 (MN)	63.4	69.8	70.0	69.3
EA_2 (MN)	70.1	63.0	70.0	69.9
EA_3 (MN)	70.0	70.0	63.0	70.0
EA_4 (MN)	69.1	70.6	70.1	64.8

It is seen the the posterior means yield a good estimate of the current axial rigidities. Certainl, the damaged elements can be identified without difficulty.

Then, the modal shapes are used in the Bayesian. Tables 2~4 list the results of Bayesian update based on the first two, three, and four modal shapes. Again, the damaged elements can be identified without difficulty. Comparing the three tables, one can see that the results improve as the number of measured modal shapes increases. This is reasonable because more information is adopted in the update process.

In all three cases, the posterior means give a good estimate of the axial rigidities for the first, third and fourth elements. However, μ''_{EA2} yields a reasonable estimate of EA_2 only when all 4 modal shapes are used.

Table 2. Posterior means based on 2 modal shapes (σ_ϕ=0.001)

damaged element	1	2	3	4
EA_1 (MN)	64.6	81.5	71.5	71.5
EA_2 (MN)	71.4	22.4	71.5	71.6
EA_3 (MN)	71.9	76.3	64.1	71.7
EA_4 (MN)	71.4	73.8	71.3	64.6

Table 3. Posterior means based on 3 modal shapes (σ_ϕ=0.001)

damaged element	1	2	3	4
EA_1 (MN)	63.2	79.0	70.3	70.9
EA_2 (MN)	70.2	33.3	70.4	71.1
EA_3 (MN)	70.2	74.8	63.3	70.9
EA_4 (MN)	70.1	71.5	70.2	64.0

Table 4. Posterior means based on 2 modal shapes

damaged element	1	2	3	4
EA_1 (MN)	63.2	69.5	70.3	70.8
EA_2 (MN)	70.1	52.2	70.3	71.0
EA_3 (MN)	70.2	67.2	63.3	70.9
EA_4 (MN)	70.1	62.9	70.2	64.0

The reason for the poor estimate of EA_2 is mainly because element 2 hardly deforms in the first three modes. Although element 2 does move in the first three modes, it moves as a rigid body basically. Even when one of the element is damaged, the first three modes do not change noticeably. Hence, the axial rigidity of element 2 has little influence on the dynamic equilibrium of the truss. On the other hand, element 2 deforms obviously in mode 4. Therefore, the inclusion of mode shape 4 improves the estimate on EA_2 greatly.

9. Comparision with Conventional Identification Methods

The main difference between the proposed method and the conventional identification methods is that the Bayesian modification is adopted in the identification process. No optimization problem is solved in the proposed method. Obviously, the proposed method has the advantage that no iterations are required, and thus is more efficient. It also avoids the problem of local minima.

Although, one can easily identify the damaged elements based on the results of Bayesian modification, the posterior means do not always give accurate estimate of the current rigidities. Therefore, the Bayesian approach is better considered as a qualitative damage detection method rather then a system identification method.

10. Conclusions

This project develops a method to access the integrity of structures. Structural tests are performed on the target structure, and the modal or dynamic data are measured. Then, the Bayesian method is adopted to incorporate test data in the modification of the distribution of structural properties. The posterior means obtain by the Bayesian approach are then used as an indicator of element damages.

Several conclusions can be drawn from this study：

1. The Bayesian update method provides a systematic way of incorporating the test data to update the distributions of the structural properties.
2. The posterior means of element properties can serve as a indicator of the current condition of the structure.
3. As the damage of the structure increases, the error of the identification results increases.
4. The measured data should reflect the change of element properties. If an element property has little effect on the measured data, the Bayesian approach does not give reasonable estimate on that property.

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