行政院國家科學委員會專題研究計畫成果報告

雙線性控制系統

Bilinear System Control

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中文摘要

本計畫主旨在研究雙線性系統的控制設計。傳統的雙線性系統都採用平方迴饋控制，以達成漸近穩定。本計畫則研發出一種新的非線性控制以達成指數穩定。此舉相當大程度地加快了系統收斂速度。

關鍵字: 雙線性控制、平方迴饋控制、非線性控制、全域穩定。

Abstract

For a bilinear system that is open-loop neutrally stable, a quadratic state feedback control has been proposed to ensure global asymptotical stability of the closed-loop system. In this project, a new nonlinear control is constructed so that the closed-loop system is not only asymptotically stable but also *exponentially* stable. The new control results in a much faster state convergence rate than the quadratic control; furthermore, it can be applied to systems with tight saturation limits on the control input.

Keywords: bilinear system, quadratic control, nonlinear control, exponential stability, global stability, saturation.

1. Introduction

In this project, we consider the control design for a bilinear system

$$\dot{x}(t) = Ax(t) + u(t)Nx(t), \qquad x(0) = x_0, \tag{1}$$

where $x(t) \in R^n$ is the system state vector, $u(t)$ is a scalar control input, and $A \in R^{n\times n}$ and $N \in R^{n\times n}$ are constant square matrices. It is assumed that there exists is a positive definite matrix Q such that

$$A^TQ + QA = 0; \tag{2}$$

in other words, the open-loop system is neutrally stable [8]. Furthermore, the pair (A, N) satisfies the following controllability assumption [9]: there exists an integer $m(\geq n-1)$ such that

$$span\{ad^k(A,N)x_0, k = 0,1,2,...,m\} = R^n \tag{3}$$

for any *nonzero* x_0 in R^n, where $ad^k(A,N)$'s are defined recursively by, $k = 0,1,2....$,

$$\begin{aligned} ad^0(A,N) &= N, \\ ad^{k+1}(A,N) &= A \cdot ad^k(A,N) - ad^k(A,N) \cdot A, \end{aligned}$$

Conventionally, quadratic feedback control [3,6,7] has been proposed for the stabilization of the system (1):

$$u(t) = -x^T(t)QNx(t), \tag{4}$$

which ensures global asymptotic stability of the closed-loop system. However, Quinn [5] has shown that the controlled system is *not* exponentially stable, and the state converges as

$$\|x(t)\| \sim \frac{1}{\sqrt{t}}. \tag{5}$$

The objective of this project is to develop a new nonlinear control which stabilizes the closed-loop system *globally* and most importantly *exponentially*. The exponential stability results in a much faster time response of the system state than in (5); furthermore, it enhances the robustness of the controlled system [1].

2. Control Design

The proposed nonlinear control is as follows:

$$u(t) = \begin{cases} -\rho \frac{x^T(t)}{\|x(t)\|} QN \frac{x(t)}{\|x(t)\|} & , \quad x(t) \neq 0, \\ 0 & , \quad x(t) = 0, \end{cases} \tag{6}$$

where ρ is a positive control gain, and Q is as in (2). Notice that the control (6) is uniformly bounded for whatever values of the state $x(t)$:

$$|u(t)| \leq \rho\ n\ q, \quad \forall t > 0, \tag{7}$$

where q and n are respectively the matrix norms of Q and N. If the bilinear system (1) is subject to the control constraint

$$|u(t)| \leq u_{max},$$

the control gain ρ in (6) will have to be chosen within the range:

$$\rho \in (0, \frac{nq}{u_{max}}). \tag{8}$$

3. Stability Analysis

Lemma 1 : If the system (1) satisfies the controllability assumption (3), and there exists a constant vector x_0 such that, for all $t \in [kT, kT+T)$,

$$x_0^T e^{A^T(t-kT)} Q N e^{A(t-kT)} x_0 \equiv 0, \tag{9}$$

then x_0 must be the null vector.

Proof : See reference [10].

Given any time interval length $T > 0$, define a scalar function $B(\cdot) : S \rightarrow R$ for the controlled system (1) and (6), where S is the unit sphere in R^n and $x(kT) \neq 0$,

$$B(\frac{x(kT)}{||x(kT)||}) \triangleq \int_{kT}^{(k+1)T} \left(\frac{x^T(t)}{||x(t)||} QN \frac{x(t)}{||x(t)||} \right)^2 dt. \tag{10}$$

Note that given Q and N, the function $B(\cdot)$ is determined by the normalized closed-loop trajectory $x(t)/||x(t)||$, $t \in [kT, kT+T)$, which is uniquely determined by its initial condition $x(kT)/||x(kT)||$. Therefore, $B(\cdot)$ in (11) is defined as a function of the initial condition $x(kT)/||x(kT)||$.

Lemma 2 : There exists some positive constant β such that

$$\inf \left[B(\frac{x(kT)}{||x(kT)||}) \right] = \beta > 0, \tag{11}$$

where the *inf(imum)* is taken over all $x(kT)/||x(kT)|| \in S$ (that is, over all $x(kT) \neq 0$).

Proof : See reference [10].

One can now prove the global exponential stability of the controlled bilinear system.

Theorem : Consider the bilinear system (1) and the nonlinear control (6) subject to the constraint (8). Given any initial condition, the controlled state $x(t)$ converges to zero *exponentially*.

Proof : Define a Lyapunov function candidate

$$V(t) = x^T(t) Q x(t),$$

where Q is as in (2). Notice that

$$x^T(t) Q x(t) \leq \bar{\lambda} ||x||^2, \tag{12}$$

where $\bar{\lambda}$ is the maximum eigenvalue of the positive definite matrix Q. The time derivative of $V(t)$ along (1) and (6) is given by

$$\begin{aligned} \dot{V}(t) &= x^T(t)(A^T Q + QA)x(t) + 2x^T(t) QNx(t) u(t) \\ &= -2\rho \left(\frac{x^T(t)}{||x(t)||} QN \frac{x(t)}{||x(t)||} \right)^2 ||x(t)||^2 \leq 0. \end{aligned} \tag{13}$$

Since $V(t)$ is non-increasing, one has

$$V(kT+T) \leq V(t), \quad \forall t \in [kT, kT+T). \tag{14}$$

Integrating the equation (17) from kT to $(k+1)T$ yields

$$\begin{aligned} & V(kT+T) - V(kT) \\ \leq\ & -2\frac{\rho}{\bar{\lambda}} V(kT+T) \int_{kT}^{(k+1)T} \left(\frac{x^T(t)}{||x(t)||} QN \frac{x(t)}{||x(t)||} \right)^2 dt, \\ \leq\ & -2\frac{\rho\beta}{\bar{\lambda}} V(kT+T), \end{aligned}$$

where the first inequality results from (16) and (18), and the second from (12) in Lemma 2. Re-arranging the last inequality gives

$$V(kT+T) \leq \frac{1}{1 + 2\rho\beta/\bar{\lambda}} V(kT); \tag{15}$$

proving that the Lyapunov function $V(kT)$ decreases exponentially to zero as k approaches infinity, and so does $x(kT)$. Q.E.D.

Remark : Notice that the Theorem holds for whatever value of the control saturation limit $u_{max} > 0$ as long as the control gain ρ satisfies (8). As a result, even if the control actuator can provide only a small amount of energy (i.e., a tight saturation limit u_{max}), the proposed control can still stabilize the system globally and exponentially. Such a property is not shared by the conventional quadratic control (4), for which the amount of energy required is proportional to the square of $||x(t)||$. Hence, large control input is required by the quadratic control (4) if $x(t)$ is far from the origin.

4. Numerical Example

Consider the bilinear system (1) with

$$A = \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}, \qquad N = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}.$$

and the initial condition $x^T(0) = [5, -2]$. Figure 1 shows the state response of the system with the conventional quadratic control (4) with $Q = 3I$, and Figure 2 the state response with the new nonlinear control (5) with $\rho = 3$, $Q = I$. It is obvious that the new nonlinear control results in a much faster time response since the state now decays exponentially.

5. Conclusions

In this project, a new nonlinear control different from the conventional quadratic feedback control is proposed to stabilize a homogeneous-in-the-state bilinear system. The new control results in exponential stability of the closed-loop system, and hence a much faster time response than with the quadratic control.

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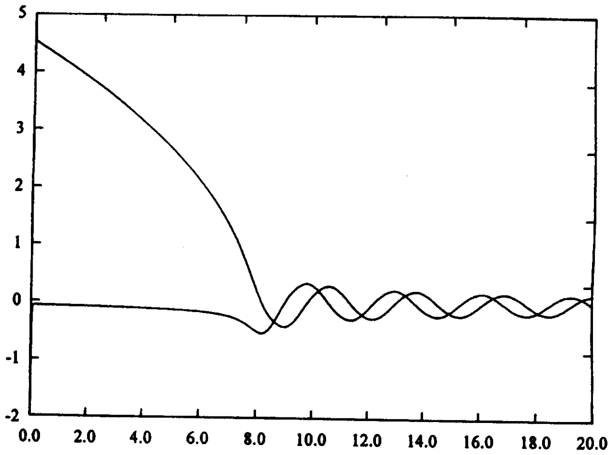

Fig. 1. State response with quadratic control.

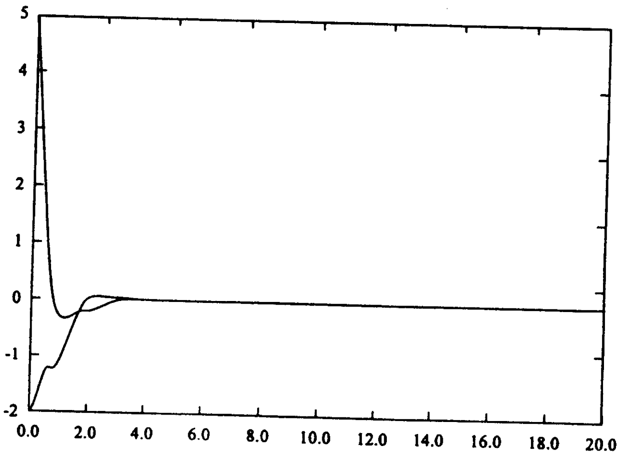

Fig. 2. State response with new nonlinear control.