行政院國家科學委員會專題研究計畫成果報告

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※　**波、流平行於多孔介質牆之弔詭的研究(Ⅱ)**　※
※　**A study on the paradox of wave and current with a parallel porous wall (Ⅱ)**　※
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執行期間： 90 年 8 月 1 日 至 91 年 7 月 31 日

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黃 良 雄

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行政院國家科學委員會專題研究計畫成果報告

波、流平行於多孔介質牆之弔詭的研究(Ⅱ)

A study on the paradox of wave and current with a parallel porous wall (Ⅱ)

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主持人: 黃良雄教授　國立臺灣大學土木工程學系

ABSTRACT

There is a paradoxical phenomenon in earlier studies such as Dalrymple et al. (1991), when the incoming water wave is parallel to a porous breakwater, the water wave permeates completely without regarding to how large the porosity of the porous breakwater is. For solving the problem of water waves obliquely impact on thin porous wall, a new boundary condition on the thin porous wall which can remedy the above mentioned paradoxical phenomenon is proposed based on the concept that the incident angle remains unchanged when the water wave permeates into the wall. According to this new boundary condition, an analytic solution of an oblique water wave impacting on a thin porous wall of any permeability is obtained. It is found that the above paradoxical phenomenon as the water wave is parallel to a thin porous wall disappears. And as the incident angle changes from 0° to 90°, the reflection coefficient and the transmission coefficient change smoothly with no discontinuities when the water wave is parallel to the thin porous wall while those in the earlier investigations do not.

Keywords: *oblique water wave, thin porous wall, new boundary condition*

摘要

以往關於斜向水波作用於多孔介質防波堤的研究中(如 Dalrymple et al. (1991))，往往出現一個有趣的弔詭現象，即當入射波之波向平行於多孔介質牆時，不論牆本身之孔隙率為何，水波都將完全滲透至牆之另一側。為了解決斜向水波作用於多孔介質薄牆之問題，本研究以水波進入多孔介質牆後入射角維持不變的觀念，提出一個新的薄牆牆面邊界條件以改正上述之弔詭現象。依據此邊界條件，可得到斜向水波作用於任意透水性之多孔介質薄牆的解析解，並且消除了波向平行於牆面時之弔詭現象。此外，本研究中入射角由0°遞增為90°時，反射係數與滲透係數皆隨之呈現漸進而和緩的變化；而當波向平行於牆面時，並未出現不連續的情況，這是以往的研究中所沒有的。

關鍵字： *斜向水波，多孔介質薄牆，新的邊界條件*

A NEW BOUNDARY CONDITION OF OBLIQUE WATER WAVES ACTING ON THIN POROUS WALLS

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ABSTRACT

There is a paradoxical phenomenon in earlier studies such as Dalrymple et al. (1991), when the incoming water wave is parallel to a porous breakwater, the water wave permeates completely without regarding to how large the porosity of the porous breakwater is. For solving the problem of water waves obliquely impact on thin porous wall, a new boundary condition on the thin porous wall which can remedy the above mentioned paradoxical phenomenon is proposed based on the concept that the incident angle remains unchanged when the water wave permeates into the wall. According to this new boundary condition, an analytic solution of an oblique water wave impacting on a thin porous wall of any permeability is obtained. It is found that the above paradoxical phenomenon as the water wave is parallel to a thin porous wall disappears. And as the incident angle changes from 0° to 90°, the reflection coefficient and the transmission coefficient change smoothly with no discontinuities when the water wave is parallel to the thin porous wall while those in the earlier investigations do not.

Keywords: *oblique water wave, thin porous wall, new boundary condition*

1. INTRODUCTION

The problem of water wave impacting on a porous wall has been studied by many investigators. Sollitt and Cross (1972) formulated and solved the problem of water wave passing through a porous medium, but due to the complex wave numbers inside the porous medium, the orthogonality of their infinite series solution is questionable. Madsen (1974) applied the method similar to Sollitt and Cross (1972) to analyze reflected and transmitted waves, but the same defect arises. Chwang (1983) used Taylor's (1956) concept of water flows through a porous screen to investigate the zero-thickness porous wavemaker problem. Later, Chwang and Li (1983), Chwang and Dong (1984), and Tyvand (1986) applied the same method in the analysis of piston-type porous wavemaker and of the problem about water wave permeating through a porous screen. Note that in Chwang (1983), the inertial reactance of flow is neglected and the porous wall is assumed zero-thickness.

Simplifying the theory of poroelasticity (see Biot (1962)) by letting the skeleton of porous medium be rigid and the fluid be incompressible, Huang (1991) obtained the governing equation of porous medium flow, similar to the one in Sollitt and Cross (1972). Chwang's (1983) study thus was extended to a finite-thickness porous medium with inertial reactance of the flow being preserved. Later, clarifying the boundary conditions for a thin porous wall with both flow inertial reactance and viscous damping, Huang et al. (1993) analyzed the wave field of a thin porous wavemaker by a simplified analytical approach.

As for an oblique water wave impacting on a porous structure, Dalrymple et al. (1991) adopted an assumption that neglects the evanescent modes of the water wave near the interfaces of the porous wall to simplify their problem. It is found that when the incoming water wave is parallel to a porous breakwater, no matter how large the porosity of the porous wall is, the water wave permeates through completely. This is indeed questionable. Actually, to the authors' knowledge, this paradoxical phenomenon can be found in all the other earlier studies of oblique wave acting on porous structures.

For the purpose of eliminating the above mentioned paradoxical phenomenon, we attempt to derive a new boundary condition for the thin porous wall impacted by an oblique wave based on the concept that the wave direction remains unchanged as it permeates into a thin porous wall in the present study. According to this new boundary condition, an analytic solution of an oblique water wave impacting on a thin porous wall of any permeability is obtained. It is found that the above paradoxical phenomenon as the water wave is parallel to a thin porous wall disappears. And as the incident angle changes from 0° to 90°, the reflection coefficient and the transmission coefficient change smoothly with no discontinuities when the water wave is parallel to the thin porous wall while those in the earlier researches do not.

2. FORMULATION OF OBLIQUE WAVE FIELD

2.1 Governing Equations

Homogeneous water wave is assumed to be inviscid, irrotational, and incompressible; and the fluid domains of the present study are shown in Figure 1.

For region (1) and region (2), we have

$$\underline{V}_j = \nabla \Phi_j, \qquad j = 1,2, \tag{1}$$

where $\underline{V}_j$ and Φ_j are velocity vectors and velocity potentials of fluid, respectively; and the subscript j denotes the flow regions (see Figure 1). And the continuity equation and linear momentum equation are

$$\nabla \cdot \underline{V}_j = 0, \qquad j = 1,2, \tag{2}$$

$$\rho \frac{\partial \underline{V}_j}{\partial t} = -\nabla p_j + \rho \underline{g}, \qquad j = 1,2. \tag{3}$$

By substituting (1) into (2), (2) becomes

$$\nabla^2 \Phi_j = 0, \qquad j = 1,2. \tag{4}$$

Then by substituting (1) into (3) and integrating (3) with respect to y, (3) becomes

$$p_j = -\rho \frac{\partial \Phi_j}{\partial t} - \rho g (y - h), \qquad j = 1,2, \tag{5a}$$

when it is not on the surface of seepage (e.g. see Muskat (1946)); and

$$p_j = 0, \qquad j = 1,2, \tag{5b}$$

when it is on the surface of seepage, where p_j is the pressure of the fluid and ρ is the density of the fluid.

For region (3) that is within the porous wall, we have

$$\underline{V}_3 = \nabla \Phi_3, \tag{6}$$

where $\underline{V}_3$ and Φ_3 are velocity vector and velocity potential of fluid inside the porous wall, respectively (see Figure 1). And, referring to Huang (1991), the continuity equation and linear momentum equation are

$$\nabla \cdot \underline{V}_3 = 0, \tag{7}$$

$$\rho \frac{\partial \underline{V}_3}{\partial t} = -\nabla p_3 + \rho \underline{g} - \frac{\mu n_0}{k} \underline{V}_3, \tag{8}$$

where μ is dynamic viscosity of fluid; n_0 and k are porosity and specific permeability of the porous wall, respectively. By substituting (6) into (7), (7) becomes

$$\nabla^2 \Phi_3 = 0. \tag{9}$$

Then by substituting (6) into (8) and integrating (8) with respect to y, (8) becomes

$$p_3 = -\rho \frac{\partial \Phi_3}{\partial t} - \rho g (y - h) - \frac{\mu n_0}{k} \Phi_3, \tag{10a}$$

when it is not on the surface of seepage; and

$$p_3 = 0, \tag{10b}$$

when it is on the surface of seepage.

2.2 Boundary Conditions

On the bottom (i.e. $y = 0$), the normal velocity equals to zero, i.e., we get

$$\frac{\partial \Phi_j}{\partial y} = 0, \qquad j = 1,2,3. \tag{11}$$

On the free surface of fluid, $y = h$ and $-\infty < x < -d/2$ for region (1) and $d/2 < x < \infty$ for region (2), we have the conventional kinematic and dynamic boundary conditions as

$$\frac{\partial \Phi_j}{\partial y} = \frac{\partial \eta_j}{\partial t}, \qquad j = 1,2, \tag{12}$$

$$\frac{\partial \Phi_j}{\partial t} + g \eta_j = 0, \qquad j = 1,2, \tag{13}$$

where η_j is the vertical deviation of water surface from the mean free surface. By combining (12) with (13), we get

$$\frac{\partial^2 \Phi_j}{\partial t^2} + g \frac{\partial \Phi_j}{\partial y} = 0, \qquad j = 1,2. \tag{14}$$

On the free surface of fluid inside the porous wall, $y = h$ and $-d/2 < x < d/2$, the kinematic and dynamic boundary conditions are

$$\frac{\partial \Phi_3}{\partial y} = n_0 \frac{\partial \eta_3}{\partial t}, \tag{15}$$

$$\frac{\partial \Phi_3}{\partial t} + \frac{\mu n_0}{\rho k} \Phi_3 + g \eta_3 = 0. \tag{16}$$

By combining (15) with (16), we get

$$\frac{\partial^2 \Phi_3}{\partial t^2} + \frac{\mu n_0}{\rho k}\frac{\partial \Phi_3}{\partial t} + \frac{g}{n_0}\frac{\partial \Phi_3}{\partial y} = 0. \tag{17}$$

At the far field, $0 < y < h$ and $x \to \pm\infty$, the radiation boundary condition is

$$\lim_{x\to\pm\infty} \Phi_j \to \text{outgoing or } 0, \quad j = 1,2. \tag{18}$$

On the interfaces of the porous wall and the fluid, $0 < y < h$ and $x = \pm d/2$, the continuities of mass flux and pressure are considered. From (1) and (6), we have

$$\frac{\partial \Phi_j}{\partial x} = n_0 \frac{\partial \Phi_3}{\partial x}, \qquad j = 1,2, \tag{19}$$

and according to (5) and (10) with surface of seepage effect being ignored for convenience, it is obtained that

$$\frac{\partial \Phi_j}{\partial t} = \frac{\partial \Phi_3}{\partial t} + \frac{\mu n_0}{\rho k}\Phi_3, \qquad j = 1,2. \tag{20}$$

2.3 Boundary Value Problem

Since the present study focuses on the problem of periodic linear water wave, we may introduce a time factor $e^{-i\omega t}$ into the time-dependent variables, where ω is the angular frequency of water wave. And according to Snell's law, in order to satisfy the matching conditions at each vertical interface, we can write

$$\Phi_j(x,y,z,t) = \mathrm{Re}\left[\phi_j(x,y)e^{i(k_0 \sin\theta z - \omega t)}\right], j = 1,2,3, \tag{21}$$

so that the z-variation of the solution in each region must be the same. In addition, a flow-efficiency parameter is defined as

$$R = \frac{reac\tan ce \;\; force}{resis\tan ce \;\; force} = \frac{\rho\omega k}{\mu n_0}, \tag{22}$$

which represents the capability for fluid to flow through the porous medium (see Huang (1991)). After getting rid of $e^{-i\omega t}$, the time-independent boundary value problem can be constructed by governing equations (4) and (9) with boundary conditions (11), (14), (17), (18), (19), and (20).

3. SOLUTION

3.1 Boundary Condition on Thin Porous Wall

When the porous wall is extremely thin (i.e., d in Figure 1 is small), only the flow fields of region (1) and region (2) need to be considered, i.e. we don't need to solve governing equation (9) with boundary conditions (11) for $j = 3$, (17), (19), and (20). Therefore, the original boundary conditions (19) and (20) on the interfaces ($x = \pm d/2$) should be rectified and combined to obtain a new boundary condition that fits the thin porous wall and is independent of ϕ_3.

We will use Sahoo et al. (2000) as an example to introduce how the conventional way is dealing with the problem of oblique waves impact on a thin porous wall. Like other earlier studies, the boundary condition on the thin porous wall given in Sahoo et al. (2000) is obtained under the assumption that, regardless of the incident angles, the wave always permeates into the porous wall in normal direction (see Figure 2). For this inference, even if the incoming water wave in region (1) is parallel to the porous wall (i.e. $\theta = 90^\circ$), there still exists a normal wave inside the porous wall and which then permeates completely into region (2). This phenomenon is indeed paradoxical and is obviously against our physical expectation when the viscous effect of water wave is so little and hence negligible. However, as the present authors know, this paradoxical result of $\theta = 90^\circ$ happens in all the earlier studies of relative problems; e.g. Darymple et al. (1991), Sahoo et al. (2000), etc.

After many thoughts, we find that since the wavelength is much longer than the thickness of the porous wall, there should be no diffraction existing. And because the water that transports the wave inside the porous wall is still the same as those in regions (1) and (2), no refraction occurs either. Hence, a new boundary condition on the thin porous wall which can remedy the above mentioned paradoxical phenomenon as $\theta = 90^\circ$ is proposed based on the concept that the incident angle θ remains unchanged when the water wave permeates into region (3) as shown in Figure 3.

Before the new boundary condition is derived, we may see from Figure 3 that because the flow in region (3) follows Darcy's law of porous media flow (i.e. the velocity vector is linearly proportional to the pressure gradient inside the porous wall), when the incoming water wave is parallel to the thin porous wall (i.e. $\theta = 90^\circ$), as long as the viscous effect of fluid on the wall is negligible, there will be no water wave penetrating through the porous wall and appearing in region (2) anymore.

We now start to derive our new boundary condition on the thin porous wall. For the oblique incoming wave as shown in Figure 3, the flow is in the direction of x' axis and passes through $\underline{x}_1\left(-\frac{d}{2},y,z_1\right)$ and $\underline{x}_2\left(\frac{d}{2},y,z_2\right)$ on the wall. Therefore, it is necessary to take velocity vectors at $\underline{x}=\underline{x}_1$ and $\underline{x}=\underline{x}_2$. And according to Huang et al. (1993), it is quite acceptable that the velocities are identical from the entrance to the exit alone a streamline inside the thin porous wall. Then, referring to Huang et al. (1993), we get

$$\left.\frac{\partial\phi_3}{\partial x'}\right|_{\left(-\frac{d}{2},y,z_1\right)}=\left.\frac{\partial\phi_3}{\partial x'}\right|_{\left(\frac{d}{2},y,z_2\right)}=\frac{1}{l}\left(\phi_3\big|_{\left(\frac{d}{2},y,z_2\right)}-\phi_3\big|_{\left(-\frac{d}{2},y,z_1\right)}\right).\tag{23}$$

Substituting $\frac{\partial\phi_3}{\partial x'}=\frac{1}{\cos\theta}\frac{\partial\phi_3}{\partial x}$ and $l=\frac{d}{\cos\theta}$ into (23), it is obtained that

$$\left.\frac{\partial\phi_3}{\partial x}\right|_{\left(-\frac{d}{2},y,z_1\right)}=\left.\frac{\partial\phi_3}{\partial x}\right|_{\left(\frac{d}{2},y,z_2\right)}=\frac{1}{d}\left(\phi_3\big|_{\left(\frac{d}{2},y,z_2\right)}-\phi_3\big|_{\left(-\frac{d}{2},y,z_1\right)}\right)\cos^2\theta.\tag{24}$$

On the other hand, referring to the continuous mass flux condition and continuous pressure condition on the interfaces (i.e. at $\underline{x}=\underline{x}_1$ and $\underline{x}=\underline{x}_2$), (19) can be rewritten as

$$\left.\frac{\partial\phi_1}{\partial x}\right|_{\left(-\frac{d}{2},y,z_1\right)}=n_0\left.\frac{\partial\phi_3}{\partial x}\right|_{\left(-\frac{d}{2},y,z_1\right)},\tag{25}$$

$$\left.\frac{\partial\phi_2}{\partial x}\right|_{\left(\frac{d}{2},y,z_2\right)}=n_0\left.\frac{\partial\phi_3}{\partial x}\right|_{\left(\frac{d}{2},y,z_2\right)},\tag{26}$$

and (20) can be expressed as

$$iR\phi_1\big|_{\left(-\frac{d}{2},y,z_1\right)}=(iR-1)\phi_3\big|_{\left(-\frac{d}{2},y,z_1\right)},\tag{27}$$

$$iR\phi_2\big|_{\left(\frac{d}{2},y,z_2\right)}=(iR-1)\phi_3\big|_{\left(\frac{d}{2},y,z_2\right)}.\tag{28}$$

(28) minus (27) gives

$$iR\left(\phi_2\big|_{\left(\frac{d}{2},y,z_2\right)}-\phi_1\big|_{\left(-\frac{d}{2},y,z_1\right)}\right)=(iR-1)\left(\phi_3\big|_{\left(\frac{d}{2},y,z_2\right)}-\phi_3\big|_{\left(-\frac{d}{2},y,z_1\right)}\right).\tag{29}$$

Now, substituting (25), (26), and (29) into (24) and getting rid of ϕ_3, the final form of boundary conditions on the interfaces are obtained as

$$\left.\frac{\partial\phi_1}{\partial x}\right|_{\left(-\frac{d}{2},y,z_1\right)}=\left.\frac{\partial\phi_2}{\partial x}\right|_{\left(\frac{d}{2},y,z_2\right)}=\frac{n_0}{d}\frac{iR}{1-iR}\left(\phi_1\big|_{\left(-\frac{d}{2},y,z_1\right)}-\phi_2\big|_{\left(\frac{d}{2},y,z_2\right)}\right)\cos^2\theta.\tag{30}$$

(30) is the boundary condition suitable for the thin porous wall impacted by the oblique water wave. It is obviously observed from (30) and (25), (26) that when $\theta=90^\circ$, there will be no normal velocity on each side of the porous wall, which satisfies Darcy's law and indeed agrees to our physical expectation.

From Figure 3, we find $z=z_1-\frac{d}{2}\tan\theta=z_2+\frac{d}{2}\tan\theta$. Hence Equation (30) can be written as

$$\left.\frac{\partial\phi_1}{\partial x}\right|_{\left(-\frac{d}{2},y,z\right)}=\left.\frac{\partial\phi_2}{\partial x}\right|_{\left(\frac{d}{2},y,z\right)}=\frac{n_0}{d}\frac{iR}{1-iR}\left(\phi_1\big|_{\left(-\frac{d}{2},y,z\right)}-\phi_2\big|_{\left(\frac{d}{2},y,z\right)}\right)\cos^2\theta\tag{30a}$$

as a good approximation when d is extremely small. Equation (30a) is the boundary condition that we are going to use. The boundary condition on the thin porous wall which is used in Sahoo et al. (2000) is similar to (30a) but without $\cos^2\theta$.

3.2 Analytical Solution

By applying (21) and (22) into the governing equation (4) and boundary conditions (11), (14), (18), and (30a), the analytical solution is easily obtained as

$$\phi_1(x,y,z)=\phi_0\left[e^{i\xi\left(x+\frac{d}{2}\right)}+A_0e^{-i\xi\left(x+\frac{d}{2}\right)}\right]\cosh(k_0y)e^{i\zeta z},\tag{31}$$

$$\phi_2(x,y,z)=\phi_0\left[e^{i\xi\left(x-\frac{d}{2}\right)}-A_0e^{i\xi\left(x-\frac{d}{2}\right)}\right]\cosh(k_0y)e^{i\zeta z},\tag{32}$$

with

$$\phi_0=-\frac{iag}{\omega\cosh(k_0h)},\tag{33}$$

$$A_0=\frac{\xi d(1-iR)}{\xi d(1-iR)+2n_0R\cos^2\theta},\tag{34}$$

where $\xi=k_0\cos\theta$, $\zeta=k_0\sin\theta$ and k_0 satisfies the dispersion relation

$$\omega^2-gk_0\tanh(k_0h)=0.\tag{35}$$

It is clearly found that when $\theta=90^\circ$,

$$\phi_1(x,y,z)=2\phi_0\cosh(k_0y)e^{ik_0z},\tag{36}$$

$$\phi_2(x,y,z)=0.\tag{37}$$

(37), again, indicates that when the incoming water wave is parallel to the porous wall, region (2) remains quiescent and the wave field does not appear. Hence the paradox which occurs when the parallel water wave acting on the thin porous wall is eliminated.

3.3 Physical Properties

Since the z-variation of analytical solutions are the same, we may study all the physical properties in the condition that $z=0$ for representation.

By (1), the nondimensional normal velocities on the interfaces ($x = \pm d/2$) are obtained as

$$\left.\frac{U_1}{-a\omega}\right|_{x=-\frac{d}{2}} = A_{U_1}\cos\omega t + B_{U_1}\sin\omega t = D_{U_1}\cos(\theta_{U_1} - \omega t), \quad (38)$$

where

$$A_{U_1} = \frac{1}{a\omega}\xi\,\mathrm{Im}(\phi_0)[1-\mathrm{Re}(A_0)]\cosh(k_0 y), \quad (39)$$

$$B_{U_1} = -\frac{1}{a\omega}\xi\,\mathrm{Im}(\phi_0)\mathrm{Im}(A_0)\cosh(k_0 y), \quad (40)$$

$$D_{U_1} = \sqrt{A_{U_1}^{\ 2} + B_{U_1}^{\ 2}}, \quad (41)$$

$$\theta_{U_1} = \tan^{-1}\left(\frac{B_{U_1}}{A_{U_1}}\right); \quad (42)$$

$$\left.\frac{U_2}{-a\omega}\right|_{x=\frac{d}{2}} = A_{U_2}\cos\omega t + B_{U_2}\sin\omega t = D_{U_2}\cos(\theta_{U_2} - \omega t), \quad (43)$$

where

$$A_{U_2} = \frac{1}{a\omega}\xi\,\mathrm{Im}(\phi_0)[1-\mathrm{Re}(A_0)]\cosh(k_0 y), \quad (44)$$

$$B_{U_2} = -\frac{1}{a\omega}\xi\,\mathrm{Im}(\phi_0)\mathrm{Im}(A_0)\cosh(k_0 y), \quad (45)$$

$$D_{U_2} = \sqrt{A_{U_2}^{\ 2} + B_{U_2}^{\ 2}}, \quad (46)$$

$$\theta_{U_2} = \tan^{-1}\left(\frac{B_{U_2}}{A_{U_2}}\right). \quad (47)$$

Similarly, by (1), the nondimensional tangential velocities on the interfaces ($x = \pm d/2$) are obtained as

$$\left.\frac{V_1}{-a\omega}\right|_{x=-\frac{d}{2}} = A_{V_1}\cos\omega t + B_{V_1}\sin\omega t = D_{V_1}\cos(\theta_{V_1} - \omega t), \quad (48)$$

where

$$A_{V_1} = \frac{1}{a\omega}k_0\,\mathrm{Im}(\phi_0)\mathrm{Im}(A_0)\sinh(k_0 y), \quad (49)$$

$$B_{V_1} = -\frac{1}{a\omega}k_0\,\mathrm{Im}(\phi_0)[1+\mathrm{Re}(A_0)]\sinh(k_0 y), \quad (50)$$

$$D_{V_1} = \sqrt{A_{V_1}^{\ 2} + B_{V_1}^{\ 2}}, \quad (51)$$

$$\theta_{V_1} = \tan^{-1}\left(\frac{B_{V_1}}{A_{V_1}}\right); \quad (52)$$

$$\left.\frac{V_2}{-a\omega}\right|_{x=\frac{d}{2}} = A_{V_2}\cos\omega t + B_{V_2}\sin\omega t = D_{V_2}\cos(\theta_{V_2} - \omega t), \quad (53)$$

where

$$A_{V_2} = -\frac{1}{a\omega}k_0\,\mathrm{Im}(\phi_0)\mathrm{Im}(A_0)\sinh(k_0 y), \quad (54)$$

$$B_{V_2} = -\frac{1}{a\omega}k_0\,\mathrm{Im}(\phi_0)[1-\mathrm{Re}(A_0)]\sinh(k_0 y), \quad (55)$$

$$D_{V_2} = \sqrt{A_{V_2}^{\ 2} + B_{V_2}^{\ 2}}, \quad (56)$$

$$\theta_{V_2} = \tan^{-1}\left(\frac{B_{V_2}}{A_{V_2}}\right). \quad (57)$$

By (13), the wave profile which takes off the incoming wave η_0 in region (1) and wave profile in region (2) are obtained as

$$\left.\frac{\eta_1}{a}\right|_{y=h} = \mathrm{Re}(A_0)\cos\left(\xi\left(x+\frac{d}{2}\right)+\omega t\right) + \mathrm{Im}(A_0)\sin\left(\xi\left(x+\frac{d}{2}\right)+\omega t\right), \quad (58)$$

$$\left.\frac{\eta_2}{a}\right|_{y=h} = \cos\left(\xi\left(x-\frac{d}{2}\right)-\omega t\right) - \mathrm{Re}(A_0)\cos\left(\xi\left(x-\frac{d}{2}\right)-\omega t\right) + \mathrm{Im}(A_0)\sin\left(\xi\left(x-\frac{d}{2}\right)-\omega t\right). \quad (59)$$

The reflection coefficient and transmission coefficient of the wave field are defined as

$$C_r = \left|\frac{\eta_1}{a}\right|^2 = \mathrm{Re}(A_0)^2 + \mathrm{Im}(A_0)^2, \quad (60)$$

$$C_t = \left|\frac{\eta_2}{a}\right|^2 = [1-\mathrm{Re}(A_0)]^2 + \mathrm{Im}(A_0)^2. \quad (61)$$

4. RESULTS

In the present study, the so-called "thin" porous wall indicates that the thickness of the porous wall, compared to the wavelength, is extremely short. Hence, due to the property of the thin porous wall, the local phenomenon of surface of seepage (see (5), (10) and refer to Muskat (1946)) is neglected.

In the following analysis, we only apply the porosity $n_0 = 0.3$, the porous Reynolds number $R = 1$ and the nondimensional wall thickness $d/2h = 0.05$ for demonstration.

Figure 4 indicates vertical distributions of normal velocity amplitudes D_{U_1} and D_{U_2} of homogeneous fluid on the thin porous wall ($x = \pm d/2$). D_{U_1} and D_{U_2} are given by (41) and (46). It is found from Figure 5 (b) that as the incident angle increases, D_{U_2} decreases monotonically. It can be seen that when the incoming water wave is parallel to the porous wall (i.e. $\theta = 90^\circ$), D_{U_2}, certainly disappears. Note that because of the mass flux continuity condition from the property of the thin porous wall, figures of D_{U_1} are identical to those of D_{U_2}.

The vertical distributions of tangential velocity amplitudes D_{V_1} and D_{V_2} of homogeneous fluid are illustrated in Figure 5. D_{V_1} and D_{V_2} are given by (51) and (56). Beside same trend as Figure 4, again, it is found that there is no tangential water wave penetrating into the original quiescent water field (region (2)) when the incoming water wave is parallel to the thin porous wall.

Figure 6 presents the wave profiles in region (1) and region (2). η_1/a and η_2/a are given by (58) and (59). By analogy with above discussions, with the increase of the incident angle, η_2/a decreases monotonically. And when the incoming water wave is parallel to the porous wall (i.e. $\theta = 90^\circ$), η_2/a vanishes. This, again, confirms our physical expectation that when one side of a thin porous wall is affected by a parallel water wave, there is no water wave penetrating into the other side (provided that the viscous effect of water is negligible).

The reflection coefficient C_r and transmission coefficient C_t are most important in the analysis of the wave field. C_r and C_t are defined by (60) and (61). The values of C_r, C_t, and C_r+C_t with respect to the incident angle θ are presented in Figure 7. It is observed that with the increase of the incident angle, C_r smoothly approaches to 1, while C_t approaches to 0 simultaneously. Different from earlier studies, both C_r and C_t display gradual and reasonable trend of curves approaching to 1 and 0 without discontinuous jumps at $\theta = 90^\circ$. This result really reveals the advantage of the present study compared to earlier ones.

5. CONCLUSIONS

Based on the concept that the wave direction remains unchanged as it permeates into the thin porous medium, the present study thus derives a new boundary condition on the interfaces of the porous wall. According to this new boundary condition an analytical solution of oblique water waves impacting on thin porous walls is obtained. The result eliminates the paradoxical phenomenon that the water wave permeates completely into the original quiescent water field when its incoming direction is parallel to the porous wall as earlier studies. And as the incident angle changes from 0° to 90°, the reflection coefficient and the transmission coefficient reasonably vary with no discontinuities when the water wave is parallel to the thin porous wall while those in the earlier studies do not.

Since the thickness of porous structures are extremely small compared to the actual wavelength, a more reasonable wave trapping analysis of harbor or coastal structure with wave direction being taken into account can be expected by applying the concept of this study.

6. ACKNOWLEDGEMENTS

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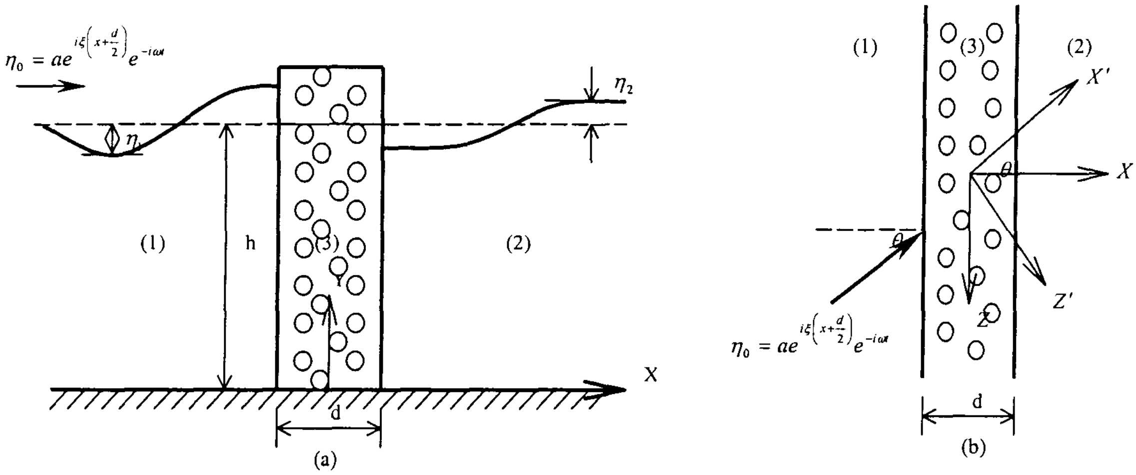

Figure 1 Schematic diagram of water wave impacting on a porous wall: (a) side view (b) top view

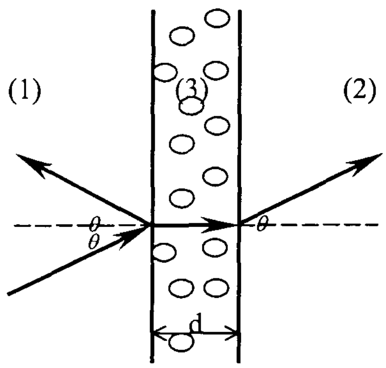

Figure 2 Schematic diagram of oblique water wave impacting on a thin porous wall in Sahoo et al. (2000)

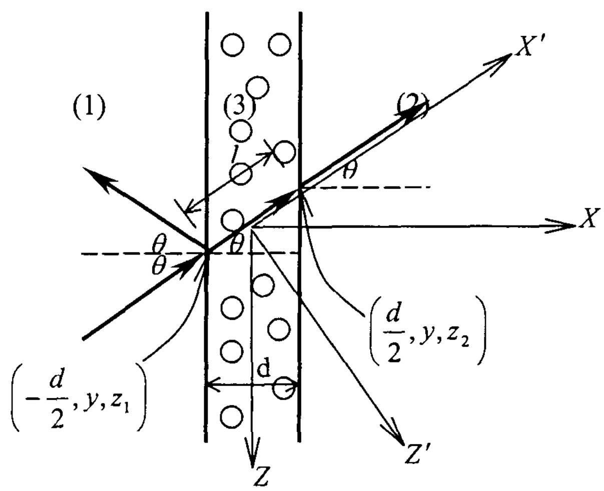

Figure 3 Schematic diagram of oblique water wave impacting on a thin porous wall in the present study

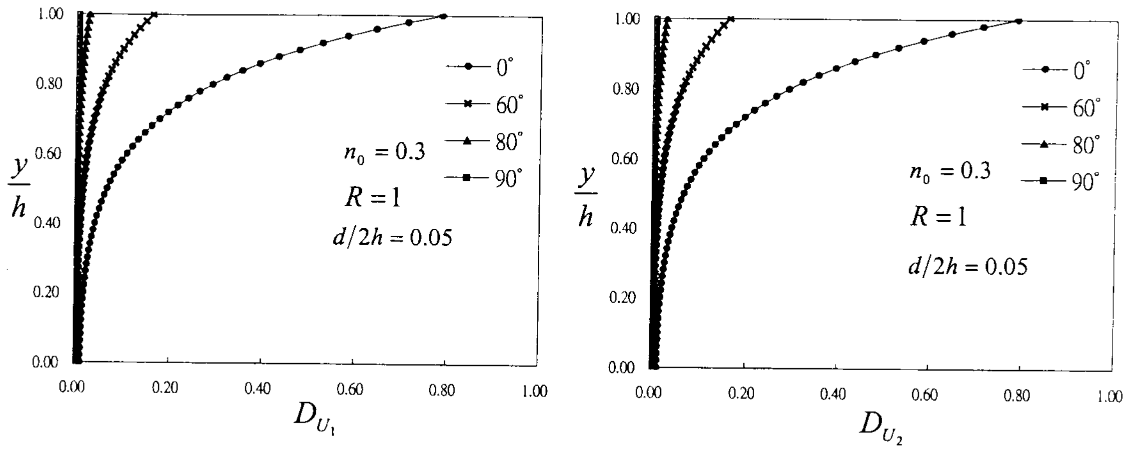

Figure 4 Distributions of normal velocity amplitudes for thin porous wall at $x = \pm d/2$: (a) D_{U_1} (b) D_{U_2}

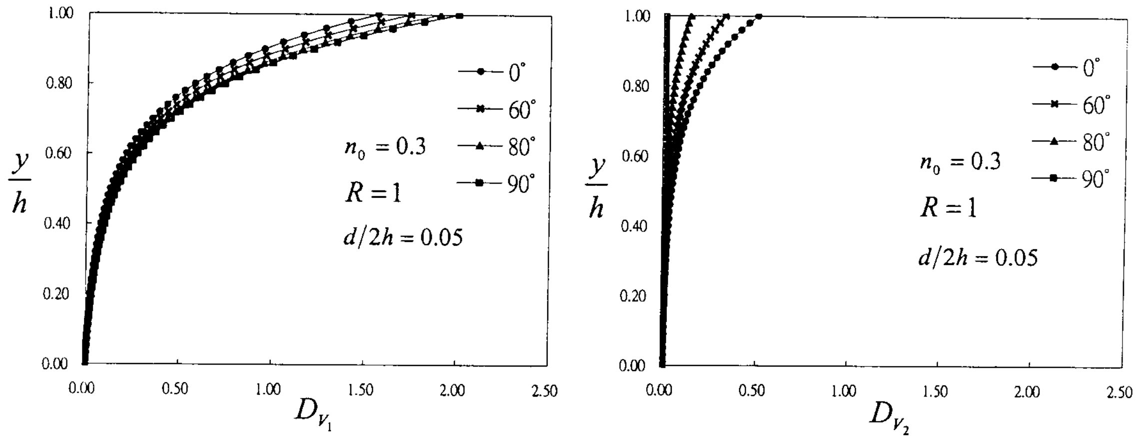

Figure 5 Distributions of tangential velocity amplitudes for thin porous wall at $x = \pm d/2$: (a) D_{V_1} (b) D_{V_2}

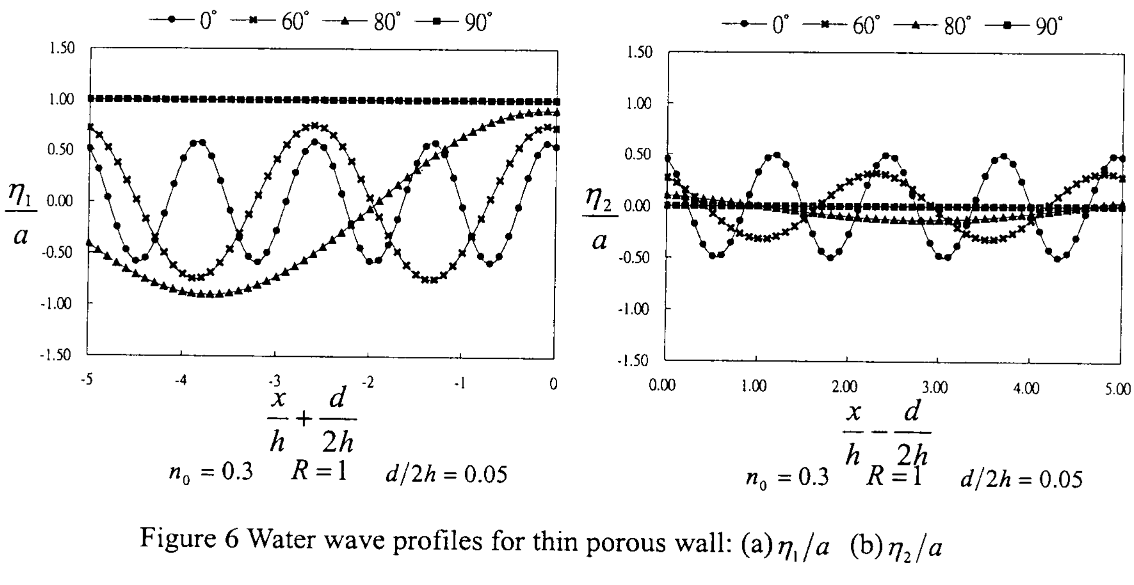

Figure 6 Water wave profiles for thin porous wall: (a) η_1/a (b) η_2/a

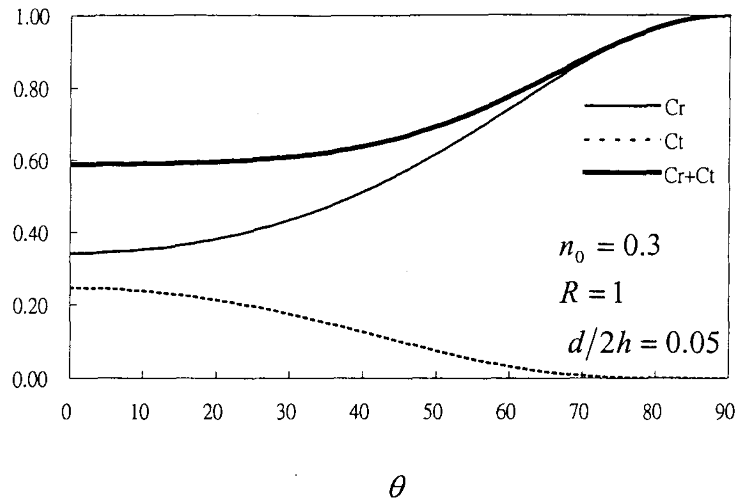

Figure 7 Variations of reflection coefficients, transmission coefficients, and their combinations with respect to incident angle θ

OBLIQUE IMPACT OF WATER WAVES ON THIN POROUS WALLS

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ABSTRACT

There is a paradoxical phenomenon in earlier studies such as Dalrymple et al. (1991), when the incoming water wave is parallel to a porous breakwater, the water wave permeates completely without regarding to how large the porosity of the porous breakwater is. For solving the problem of water waves obliquely impact on thin porous wall, a new boundary condition on the thin porous wall which can remedy the above mentioned paradoxical phenomenon is proposed based on the concept that the incident angle remains unchanged when the water wave permeates into the wall. According to this new boundary condition, an analytic solution of an oblique water wave impacting on a thin porous wall of any permeability is obtained. It is found that the above paradoxical phenomenon as the water wave is parallel to a thin porous wall disappears. And as the incident angle changes from 0° to 90°, the reflection coefficient and the transmission coefficient change smoothly with no discontinuities

when the water wave is parallel to the thin porous wall while those in the earlier investigations do not.

Keywords: *oblique water wave, thin porous wall, new boundary condition*

1. INTRODUCTION

The problem of water wave impacting on a porous wall has been studied by many investigators. Sollitt and Cross (1972) formulated and solved the problem of water wave passing through a porous medium, but due to the complex wave numbers inside the porous medium, the orthogonality of their infinite series solution is questionable. Madsen (1974) applied the method similar to Sollitt and Cross (1972) to analyze reflected and transmitted waves, but the same defect arises. Chwang (1983) used Taylor's (1956) concept of water flows through a porous screen to investigate the zero-thickness porous wavemaker problem. Later, Chwang and Li (1983), Chwang and Dong (1984), and Tyvand (1986) applied the same method in the analysis of piston-type porous wavemaker and of the problem about water wave permeating through a porous screen. Note that in Chwang (1983), the inertial reactance of flow is neglected and the porous wall is assumed zero-thickness.

Simplifying the theory of poroelasticity (see Biot (1962)) by letting the skeleton of porous medium be rigid and the fluid be incompressible, Huang (1991) obtained the governing equation of porous medium flow, similar to the one in Sollitt and Cross (1972). Chwang's (1983) study thus was extended to a finite-thickness porous medium with inertial reactance of the flow being preserved. Later, clarifying the boundary conditions for a thin porous wall with both flow inertial reactance and viscous damping, Huang et al. (1993) analyzed the wave field of a thin porous wavemaker by a simplified analytical approach.

As for an oblique water wave impacting on a porous structure, Dalrymple et al. (1991) adopted an assumption that neglects the evanescent modes of the water wave near the interfaces of the porous wall to simplify their problem. It is found that when the incoming water wave is parallel to a porous breakwater, no matter how large the porosity of the porous wall is, the water wave permeates through completely. This is indeed questionable. Actually, to the authors' knowledge, this paradoxical phenomenon can be found in all the other earlier studies of oblique wave acting on porous structures.

For the purpose of eliminating the above mentioned paradoxical phenomenon, we attempt to derive a new boundary condition for the thin porous wall impacted by an oblique wave based on the concept that the wave direction remains unchanged as it permeates into a thin porous wall in the present study. According to this new boundary condition, an analytic solution of an oblique water wave impacting on a thin porous wall of any permeability is obtained. It is found that the above paradoxical phenomenon as the water wave is parallel to a thin porous wall disappears. And as the incident angle changes from 0° to 90°, the reflection coefficient and the transmission coefficient change smoothly with no discontinuities when the water wave is parallel to the thin porous wall while those in the earlier researches do not.

2. FORMULATION OF OBLIQUE WAVE FIELD

2.1 Governing Equations

Homogeneous water wave is assumed to be inviscid, irrotational, and incompressible; and the fluid domains of the present study are shown in Figure 1.

For region (1) and region (2), we have

$$\underline{V}_j = \nabla\Phi_j, \qquad j = 1,2, \tag{1}$$

where $\underline{V}_j$ and Φ_j are velocity vectors and velocity potentials of fluid, respectively; and the subscript j denotes the flow regions (see Figure 1). And the continuity equation and linear momentum equation are

$$\nabla \cdot \underline{V}_j = 0, \qquad j = 1,2, \tag{2}$$

$$\rho\frac{\partial \underline{V}_j}{\partial t} = -\nabla p_j + \rho \underline{g}, \qquad j = 1,2. \tag{3}$$

By substituting (1) into (2), (2) becomes

$$\nabla^2 \Phi_j = 0, \qquad j = 1,2. \tag{4}$$

Then by substituting (1) into (3) and integrating (3) with respect to y, (3) becomes

$$p_j = -\rho \frac{\partial \Phi_j}{\partial t} - \rho g (y - h), \qquad j = 1,2, \tag{5a}$$

when it is not on the surface of seepage (e.g. see Muskat (1946)); and

$$p_j = 0, \qquad j = 1,2, \tag{5b}$$

when it is on the surface of seepage, where p_j is the pressure of the fluid and ρ is the density of the fluid.

For region (3) that is within the porous wall, we have

$$\underline{V}_3 = \nabla \Phi_3, \tag{6}$$

where $\underline{V}_3$ and Φ_3 are velocity vector and velocity potential of fluid inside the porous wall, respectively (see Figure 1). And, referring to Huang (1991), the continuity equation and linear momentum equation are

$$\nabla \cdot \underline{V}_3 = 0, \tag{7}$$

$$\rho \frac{\partial \underline{V}_3}{\partial t} = -\nabla p_3 + \rho \underline{g} - \frac{\mu n_0}{k} \underline{V}_3, \tag{8}$$

where μ is dynamic viscosity of fluid; n_0 and k are porosity and specific permeability of the porous wall, respectively. By substituting (6) into (7), (7) becomes

$$\nabla^2 \Phi_3 = 0. \tag{9}$$

Then by substituting (6) into (8) and integrating (8) with respect to y, (8) becomes

$$p_3 = -\rho \frac{\partial \Phi_3}{\partial t} - \rho g(y-h) - \frac{\mu n_0}{k}\Phi_3, \tag{10a}$$

when it is not on the surface of seepage; and

$$p_3 = 0, \tag{10b}$$

when it is on the surface of seepage.

2.2 Boundary Conditions

On the bottom (i.e. $y = 0$), the normal velocity equals to zero, i.e., we get

$$\frac{\partial \Phi_j}{\partial y} = 0, \qquad j = 1,2,3. \tag{11}$$

On the free surface of fluid, $y = h$ and $-\infty < x < -d/2$ for region (1) and $d/2 < x < \infty$ for region (2), we have the conventional kinematic and dynamic boundary conditions as

$$\frac{\partial \Phi_j}{\partial y} = \frac{\partial \eta_j}{\partial t}, \qquad j = 1,2, \tag{12}$$

$$\frac{\partial \Phi_j}{\partial t} + g\eta_j = 0, \qquad j = 1,2, \tag{13}$$

where η_j is the vertical deviation of water surface from the mean free surface. By combining (12) with (13), we get

$$\frac{\partial^2 \Phi_j}{\partial t^2} + g\frac{\partial \Phi_j}{\partial y} = 0, \qquad j = 1,2. \tag{14}$$

On the free surface of fluid inside the porous wall, $y = h$ and $-d/2 < x < d/2$, the kinematic and dynamic boundary conditions are

$$\frac{\partial \Phi_3}{\partial y} = n_0 \frac{\partial \eta_3}{\partial t}, \tag{15}$$

$$\frac{\partial \Phi_3}{\partial t} + \frac{\mu n_0}{\rho k}\Phi_3 + g\eta_3 = 0. \tag{16}$$

By combining (15) with (16), we get

$$\frac{\partial^2 \Phi_3}{\partial t^2} + \frac{\mu n_0}{\rho k}\frac{\partial \Phi_3}{\partial t} + \frac{g}{n_0}\frac{\partial \Phi_3}{\partial y} = 0. \tag{17}$$

At the far field, $0 < y < h$ and $x \to \pm\infty$, the radiation boundary condition is

$$\lim_{x \to \pm\infty} \Phi_j \to \text{outgoing or } 0 \qquad j = 1,2 \tag{18}$$

On the interfaces of the porous wall and the fluid, $0 < y < h$ and $x = \pm d/2$, the continuities of mass flux and pressure are considered. From (1) and (6), we have

$$\frac{\partial \Phi_j}{\partial x} = n_0 \frac{\partial \Phi_3}{\partial x}, \qquad j = 1,2, \tag{19}$$

and according to (5) and (10) with surface of seepage effect being ignored for convenience, it is obtained that

$$\frac{\partial \Phi_j}{\partial t} = \frac{\partial \Phi_3}{\partial t} + \frac{\mu n_0}{\rho k}\Phi_3, \qquad j = 1,2. \tag{20}$$

2.3 Boundary Value Problem

Since the present study focuses on the problem of periodic linear water wave, we may introduce a time factor $e^{-i\omega t}$ into the time-dependent variables, where ω is the angular frequency of water wave. And according to Snell's law, in order to satisfy the matching conditions at each vertical interface, we can write

$$\Phi_j(x,y,z,t) = \mathrm{Re}\left[\phi_j(x,y)e^{i(k_0 \sin\theta z - \omega t)}\right], \qquad j = 1,2,3, \tag{21}$$

so that the z-variation of the solution in each region must be the same. In addition, a flow-efficiency parameter is defined as

$$R = \frac{reactance\ force}{resistance\ force} = \frac{\rho\omega k}{\mu n_0}, \tag{22}$$

which represents the capability for fluid to flow through the porous medium (see Huang (1991)). After getting rid of $e^{-i\omega t}$, the time-independent boundary value problem can be constructed by governing equations (4) and (9) with boundary conditions (11), (14), (17), (18), (19), and (20).

3. SOLUTION

3.1 Boundary Condition on Thin Porous Wall

When the porous wall is extremely thin (i.e., d in Figure 1 is small), only the flow fields of region (1) and region (2) need to be considered, i.e. we don't need to solve governing equation (9) with boundary conditions (11) for $j = 3$, (17), (19), and (20). Therefore, the original boundary conditions (19) and (20) on the interfaces ($x = \pm d/2$) should be rectified and combined to obtain a new boundary condition that fits the thin porous wall and is independent of ϕ_3.

We will use Sahoo et al. (2000) as an example to introduce how the conventional way is dealing with the problem of oblique waves impact on a thin porous wall. Like other earlier studies, the boundary condition on the thin porous wall given in Sahoo et al. (2000) is obtained under the assumption that, regardless of the incident angles, the wave always permeates into the porous wall in normal direction (see Figure 2). For this inference, even if the incoming water wave in region (1) is parallel to the porous wall (i.e. $\theta = 90^\circ$), there still exists a normal wave inside the porous wall and which then permeates completely into region (2). This phenomenon is indeed paradoxical and is obviously against our physical expectation when the viscous effect of water wave is so little and hence negligible. However, as the present authors know, this paradoxical result of $\theta = 90^\circ$ happens in all the earlier studies of relative problems; e.g. Darymple et al. (1991), Sahoo et al. (2000), etc.

After many thoughts, we find that since the wavelength is much longer than the thickness of the porous wall, there should be no diffraction existing. And because the water that transports the wave inside the porous wall is still the same as those in regions (1) and (2), no refraction occurs either. Hence, a new boundary condition on the thin porous wall which can remedy the above mentioned paradoxical phenomenon as $\theta = 90^\circ$ is proposed based on the concept that the incident angle θ remains unchanged when the water wave permeates into region (3) as shown in Figure 3.

Before the new boundary condition is derived, we may see from Figure 3 that because the flow in region (3) follows Darcy's law of porous media flow (i.e. the velocity vector is linearly proportional to the pressure gradient inside the porous wall), when the incoming water wave is parallel to the thin porous wall (i.e. $\theta = 90^\circ$), as long as the viscous effect of fluid on the wall is negligible, there will be no water wave penetrating through the porous wall and appearing in region (2) anymore.

We now start to derive our new boundary condition on the thin porous wall. For the oblique incoming wave as shown in Figure 3, the flow is in the direction of x' axis and passes through $\underline{x}_1\left(-\frac{d}{2}, y, z_1\right)$ and $\underline{x}_2\left(\frac{d}{2}, y, z_2\right)$ on the wall. Therefore, it is necessary to take velocity vectors at $\underline{x} = \underline{x}_1$ and $\underline{x} = \underline{x}_2$. And according to Huang et al. (1993), it is quite acceptable that the velocities are identical from the entrance to the exit alone a streamline inside the thin porous wall. Then, referring to Huang et al. (1993), we get

$$\left.\frac{\partial \phi_3}{\partial x'}\right|_{\left(-\frac{d}{2}, y, z_1\right)} = \left.\frac{\partial \phi_3}{\partial x'}\right|_{\left(\frac{d}{2}, y, z_2\right)} = \frac{1}{l}\left(\phi_3\big|_{\left(\frac{d}{2}, y, z_2\right)} - \phi_3\big|_{\left(-\frac{d}{2}, y, z_1\right)} \right). \tag{23}$$

Substituting $\frac{\partial \phi_3}{\partial x'} = \frac{1}{\cos\theta}\frac{\partial \phi_3}{\partial x}$ and $l = \frac{d}{\cos\theta}$ into (23), it is obtained that

$$\left.\frac{\partial \phi_3}{\partial x}\right|_{\left(-\frac{d}{2}, y, z_1\right)} = \left.\frac{\partial \phi_3}{\partial x}\right|_{\left(\frac{d}{2}, y, z_2\right)} = \frac{1}{d}\left(\phi_3\big|_{\left(\frac{d}{2}, y, z_2\right)} - \phi_3\big|_{\left(-\frac{d}{2}, y, z_1\right)} \right)\cos^2\theta. \tag{24}$$

On the other hand, referring to the continuous mass flux condition and continuous pressure condition on the interfaces (i.e. at $\underline{x} = \underline{x}_1$ and $\underline{x} = \underline{x}_2$), (19) can be rewritten as

$$\left.\frac{\partial \phi_1}{\partial x}\right|_{\left(-\frac{d}{2},y,z_1\right)} = n_0 \left.\frac{\partial \phi_3}{\partial x}\right|_{\left(-\frac{d}{2},y,z_1\right)}, \tag{25}$$

$$\left.\frac{\partial \phi_2}{\partial x}\right|_{\left(\frac{d}{2},y,z_2\right)} = n_0 \left.\frac{\partial \phi_3}{\partial x}\right|_{\left(\frac{d}{2},y,z_2\right)}, \tag{26}$$

and (20) can be expressed as

$$iR\phi_1\big|_{\left(-\frac{d}{2},y,z_1\right)} = (iR-1)\phi_3\big|_{\left(-\frac{d}{2},y,z_1\right)}, \tag{27}$$

$$iR\phi_2\big|_{\left(\frac{d}{2},y,z_2\right)} = (iR-1)\phi_3\big|_{\left(\frac{d}{2},y,z_2\right)}. \tag{28}$$

(28) minus (27) gives

$$iR\left(\phi_2\big|_{\left(\frac{d}{2},y,z_2\right)} - \phi_1\big|_{\left(-\frac{d}{2},y,z_1\right)}\right) = (iR-1)\left(\phi_3\big|_{\left(\frac{d}{2},y,z_2\right)} - \phi_3\big|_{\left(-\frac{d}{2},y,z_1\right)}\right). \tag{29}$$

Now, substituting (25), (26), and (29) into (24) and getting rid of ϕ_3, the final form of boundary conditions on the interfaces are obtained as

$$\left.\frac{\partial \phi_1}{\partial x}\right|_{\left(-\frac{d}{2},y,z_1\right)} = \left.\frac{\partial \phi_2}{\partial x}\right|_{\left(\frac{d}{2},y,z_2\right)} = \frac{n_0}{d}\frac{iR}{1-iR}\left(\phi_1\big|_{\left(-\frac{d}{2},y,z_1\right)} - \phi_2\big|_{\left(\frac{d}{2},y,z_2\right)}\right)\cos^2\theta. \tag{30}$$

(30) is the boundary condition suitable for the thin porous wall impacted by the oblique water wave. It is obviously observed from (30) and (25), (26) that when $\theta = 90^\circ$, there will be no normal velocity on each side of the porous wall, which satisfies Darcy's law and indeed agrees to our physical expectation.

From Figure 3, we find $z = z_1 - \frac{d}{2}\tan\theta = z_2 + \frac{d}{2}\tan\theta$. Hence Equation (30) can be written as

$$\left.\frac{\partial \phi_1}{\partial x}\right|_{\left(-\frac{d}{2},y,z\right)} = \left.\frac{\partial \phi_2}{\partial x}\right|_{\left(\frac{d}{2},y,z\right)} = \frac{n_0}{d}\frac{iR}{1-iR}\left(\phi_1\Big|_{\left(-\frac{d}{2},y,z\right)} - \phi_2\Big|_{\left(\frac{d}{2},y,z\right)}\right)\cos^2\theta \tag{30a}$$

as a good approximation when d is extremely small. Equation (30a) is the boundary condition that we are going to use. The boundary condition on the thin porous wall which is used in Sahoo et al. (2000) is similar to (30a) but without $\cos^2\theta$.

3.2 Analytical Solution

By applying (21) and (22) into the governing equation (4) and boundary conditions (11), (14), (18), and (30a), the analytical solution is easily obtained as

$$\phi_1(x,y,z) = \phi_0\left[e^{i\xi\left(x+\frac{d}{2}\right)} + A_0 e^{-i\xi\left(x+\frac{d}{2}\right)}\right]\cosh(k_0 y)e^{i\zeta z}, \tag{31}$$

$$\phi_2(x,y,z) = \phi_0\left[e^{i\xi\left(x-\frac{d}{2}\right)} - A_0 e^{i\xi\left(x-\frac{d}{2}\right)}\right]\cosh(k_0 y)e^{i\zeta z}, \tag{32}$$

with

$$\phi_0 = -\frac{iag}{\omega\cosh(k_0 h)}, \tag{33}$$

$$A_0 = \frac{\xi d(1-iR)}{\xi d(1-iR) + 2n_0 R\cos^2\theta}, \tag{34}$$

where $\xi = k_0\cos\theta$, $\zeta = k_0\sin\theta$ and k_0 satisfies the dispersion relation

$$\omega^2 - gk_0 \tanh(k_0 h) = 0. \tag{35}$$

It is clearly found that when $\theta = 90^\circ$,

$$\phi_1(x, y, z) = 2\phi_0 \cosh(k_0 y) e^{ik_0 z}, \tag{36}$$

$$\phi_2(x, y, z) = 0. \tag{37}$$

(37), again, indicates that when the incoming water wave is parallel to the porous wall, region (2) remains quiescent and the wave field does not appear. Hence the paradox which occurs when the parallel water wave acting on the thin porous wall is eliminated.

3.3 Physical Properties

Since the z-variation of analytical solutions are the same, we may study all the physical properties in the condition that $z = 0$ for representation.

By (5), the nondimensional perturbed pressures on the interfaces ($x = \pm d/2$) are obtained as

$$\left.\frac{p_1}{\rho\omega^2 ah}\right|_{x=-\frac{d}{2}} = A_{p_1} \cos\omega t + B_{p_1} \sin\omega t = D_{p_1} \cos(\theta_{p_1} - \omega t), \tag{38}$$

where

$$A_{p_1} = -\frac{1}{\omega ah} \mathrm{Im}(\phi_0)[1 + \mathrm{Re}(A_0)]\cosh(k_0 y), \tag{39}$$

$$B_{p_1} = -\frac{1}{\omega ah} \mathrm{Im}(\phi_0)\mathrm{Im}(A_0)\cosh(k_0 y), \tag{40}$$

$$D_{p_1} = \sqrt{A_{p_1}{}^2 + B_{p_1}{}^2}, \tag{41}$$

$$\theta_{p_1} = \tan^{-1}\left(\frac{B_{p_1}}{A_{p_1}}\right); \tag{42}$$

$$\left.\frac{p_2}{\rho\omega^2 ah}\right|_{x=\frac{d}{2}} = A_{p_2}\cos\omega t + B_{p_2}\sin\omega t = D_{p_2}\cos(\theta_{p_2} - \omega t), \tag{43}$$

where

$$A_{p_2} = -\frac{1}{\omega ah}\operatorname{Im}(\phi_0)[1 - \operatorname{Re}(A_0)]\cosh(k_0 y), \tag{44}$$

$$B_{p_2} = \frac{1}{\omega ah}\operatorname{Im}(\phi_0)\operatorname{Im}(A_0)\cosh(k_0 y), \tag{45}$$

$$D_{p_2} = \sqrt{A_{p_2}{}^2 + B_{p_2}{}^2}, \tag{46}$$

$$\theta_{p_2} = \tan^{-1}\left(\frac{B_{p_2}}{A_{p_2}}\right), \tag{47}$$

where Re and Im indicate real and imaginary parts, respectively.

By (1), the nondimensional normal velocities on the interfaces ($x = \pm d/2$) are obtained as

$$\left.\frac{U_1}{-a\omega}\right|_{x=-\frac{d}{2}} = A_{U_1}\cos\omega t + B_{U_1}\sin\omega t = D_{U_1}\cos(\theta_{U_1} - \omega t), \tag{48}$$

where

$$A_{U_1} = \frac{1}{a\omega}\xi\operatorname{Im}(\phi_0)[1 - \operatorname{Re}(A_0)]\cosh(k_0 y), \tag{49}$$

$$B_{U_1} = -\frac{1}{a\omega}\xi\,\mathrm{Im}(\phi_0)\mathrm{Im}(A_0)\cosh(k_0 y), \tag{50}$$

$$D_{U_1} = \sqrt{A_{U_1}{}^2 + B_{U_1}{}^2}, \tag{51}$$

$$\theta_{U_1} = \tan^{-1}\left(\frac{B_{U_1}}{A_{U_1}}\right); \tag{52}$$

$$\left.\frac{U_2}{-a\omega}\right|_{x=\frac{d}{2}} = A_{U_2}\cos\omega t + B_{U_2}\sin\omega t = D_{U_2}\cos(\theta_{U_2} - \omega t), \tag{53}$$

where

$$A_{U_2} = \frac{1}{a\omega}\xi\,\mathrm{Im}(\phi_0)[1 - \mathrm{Re}(A_0)]\cosh(k_0 y), \tag{54}$$

$$B_{U_2} = -\frac{1}{a\omega}\xi\,\mathrm{Im}(\phi_0)\mathrm{Im}(A_0)\cosh(k_0 y), \tag{55}$$

$$D_{U_2} = \sqrt{A_{U_2}{}^2 + B_{U_2}{}^2}, \tag{56}$$

$$\theta_{U_2} = \tan^{-1}\left(\frac{B_{U_2}}{A_{U_2}}\right). \tag{57}$$

Similarly, by (1), the nondimensional tangential velocities on the interfaces ($x = \pm d/2$) are obtained as

$$\left.\frac{V_1}{-a\omega}\right|_{x=-\frac{d}{2}} = A_{V_1}\cos\omega t + B_{V_1}\sin\omega t = D_{V_1}\cos(\theta_{V_1} - \omega t), \tag{58}$$

where

$$A_{V_1} = \frac{1}{a\omega}k_0\,\mathrm{Im}(\phi_0)\mathrm{Im}(A_0)\sinh(k_0 y), \tag{59}$$

$$B_{V_1} = -\frac{1}{a\omega} k_0 \operatorname{Im}(\phi_0)[1 + \operatorname{Re}(A_0)]\sinh(k_0 y), \tag{60}$$

$$D_{V_1} = \sqrt{{A_{V_1}}^2 + {B_{V_1}}^2}, \tag{61}$$

$$\theta_{V_1} = \tan^{-1}\left(\frac{B_{V_1}}{A_{V_1}}\right); \tag{62}$$

$$\left.\frac{V_2}{-a\omega}\right|_{x=\frac{d}{2}} = A_{V_2}\cos\omega t + B_{V_2}\sin\omega t = D_{V_2}\cos(\theta_{V_2} - \omega t), \tag{63}$$

where

$$A_{V_2} = -\frac{1}{a\omega} k_0 \operatorname{Im}(\phi_0)\operatorname{Im}(A_0)\sinh(k_0 y), \tag{64}$$

$$B_{V_2} = -\frac{1}{a\omega} k_0 \operatorname{Im}(\phi_0)[1 - \operatorname{Re}(A_0)]\sinh(k_0 y), \tag{65}$$

$$D_{V_2} = \sqrt{{A_{V_2}}^2 + {B_{V_2}}^2}, \tag{66}$$

$$\theta_{V_2} = \tan^{-1}\left(\frac{B_{V_2}}{A_{V_2}}\right). \tag{67}$$

By (13), the wave profile which takes off the incoming wave η_0 in region (1) and wave profile in region (2) are obtained as

$$\left.\frac{\eta_1}{a}\right|_{y=h} = \operatorname{Re}(A_0)\cos\left(\xi\left(x + \frac{d}{2}\right) + \omega t\right) + \operatorname{Im}(A_0)\sin\left(\xi\left(x + \frac{d}{2}\right) + \omega t\right), \tag{68}$$

$$\begin{aligned}\left.\frac{\eta_2}{a}\right|_{y=h} &= \cos\left(\xi\left(x - \frac{d}{2}\right) - \omega t\right) - \operatorname{Re}(A_0)\cos\left(\xi\left(x - \frac{d}{2}\right) - \omega t\right) \\ &+ \operatorname{Im}(A_0)\sin\left(\xi\left(x - \frac{d}{2}\right) - \omega t\right).\end{aligned} \tag{69}$$

The reflection coefficient and transmission coefficient of the wave field are defined as

$$C_r = \left|\frac{\eta_1}{a}\right|^2 = \mathrm{Re}(A_0)^2 + \mathrm{Im}(A_0)^2, \tag{70}$$

$$C_t = \left|\frac{\eta_2}{a}\right|^2 = [1 - \mathrm{Re}(A_0)]^2 + \mathrm{Im}(A_0)^2. \tag{71}$$

4. RESULTS

In the present study, the so-called "thin" porous wall indicates that the thickness of the porous wall, compared to the wavelength, is extremely short. Hence, due to the property of the thin porous wall, the local phenomenon of surface of seepage (see (5), (10) and refer to Muskat (1946)) is neglected.

In the following analysis, we only apply the porosity $n_0 = 0.3$ and the nondimensional wall thickness $d/2h = 0.05$ for demonstration.

Figure 4 reveals vertical distributions of perturbed pressure amplitudes D_{p_1} and D_{p_2} of homogeneous fluid on the interfaces of the thin porous wall ($x = \pm d/2$). D_{p_1} and D_{p_2} are given by (41) and (46). Since it will be easier for the water wave to pass from region (1) to region (2) as the porous Reynolds number R grows, it is observed that D_{p_2} increases gradually when R becomes larger. And as we expect, as the incident angle increases, D_{p_2} decreases monotonically; when the incoming water wave is parallel to the porous wall (i.e. $\theta = 90^\circ$), D_{p_2} vanishes (i.e. when one side of a thin porous wall is affected by a parallel water wave, there is no water wave penetrating into the other side). This result agrees with our physical expectation when viscous effect of fluid on the wall is negligible.

Figure 5 indicates vertical distributions of normal velocity amplitudes D_{U_1} and D_{U_2} of homogeneous fluid on the thin porous wall ($x = \pm d/2$). D_{U_1} and D_{U_2} are given by (51) and (56). It is found from Figure 5 (b) that D_{U_2} shows similar trend as D_{p_2} does in Figure 4 (b) as R and θ vary. It can be seen that when the incoming water wave is parallel to the porous wall (i.e. $\theta = 90^\circ$), D_{U_2}, certainly disappears. Note that because of the mass flux continuity condition from the property of the thin porous wall, figures of D_{U_1} are identical to those of D_{U_2}.

The vertical distributions of tangential velocity amplitudes D_{V_1} and D_{V_2} of homogeneous fluid are illustrated in Figure 6. D_{V_1} and D_{V_2} are given by (61) and (66). Beside same trend as Figure 5, again, it is found that there is no tangential water wave penetrating into the original quiescent water field (region (2)) when the incoming water wave is parallel to the thin porous wall.

Figure 7 presents the wave profiles in region (1) and region (2). η_1/a and η_2/a are given by (68) and (69). By analogy with above discussions, because it is easier for the water wave to pass from region (1) to region (2) as the porous Reynolds number

R grows, it is found that η_2/a increases gradually when R becomes larger. Besides, with the increase of the incident angle, η_2/a decreases monotonically. And when the incoming water wave is parallel to the porous wall (i.e. $\theta = 90^\circ$), η_2/a vanishes. This, again, confirms our physical expectation that when one side of a thin porous wall is affected by a parallel water wave, there is no water wave penetrating into the other side (provided that the viscous effect of water is negligible).

The reflection coefficient C_r and transmission coefficient C_t are most important in the analysis of the wave field. C_r and C_t are defined by (70) and (71). Figure 8 shows C_r, C_t, and their combination C_r+C_t with respect to the variations of R. Note that there is a minimum value of C_r+C_t when R is closed to 1 and θ is small. Also notice that in general C_r+C_t is less than 1 due to the damping effect of the porous wall except when there is no wave passing through the porous wall (i.e. $\theta = 90^\circ$). Based on the definition of C_r+C_t, Figure 8 indeed demonstrates the fact that the wave energy (excluding the incoming wave) plus the damping loss due to the thin porous wall is conserved at any incident angle.

The values of C_r, C_t, and C_r+C_t with respect to the incident angle θ are presented in Figure 9. Comparing to Figure 8, Figure 9 can be seen more distinctly how C_r and C_t change owing to the variations of θ. It is observed that with the increase of the incident angle, C_r smoothly approaches to 1, while C_t approaches to 0 simultaneously. Different from earlier studies, both C_r and C_t display gradual and reasonable trend of curves approaching to 1 and 0 without discontinuous jumps at $\theta = 90^\circ$. This result really reveals the advantage of the present study compared to earlier ones.

5. CONCLUSIONS

Based on the concept that the wave direction remains unchanged as it permeates into the thin porous medium, the present study thus derives a new boundary condition on the interfaces of the porous wall. According to this new boundary condition an analytical solution of oblique water waves impacting on thin porous walls is obtained. The result eliminates the paradoxical phenomenon that the water wave permeates completely into the original quiescent water field when its incoming direction is parallel to the porous wall as earlier studies. And as the incident angle changes from 0° to 90°, the reflection coefficient and the transmission coefficient reasonably vary with no discontinuities when the water wave is parallel to the thin porous wall while those in the earlier studies do not.

Since the thickness of porous structures are extremely small compared to the actual wavelength, a more reasonable wave trapping analysis of harbor or coastal structure with wave direction being taken into account can be expected by applying the concept of this study.

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NOTATION

C_r : reflection coefficient

C_t : transmission coefficient

D_{p_j} : nondimensional amplitude of perturbed pressure in region j; $j = 1,2$

D_{U_j} : nondimensional amplitude of normal velocity in region j; $j = 1,2$

D_{V_j} : nondimensional amplitude of tangential velocity in region j; $j = 1,2$

d : thickness of porous wall

g : gravitational acceleration

h : undisturbed water depth

k : specific permeability of porous medium

k_0 : wave number of propagating wave

n_0 : porosity of porous medium

p_j : perturbed pressure in region j; $j = 1,2,3$

R : porous Reynolds number

U_j : normal velocity in region j; $j = 1,2$

V_j : tangential velocity in region j; $j = 1,2$

Φ_j : velocity potential of flow in region j; $j = 1,2,3$

ϕ_j : velocity potential of flow in region j; $j = 1,2,3$

$$\Phi_j(x,y,z,t) = \mathrm{Re}\left[\phi_j(x,y)e^{i(k_0 \sin\theta z - \omega t)}\right]$$

η_0 : wave profile of incoming wave

η_1 : wave profile in region (1) with incoming wave being excluded

η_2 : wave profile in region (2)

μ : dynamic viscosity of fluid

θ : angle of incidence

ρ : density of fluid

ω : angular frequency of water wave

ξ : modified wave number of propagating wave; $\xi = k_0 \cos\theta$

ζ : modified wave number of propagating wave; $\zeta = k_0 \sin\theta$

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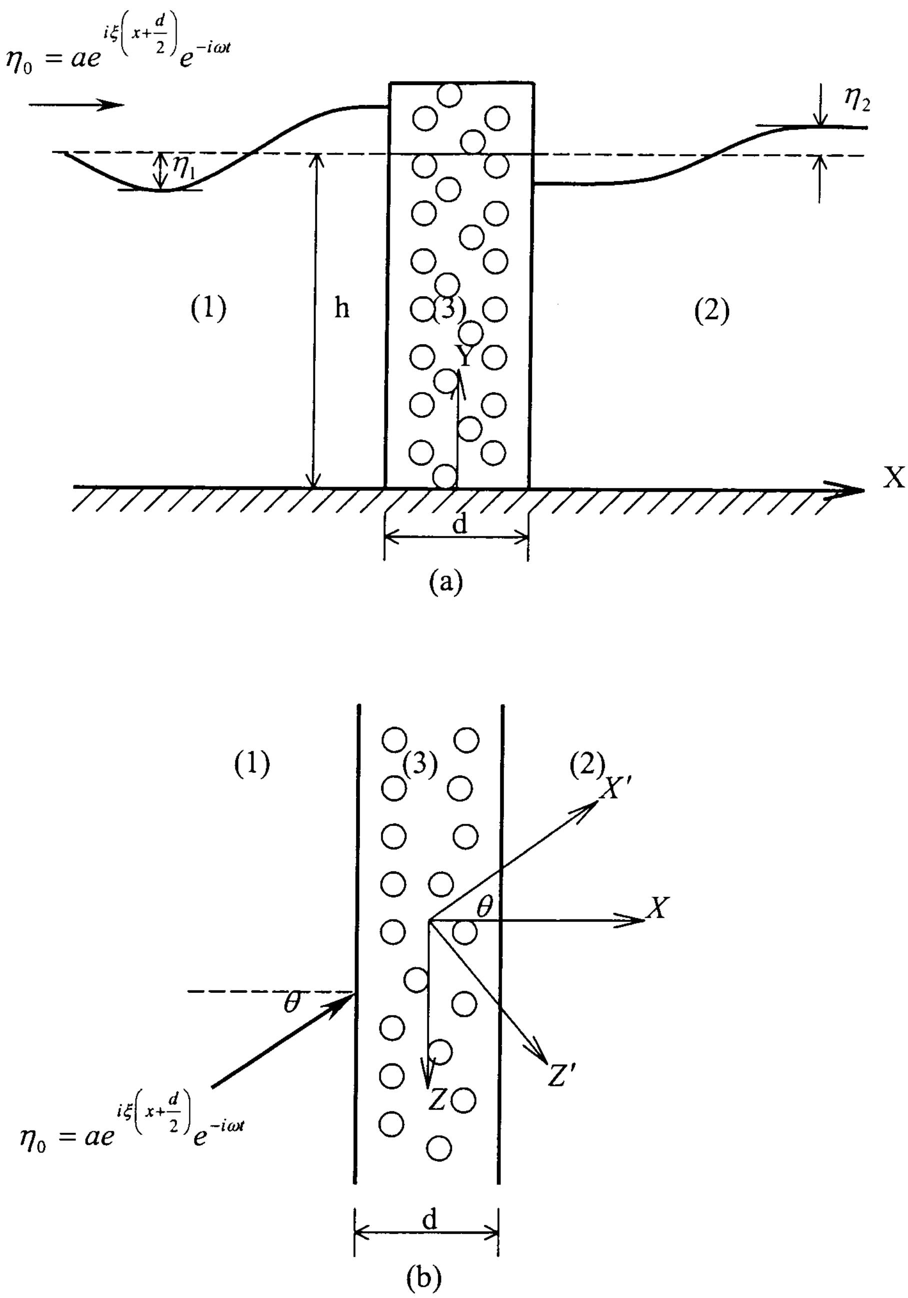


Figure 1 Schematic diagram of water wave impacting on a porous wall: (a) side view (b) top view

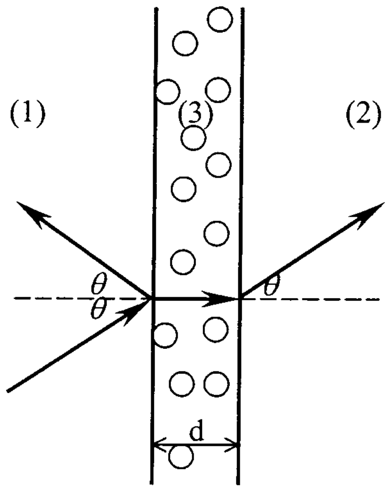


Figure 2 Schematic diagram of oblique water wave impacting on a thin porous wall in Sahoo et al. (2000)

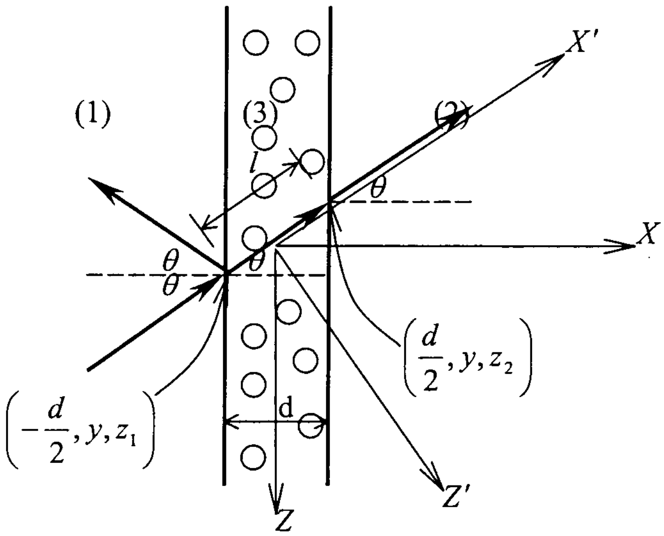
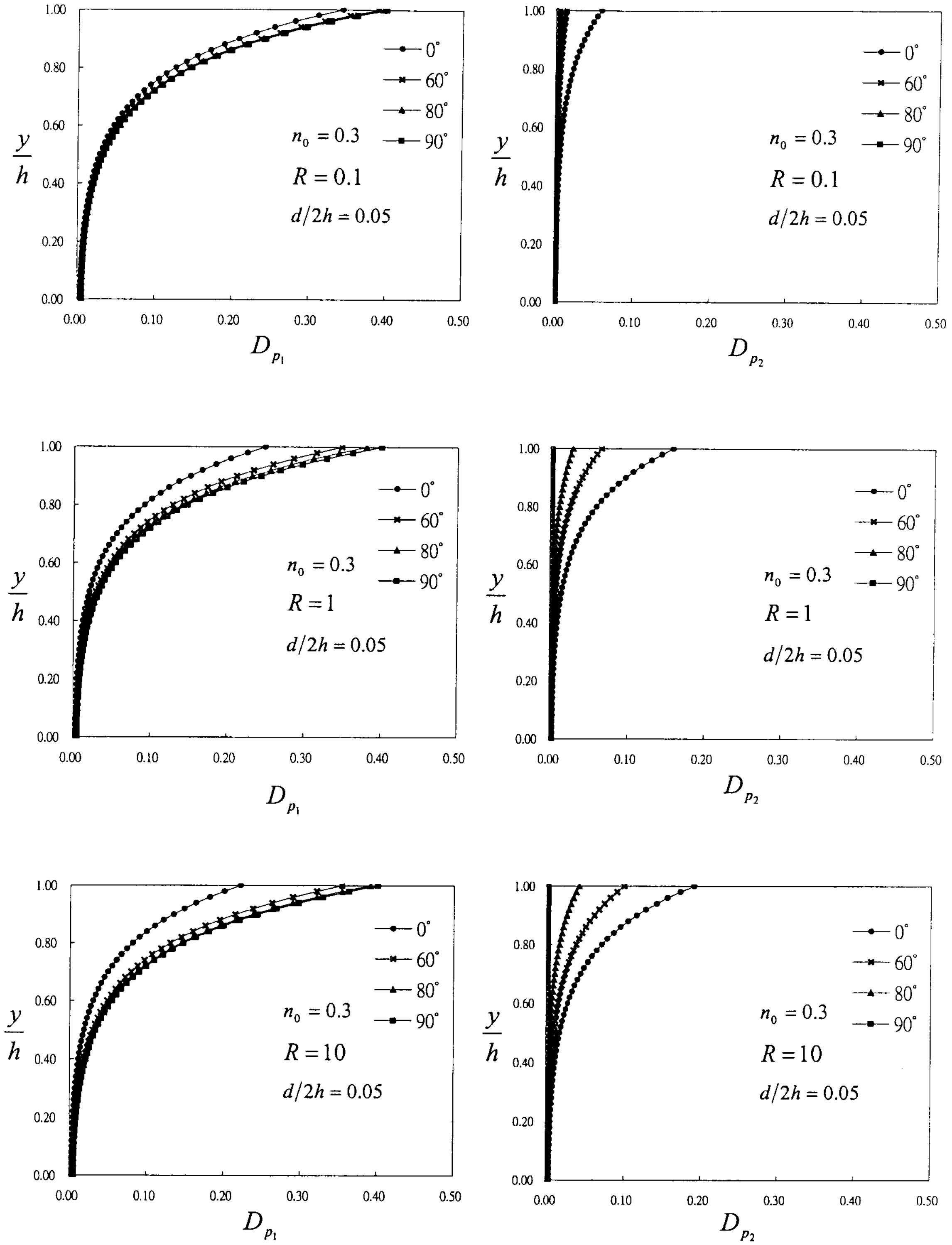
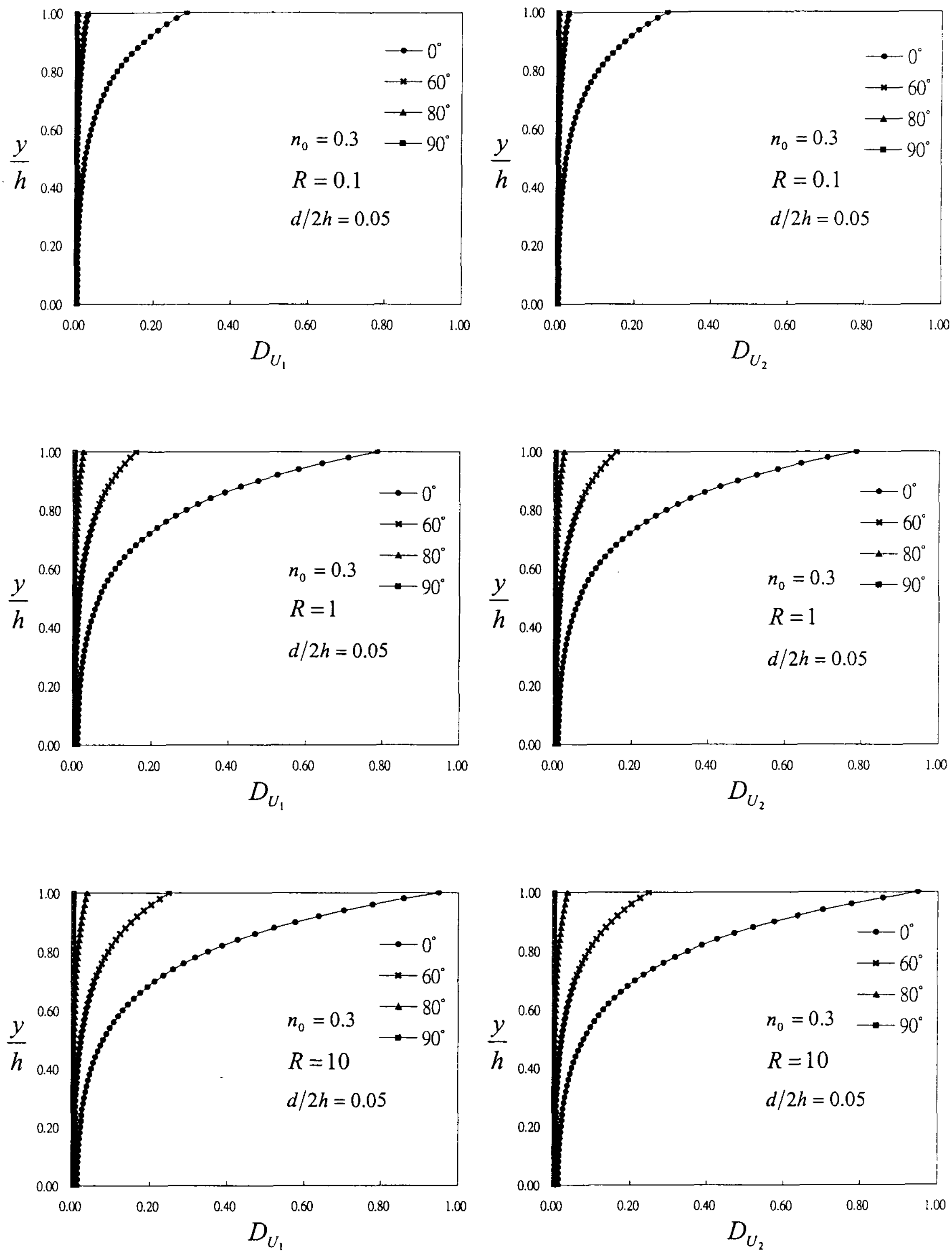


Figure 3 Schematic diagram of oblique water wave impacting on a thin porous wall in the present study



(a) (b)

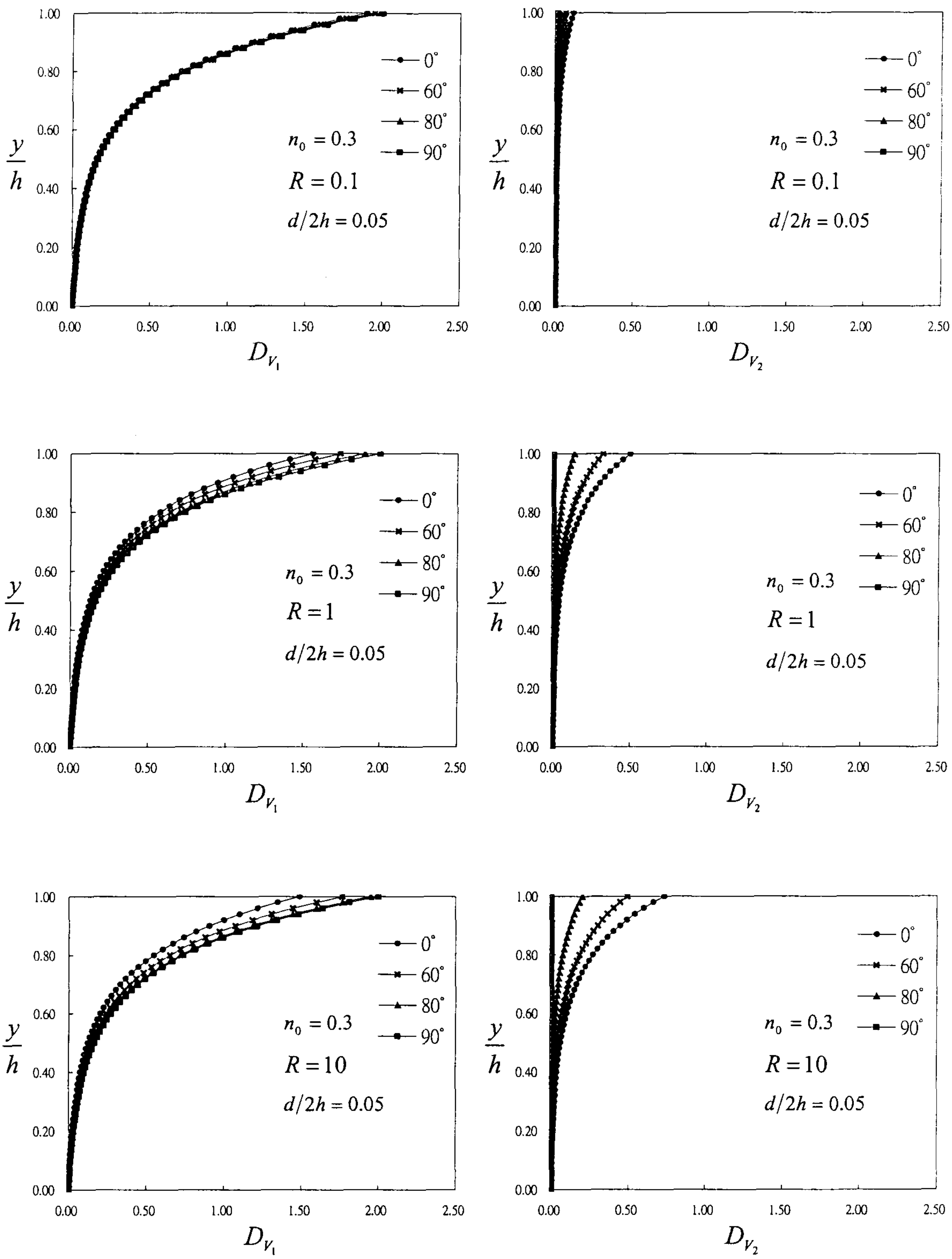
Figure 4 Distributions of perturbed pressure amplitudes for thin porous wall at $x = \pm d/2$: (a) D_{p_1} (b) D_{p_2}



(a) (b)

Figure 5 Distributions of normal velocity amplitudes for thin porous wall

at $x = \pm d/2$: (a) D_{U_1} (b) D_{U_2}



(a) (b)

Figure 6 Distributions of tangential velocity amplitudes for thin porous wall at $x = \pm d/2$: (a) D_{V_1} (b) D_{V_2}

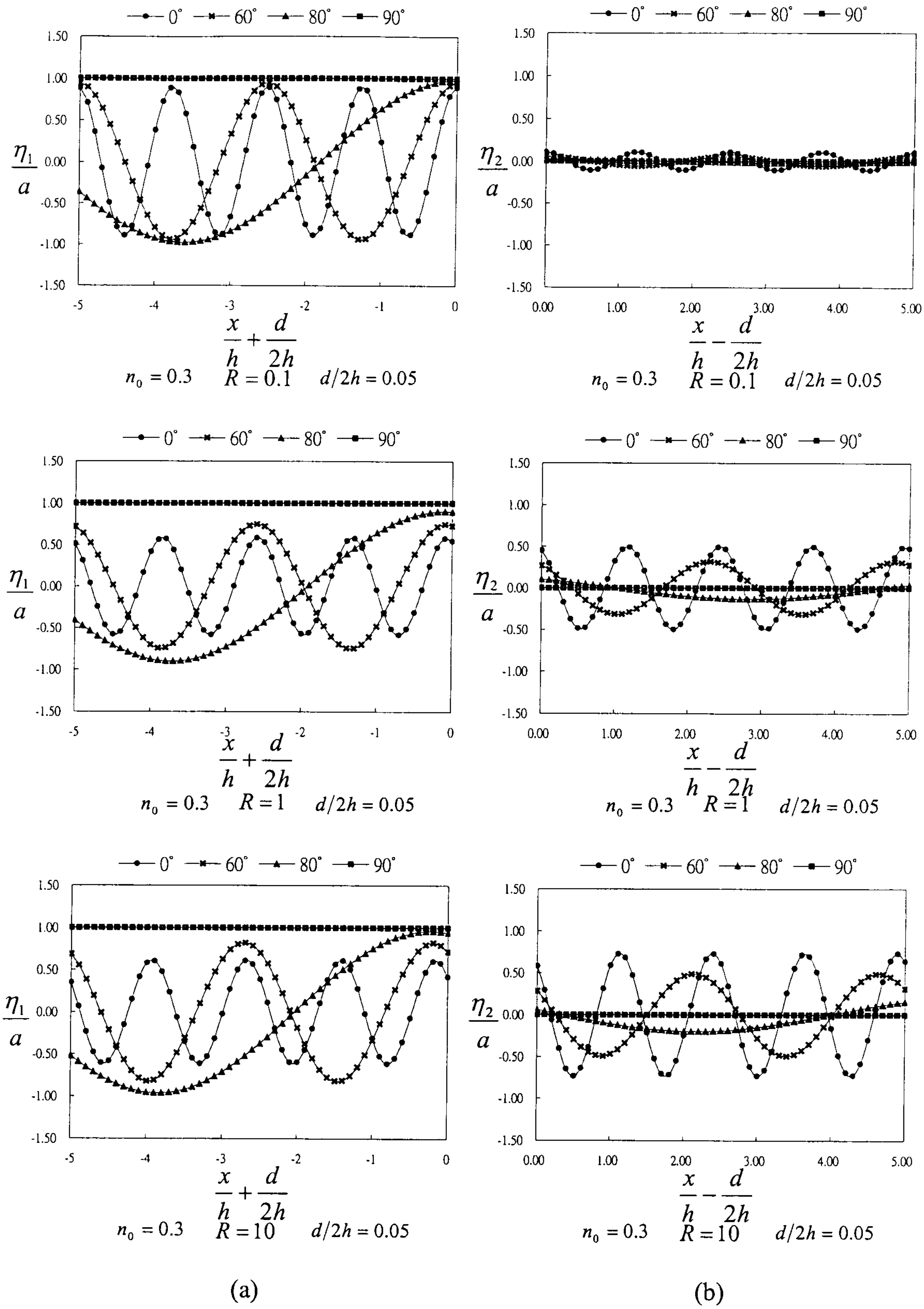


Figure 7 Water wave profiles for thin porous wall: (a) η_1/a (b) η_2/a

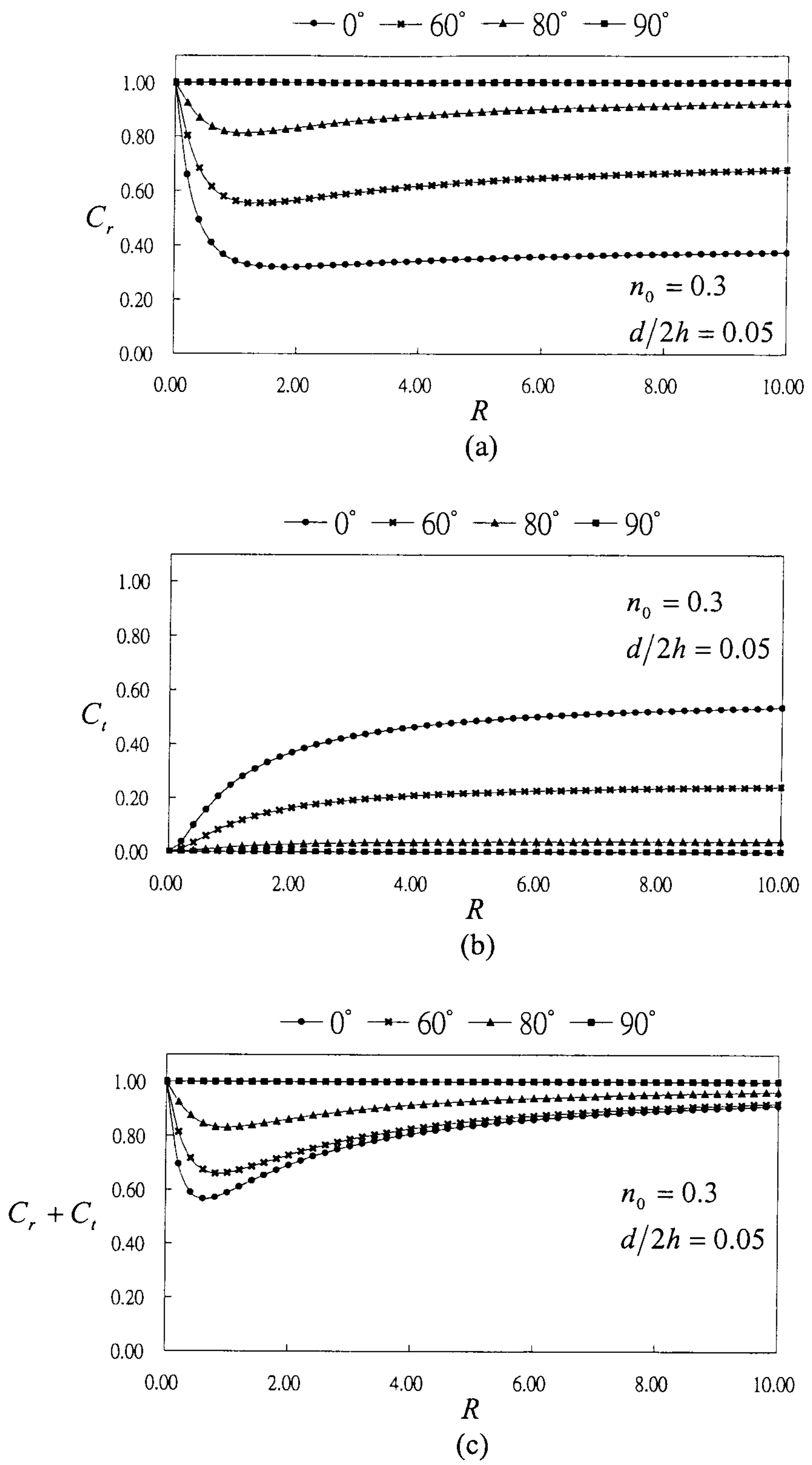


Figure 8 Variations of reflection coefficients, transmission coefficients, and their combinations with respect to porous Reynolds number R : (a) C_r (b) C_t (c) $C_r + C_t$

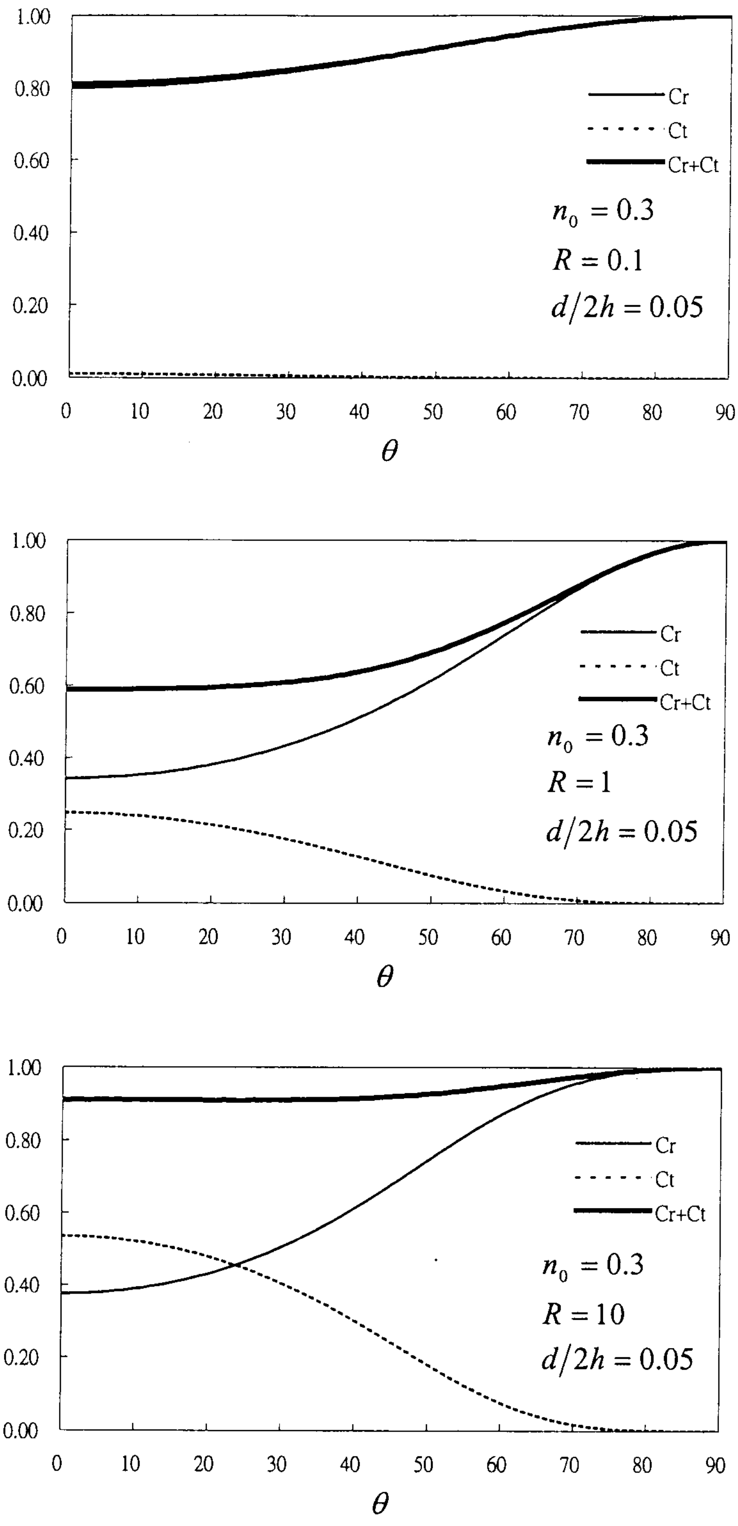


Figure 9 Variations of reflection coefficients, transmission coefficients, and their combinations with respect to incident angle θ