Novel Systolic Array Design for the Discrete Hartley Transform with High Throughput Rate

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Abstract---In this paper, three attractive features for the DHT transform are derived, including (1)the Single/ Double Data Folding ; (2) the constructive feature and (3) the data reusage. By using CORDIC algorithm, The DHT Transform Kernel h_n*[cosθ+ sinθ] and conjugate kernel h_n*[cosθ-sinθ] can be simultanously determined. With these favorable properties, a novel systolic array design for the Discrete Hartley Transform has been designed with eight times throughput of the linear systolic array designed by [1].

1. INTRODUCTION

The Discrete Hartley Transform [DHT], which involves only real arithmatic, is very popular in the field of digital signal processing as an alternative method to the Discrete Fourier Transform [DFT]. In addition, the inverse DHT has the same form as the forward DHT[2]. Therefore, one can use a single program or a single architecture for both the forward and inverse transform computation. With its favorable properties, the DHT has been applied in the spectral analysis of the real signal, autocorrection of 2-D images, convolution computation [3], data compression, the adaptive digital filter and fast interpolation.

Many previous work [2-6] focus on reducing the multiplication and addition counts. But the major disadvange of these algorithms is the difficulties of shuffle communication between stages when they are implemented to systolic architecture with large data sequence N.

The systolic array is made up of simple basic cells that connected regularity and locality. By pumping data through the array rhythmically, one can achieve high throughput with balance memory bandwidth. On the hand, the multiplication in the DHT involves the triangular function. In general, these operations cannot be executed efficiently in the general purposed computer. However, with the advancement in the VLSI technology the COordinate Rotations DIgital Computer (CORDIC) algorithm [1][7-8] provide an opportunity in building a high throughput and cost effective systolic array for the DHT transform.

2. CORDIC ALGORITHM

The CORDIC algorithm was first introduced by Volder[7] and generalized by Walther[9]. The major advantage of the algorithm is given by the fact that can be realized as a sequence of additions/ subtractions and shift operations.

In the DHT computation, the triangular functions are involved. In order to facilitate these operations, the CORDIC will operate in the circular system. Fig. 1 shows the rotation function computed by the CORDIC algorithm in the system. It is obvious from Fig. 1 that by setting $x_n=A_n$, $y_n=B_n$ and $Z_n=\theta=(2\pi mn/N)$ one can obtain $A_n\sin(2\pi mn/N)+B_n\cos(2\pi mn/N)$ from cordic$Y_{m,n}$ and obtain another co-result $A_n\cos(2\pi mn/N)-B_n\sin(2\pi mn/N)$ from output cordic$X_{m,n}$. Cordic$Y_{m,n}$ and cordic$X_{m,n}$ are mutual co-result with each other in the same hardware. Hence, we can define the cordicCO$X_{m,n}$ symbol that presents the same thing as cordic$Y_{m,n}$. And the cordicCO$Y_{m,n}$ symbol is defined for cordic$X_{m,n}$.

3.PROPERTIES OF DISCRETE HARTLEY TRANSFORM

The Discrete Hartley Transform (DHT) can be written as

$H_m=\sum_{n=0}^{N-1} h_n cas(\frac{2\pi}{N}mn)$ where m=0,1,2...,N-1 and

$$cas[x]=\cos[x]+\sin[x] \quad ....(1)$$

3.A The Data Folding Feature of Hm: There are two folding types which are derived from the DHT transform: (1) Single Data Folding (SDF) folds Hm as below , when N= 2*X and X >=1.

$$H_m=\sum_{n=0}^{N/2-1}[(h_n+(-1)^m h_{n+N/2})cas(\frac{2\pi}{N}mn)] \quad ....(2)$$ and

(2) Double Data Folding (DDF) that is done in N=4*X, and X >=1. The folded H_m will be

$$=\sum_{n=0}^{N/4-1}[(h_n+(-1)^m h_{n+N/2}+h_{n+N/4}+(-1)^m h_{n+3N/4})\cos(\frac{2\pi}{N}mn)+(h_n+(-1)^m h_{n+N/2}+h_{n+N/4}+(-1)^m h_{n+3N/4})\sin(\frac{2\pi}{N}mn)],$$
when m mod 4 = 0

$$=\sum_{n=0}^{N/4-1}[(h_n+(-1)^m h_{n+N/2}+h_{n+N/4}+(-1)^m h_{n+3N/4})\cos(\frac{2\pi}{N}mn)+(h_n+(-1)^m h_{n+N/2}-h_{n+N/4}-(-1)^m h_{n+3N/4})\sin(\frac{2\pi}{N}mn)],$$
when m mod 4 = 1

$$=\sum_{n=0}^{N/4-1}[(h_n+(-1)^m h_{n+N/2}-h_{n+N/4}-(-1)^m h_{n+3N/4})\cos(\frac{2\pi}{N}mn)+(h_n+(-1)^m h_{n+N/2}-h_{n+N/4}-(-1)^m h_{n+3N/4})\sin(\frac{2\pi}{N}mn)],$$
when m mod 4 = 2

$$=\sum_{n=0}^{N/4-1}[(h_n+(-1)^m h_{n+N/2}-h_{n+N/4}-(-1)^m h_{n+3N/4})\cos(\frac{2\pi}{N}mn)+(h_n+(-1)^m h_{n+N/2}+h_{n+N/4}+(-1)^m h_{n+3N/4})\sin(\frac{2\pi}{N}mn)],$$
when m mod 4 = 3
............................(3)

3.B The Constructive Feature: We can also divide H_k into four sub_summatioms as below :

$$H_k=\{\sum_{n=0}^{C/4-1}[A_{4n}\sin(\frac{2\pi}{N}k(4n))+B_{4n}\cos(\frac{2\pi}{N}k(4n))]+\sum_{n=0}^{C/4-1}[A_{4n+2}\sin(\frac{2\pi}{N}k(4n+2))+B_{4n+2}\cos(\frac{2\pi}{N}k(4n+2))]\}+\{\sum_{n=0}^{C/4-1}[A_{4n+1}\sin(\frac{2\pi}{N}k(4n+1))+B_{4n+1}\cos(\frac{2\pi}{N}k(4n+1))]+\sum_{n=0}^{C/4-1}[A_{4n+3}\sin(\frac{2\pi}{N}k(4n+3))+B_{4n+3}\cos(\frac{2\pi}{N}k(4n+3))]\} \quad ....(4)$$

where k is the number of m, m+N/2, m+N/4 and m+3N/4. Apply the following

$$cordicY_{m,n}=cordicCOX_{m,n}=A_n\sin(\frac{2\pi}{N}mn)+B_n\cos(\frac{2\pi}{N}mn)$$

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and

$cordicX_{m,n} = cordicCOY_{m,n} = A_n \cos(\frac{2\pi}{N}mn) - B_n \sin(\frac{2\pi}{N}mn)$

in H_K, which can be rewritten as follows :

$$H_m = \{\sum_{n=0}^{C/4-1} cordicY_{m,4n} + \sum_{n=0}^{C/4-1} cordicY_{m,4n+2}\} + \{\sum_{n=0}^{C/4-1} cordicY_{m,4n+1} + \sum_{n=0}^{C/4-1} cordicY_{m,4n+3}\} \ldots\ldots\ldots\ldots (5a)$$

$$H_{m+N/2} = \{\sum_{n=0}^{C/4-1} cordicY_{m,4n} + \sum_{n=0}^{C/4-1} cordicY_{m,4n+2}\} - \{\sum_{n=0}^{C/4-1} cordicY_{m,4n+1} + \sum_{n=0}^{C/4-1} cordicY_{m,4n+3}\} \ldots\ldots\ldots\ldots (5b)$$

$$H_{m+N/4} = \{\sum_{n=0}^{C/4-1} cordicY_{m,4n} - \sum_{n=0}^{C/4-1} cordicY_{m,4n+2}\} + \{\sum_{n=0}^{C/4-1} cordicCOY_{m,4n+1} - \sum_{n=0}^{C/4-1} cordicCOY_{m,4n+3}\} \ldots (5c)$$

$$H_{m+3N/4} = \{\sum_{n=0}^{C/4-1} cordicY_{m,4n} - \sum_{n=0}^{C/4-1} cordicY_{m,4n+2})\} - \{\sum_{n=0}^{C/4-1} cordicCOY_{m,4n+1} - \sum_{n=0}^{C/4-1} cordicCOY_{m,4n+3}\} \ldots (5d)$$

Certainly, Eqs. (5a-5d) indicate that six constructive parts, $\sum_{n=0}^{C/4-1} cordicY_{m,4n}$, $\sum_{n=0}^{C/4-1} cordicY_{m,4n+2}$, $\sum_{n=0}^{C/4-1} cordicY_{m,4n+1}$, $\sum_{n=0}^{C/4-1} cordicY_{m,4n+3}$, $\sum_{n=0}^{C/4-1} cordicCOY_{m,4n+1}$ and $\sum_{n=0}^{C/4-1} cordicCOY_{m,4n+3}$, are used to construct H_m, $H_{m+N/2}$, $H_{m+N/4}$ and $H_{m+3N/4}$. As the pair of $cordicY_{m,4n+1}$ and $cordicCOY_{m,4n+1}$ and the pair of $cordicY_{m,4n+3}$ and $cordicCOY_{m,4n+3}$ are a mutual co_result, two results that come out of the same computing hardware concurrently. Hence, only four parts of computing hardware are needed to construct them. Obviously, by using this constructive feature it is possible to achieve four times throughput.

3.C The Co-computing between H_{N-m} and H_m: It is easy to show that

$$H_{N-m} = \{\sum_{c=0}^{C/4-1} [A_{4n} \cos(\frac{2\pi}{N}m(4n)) - B_{4n} \sin(\frac{2\pi}{N}m(4n))] + \sum_{c=0}^{C/4-1} [A_{4n+2} \cos(\frac{2\pi}{N}m(4n+2)) - B_{4n+2} \sin(\frac{2\pi}{N}m(4n+2))]\} + \{\sum_{c=0}^{C/4-1} [A_{4n+1} \cos(\frac{2\pi}{N}m(4n+1)) - B_{4n+1} \sin(\frac{2\pi}{N}m(4n+1))] + \sum_{c=0}^{C/4-1} [A_{4n+3} \cos(\frac{2\pi}{N}m(4n+3)) - B_{4n+3} \sin(\frac{2\pi}{N}m(4n+3))]\} .. (6a)$$

In fact, we use the CORDIC technology to compute H_m, and find that

$$H_{N-m} = \{\sum_{n=0}^{C/4-1} cordicCOY_{m,4n} - \sum_{n=0}^{C/4-1} cordicCOY_{m,4n+2}\} + \{\sum_{n=0}^{C/4-1} cordicCOY_{m,4n+1} + \sum_{n=0}^{C/4-1} cordicCOY_{m,4n+3}\} \ldots (6b)$$

This equation indicates that by using CORDIC technology the computing of H_{N-m} is concurrent with the computing of H_m. To sum up, by using this second constructive feature it is almost to achieve eight times throughput.

4.THE PROPOSED SYSTOLIC ARRAY

In consequence, three tasks must be implemented to achieve the DHT transform. They are data folding, CORDIC computing and constructing. First, the DDF (SDF) cell is designed to implement the data folding task. Fig. 2 shows the inner structure of DDF (SDF) cell. Second, a CORDIC processor kernel is developed to meet the requirements of DHT computation. The structure of the kernel CORDIC processor is shown in Fig. 3. Third, two CONStructure cells are implemented to construct H_m, $H_{m+N/4}$, $H_{m+N/2}$, $H_{m+3N/4}$, H_{N-m}, $H_{N-(m+N/4)}$, $H_{N-(m+N/2)}$ and $H_{N-(m+3N/4)}$. Fig. 4 shows the structure of the CONS cell.

By using these three cell types and their features, a novel systolic array for the DHT can be obtained. Fig. 5 shows the proposed architecture with N= 8. The SDF cell is used to do single data folding before CORDIC computing and is followed by the CONStructure cell.

The data flow of the proposed systolic array for DHT computation is shown in Fig. 6. After the latency, the DHT transform results of H0, H2, H4 and H6 are obtained from the CONStructure cell-1 at the end of clock and another H0, H6, H4 and H2 are obtained from the CONStructure cell-2 at the same clock. In the next clock, another eight DHT transform results would be achieved. Therefore, this proposed systolic array can obtain eight DHT transforms per clock after the latency. This proposed array latency time consists of three parts : one data folding clock, stage clocks and one result constructive clock. Incidentally, stage clocks need to compute each constructive part of H_m and can be represented as :

stage clocks = N /(data folded factor*constructive factor)

data folded factor = 2 , for single data folded and
= 4 , for double data folded.

constructive factor = 8 .

Fig. 7 shows another proposed architecture with N= 32. The data flow of the proposed systolic array for DHT computation is shown in Fig. 8. This proposed systolic array can also get eight DHT transforms per clock after the latency. Fig. 8 also shows that the data introduce time is 5 clocks, and this time can be formulated as (N/8)+1.

5. PERFORMANCE ANALYSIS

In previous studies [1], N PEes are used to design a linear systolic array for the DHT transform, and there are N latency clocks, and one transform per clock throughput of the DHT. In our proposed systolic array, the latency is only (N/2*4)+2 [(N/4*4)+2] clocks for the SDF [DDF] mode , and the throughput of the DHT transform is eight DHT transforms per clock. A comparison between [1] and our proposal are listed in Table 1. The throughput rate is defined by the number of the DHT transformation in one clock. The turnaround time represents the total clocks needed to complete the whole DHT transform, the latency time are included. Another measurement, shown in column 8, is the throughput of the CORDIC unit. It can be defined as follows:

$$\textit{Throughput of CORDIC} = \frac{N\ \text{DHT}}{(\textit{Data}\ \text{introduce time})(\text{Number of CORDIC used})}$$

The throughput of CORDIC in [1] is 1/N. In our proposal, the throughput of the SDF [DDF] mode is 16/(8+N) [32/(8+N)].

6. CONCLUSIONS

Three attractive features of the DHT transform have been derived in this paper. By slightly modifing the CORDIC processor, three types of processing kernels are developed to fit the computation requirement. Using these processing kernels, a high throughput and cost effective architecture have been designed for the 1-D DHT. Almost eight times performance of [1] are obtained, while much cheaper hardware is implemented. Due to its regularity and simplicity, the proposed architecture will be very suitable for VLSI implementation.

7. REFERENCES

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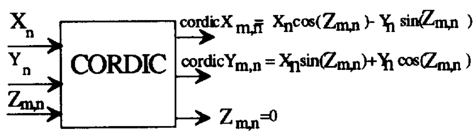

Fig. 1 Rotation function computed by the CORDIC algorithm in the circular mode.

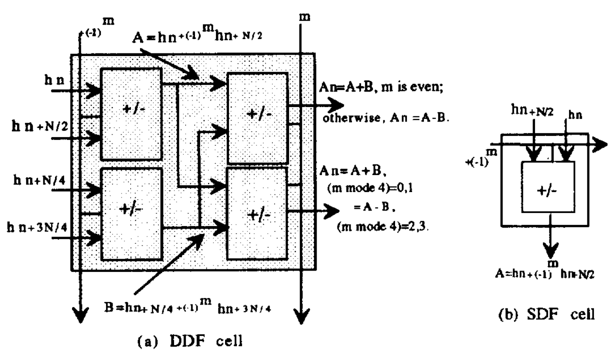

Fig. 2 The inner structure of the DDF cell and the SDF cell.

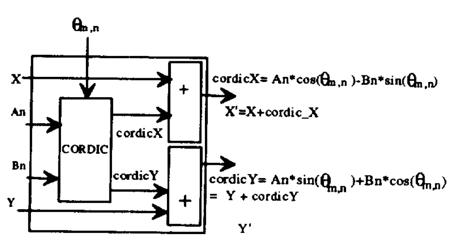

Fig. 3 The inner structure of CORDIC processor

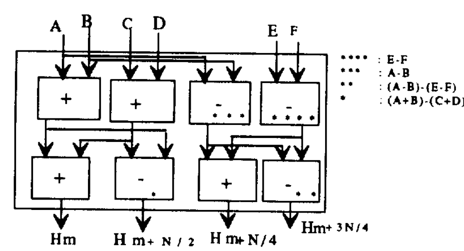

Fig. 4 The CONStructure cell.

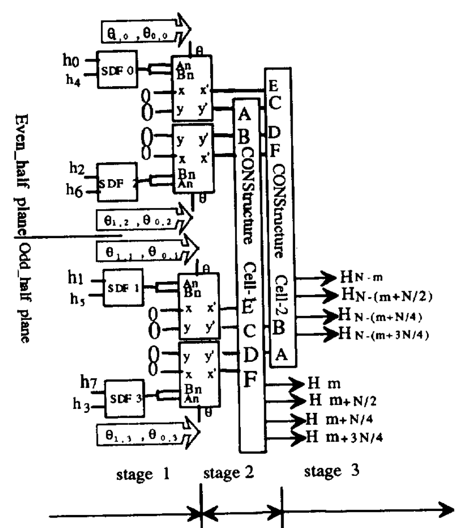

Fig. 5 A novel systolic array designed for DHT transform, when N=8, where m=0,1,2,..., N/4)-1 and $\theta_{m,n}=(2\pi mn/N)$.

TABLE 1. The comparison between [1] and our proposed systolic array.

	Latency	Throughput rate	Implementation of h*cas[x]	Turnaround time	CORDIC cells	SDF or DDF	Data introduce time	Throughput of CORDIC (1)
[1]	N	1	CORDIC	2N-1	N		N	1/N
SDF proposed	(N/8)+2	8	Modified CORDIC	(3N/8)+1	N/2	N/2 SDF	(N/8)+1	16/(8+N)
DDF proposed	(N/16) +2	8	Modified CORDIC	(5N/16)+1	N/4	N/4 DDF	(N/8)+1	32/(8+N)

(1) unit of Throughput of CORDIC is DHT/ cordic-clock.

PE Type	Clock 1	Clock 2	Clock 3	Clock 4	....
SDF 0	$A=h_0+h_4$	$A=h_0-h_4$	$A=x_0+x_4$	$A=x_0-x_4$	....
Cordic 0	NOP	$A(\sin\theta_{0,0}+\cos\theta_{0,0})$ $A(\cos\theta_{0,0}-\sin\theta_{0,0})$	$A(\sin\theta_{1,0}+\cos\theta_{1,0})$ $A(\cos\theta_{1,0}-\sin\theta_{1,0})$	$A(\sin\theta_{0,0}+\cos\theta_{0,0})$ $A(\cos\theta_{0,0}-\sin\theta_{0,0})$	....
SDF 2	$A=h_2+h_6$	$A=h_2-h_6$	$A=x_2+x_6$	$A=x_2-x_6$	....
Cordic 2	NOP	$A(\sin\theta_{0,2}+\cos\theta_{0,2})$ $A(\cos\theta_{0,2}-\sin\theta_{0,2})$	$A(\sin\theta_{1,2}+\cos\theta_{1,2})$ $A(\cos\theta_{1,2}-\sin\theta_{1,2})$	$A(\sin\theta_{0,2}+\cos\theta_{0,2})$ $A(\cos\theta_{0,2}-\sin\theta_{0,2})$	....
SDF 1	$A=h_1+h_5$	$A=h_1-h_5$	$A=x_1+x_5$	$A=x_1-x_5$	....
Cordic 1	NOP	$A(\sin\theta_{0,1}+\cos\theta_{0,1})$ $A(\cos\theta_{0,1}-\sin\theta_{0,1})$	$A(\sin\theta_{1,1}+\cos\theta_{1,1})$ $A(\cos\theta_{1,1}-\sin\theta_{1,1})$	$A(\sin\theta_{0,1}+\cos\theta_{0,1})$ $A(\cos\theta_{0,1}-\sin\theta_{0,1})$	....
SDF 3	$A=h_3+h_7$	$A=h_3-h_7$	$A=x_3+x_7$	$A=x_3-x_7$	....
Cordic 3	NOP	$A(\sin\theta_{0,3}+\cos\theta_{0,3})$ $A(\cos\theta_{0,3}-\sin\theta_{0,3})$	$A(\sin\theta_{1,3}+\cos\theta_{1,3})$ $A(\cos\theta_{1,3}-\sin\theta_{1,3})$	$A(\sin\theta_{0,3}+\cos\theta_{0,3})$ $A(\cos\theta_{0,3}-\sin\theta_{0,3})$	....
CONS cell-1	NOP	NOP	$H_0H_4H_2H_6$	$H_1H_5H_3H_7$	....
CONS cell-2	NOP	NOP	$H_0H_4H_6H_2$	$H_7H_3H_5H_1$	....

Fig. 6 The data flow of the proposed systolic array for the DHT with N=8. It shows that the throughput is eight transforms per clock and the latency is 3, (N/2*4)+2.

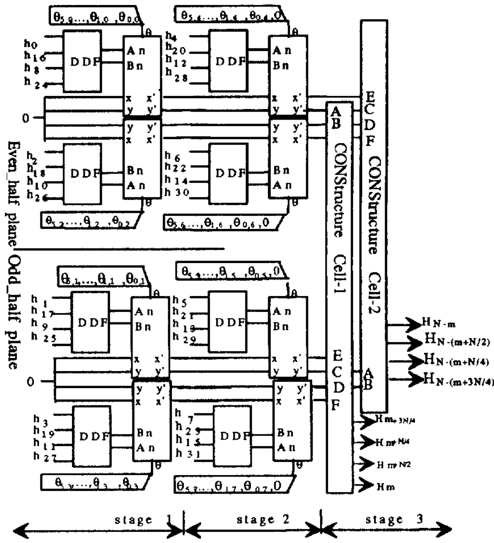

Fig. 7 A novel systolic array designed for DHT transform, when N=32, where m=0,1,2,...,(N/4)-1 and $\theta_{m,n}=(2\pi mn/N)$.

PE Type	Clock 1	Clock 2	Clock 3	Clock 4	Clock 5	Clock 6	Clock 7	Clock 8	Clock 9	...
DDF 0 A_n = B_n =	$(h_0+h_{16})+(h_8+h_{24})$ $(h_0+h_{16})+(h_8+h_{24})$	$(h_0-h_{16})-(h_8-h_{24})$ $(h_0-h_{16})+(h_8-h_{24})$	$(h_0+h_{16})-(h_8+h_{24})$ $(h_0+h_{16})-(h_8+h_{24})$	$(h_0-h_{16})+(h_8-h_{24})$ $(h_0-h_{16})-(h_8-h_{24})$	$(h_0+h_{16})+(h_8+h_{24})$ $(h_0+h_{16})+(h_8+h_{24})$	$(x_0+x_{16})+(x_8+x_{24})$ $(x_0+x_{16})+(x_8+x_{24})$	$(x_0-x_{16})-(x_8-x_{24})$ $(x_0-x_{16})+(x_8-x_{24})$	$(x_0+x_{16})-(x_8+x_{24})$ $(x_0+x_{16})-(x_8+x_{24})$	$(x_0-x_{16})+(x_8-x_{24})$ $(x_0-x_{16})-(x_8-x_{24})$	...
CORDIC 0	NOP	CORDIC(0,0)+Y	CORDIC(1,0)+Y	CORDIC(2,0)+Y	CORDIC(3,0)+Y	CORDIC(4,0)+Y	CORDIC(0,0)+Y	CORDIC(1,0)+Y	CORDIC(2,0)+Y	...
DDF 2 A_n = B_n =	$(h_2+h_{18})+(h_{10}+h_{26})$ $(h_2+h_{18})+(h_{10}+h_{26})$	$(h_2-h_{18})-(h_{10}-h_{26})$ $(h_2-h_{18})+(h_{10}-h_{26})$	$(h_2+h_{18})-(h_{10}+h_{26})$ $(h_2+h_{18})-(h_{10}+h_{26})$	$(h_2-h_{18})+(h_{10}-h_{26})$ $(h_2-h_{18})-(h_{10}-h_{26})$	$(h_2+h_{18})+(h_{10}+h_{26})(h_2$ $+h_{18})+(h_{10}+h_{26})$	$(x_2+x_{18})+(x_{10}+x_{26})$ $(x_2+x_{18})+(x_{10}+x_{26})$	$(x_2-x_{18})-(x_{10}-x_{26})$ $(x_2-x_{18})+(x_{10}-x_{26})$	$(x_2+x_{18})-(x_{10}+x_{26})$ $(x_2+x_{18})-(x_{10}+x_{26})$	$(x_2-x_{18})+(x_{10}-x_{26})$ $(x_2-x_{18})-(x_{10}-x_{26})$	...
CORDIC 2	NOP	CORDIC(0,2)+Y	CORDIC(1,2)+Y	CORDIC(2,2)+Y	CORDIC(3,2)+Y	CORDIC(4,2)+Y	CORDIC(0,2)+Y	CORDIC(1,2)+Y	CORDIC(2,2)+Y	...
DDF 1 A_n = B_n =	$(h_1+h_{17})+(h_9+h_{25})$ $(h_1+h_{17})+(h_9+h_{25})$	$(h_1-h_{17})-(h_9-h_{25})$ $(h_1-h_{17})+(h_9-h_{25})$	$(h_1+h_{17})-(h_9+h_{25})$ $(h_1+h_{17})-(h_9+h_{25})$	$(h_1-h_{17})+(h_9-h_{25})$ $(h_1-h_{17})-(h_9-h_{25})$	$(h_1+h_{17})+(h_9+h_{25})$ $(h_1+h_{17})+(h_9+h_{25})$	$(x_1+x_{17})+(x_9+x_{25})$ $(x_1+x_{17})+(x_9+x_{25})$	$(x_1-x_{17})-(x_9-x_{25})$ $(x_1-x_{17})+(x_9-x_{25})$	$(x_1+x_{17})-(x_9+x_{25})$ $(x_1+x_{17})-(x_9+x_{25})$	$(x_1-x_{17})+(x_9-x_{25})$ $(x_1-x_{17})-(x_9-x_{25})$	...
CORDIC 1	NOP	CORDIC(0,1)+Y	CORDIC(1,1)+Y	CORDIC(2,1)+Y	CORDIC(3,1)+Y	CORDIC(4,1)+Y	CORDIC(0,1)+Y	CORDIC(1,1)+Y	CORDIC(2,1)+Y	...
DDF 3 A_n = B_n =	$(h_3+h_{19})+(h_{11}+h_{27})$ $(h_3+h_{19})+(h_{11}+h_{27})$	$(h_3-h_{19})-(h_{11}-h_{27})$ $(h_3-h_{19})+(h_{11}-h_{27})$	$(h_3+h_{19})-(h_{11}+h_{27})$ $(h_3+h_{19})-(h_{11}+h_{27})$	$(h_3-h_{19})+(h_{11}-h_{27})$ $(h_3-h_{19})-(h_{11}-h_{27})$	$(h_3+h_{19})+(h_{11}+h_{27})$ $(h_3+h_{19})+(h_{11}+h_{27})$	$(x_3+x_{19})+(x_{11}+x_{27})$ $(x_3+x_{19})+(x_{11}+x_{27})$	$(x_3-x_{19})-(x_{11}-x_{27})$ $(x_3-x_{19})+(x_{11}-x_{27})$	$(x_3+x_{19})-(x_{11}+x_{27})$ $(x_3+x_{19})-(x_{11}+x_{27})$	$(x_3-x_{19})+(x_{11}-x_{27})$ $(x_3-x_{19})-(x_{11}-x_{27})$	...
CORDIC 3	NOP	CORDIC(0,3)+Y	CORDIC(1,3)+Y	CORDIC(2,3)+Y	CORDIC(3,3)+Y	CORDIC(4,3)+Y	CORDIC(0,3)+Y	CORDIC(1,3)+Y	CORDIC(2,3)+Y	...
DDF 4 A_n = B_n =	NOP	$(h_4+h_{20})+(h_{12}+h_{28})$ $(h_4+h_{20})+(h_{12}+h_{28})$	$(h_4-h_{20})-(h_{12}-h_{28})$ $(h_4-h_{20})+(h_{12}-h_{28})$	$(h_4+h_{20})-(h_{12}+h_{28})$ $(h_4+h_{20})-(h_{12}+h_{28})$	$(h_4-h_{20})+(h_{12}-h_{28})$ $(h_4-h_{20})-(h_{12}-h_{28})$	$(h_4+h_{20})+(h_{12}+h_{28})$ $(h_4+h_{20})+(h_{12}+h_{28})$	$(x_4+x_{20})+(x_{12}+x_{28})$ $(x_4+x_{20})+(x_{12}+x_{28})$	$(x_4-x_{20})-(x_{12}-x_{28})$ $(x_4-x_{20})+(x_{12}-x_{28})$	$(x_4+x_{20})-(x_{12}+x_{28})$ $(x_4+x_{20})-(x_{12}+x_{28})$	...
CORDIC 4	NOP	NOP	CORDIC(0,4)+Y	CORDIC(1,4)+Y	CORDIC(2,4)+Y	CORDIC(3,4)+Y	CORDIC(4,4)+Y	CORDIC(0,4)+Y	CORDIC(1,4)+Y	...
DDF 6 A_n = B_n =	NOP	$(h_6+h_{22})+(h_{14}+h_{30})$ $(h_6+h_{22})+(h_{14}+h_{30})$	$(h_6-h_{22})-(h_{14}-h_{30})$ $(h_6-h_{22})+(h_{14}-h_{30})$	$(h_6+h_{22})-(h_{14}+h_{30})$ $(h_6+h_{22})-(h_{14}+h_{30})$	$(h_6-h_{22})+(h_{14}-h_{30})$ $(h_6-h_{22})-(h_{14}-h_{30})$	$(h_6+h_{22})+(h_{14}+h_{30})$ $(h_6+h_{22})+(h_{14}+h_{30})$	$(x_6+x_{22})+(x_{14}+x_{30})$ $(x_6+x_{22})+(x_{14}+x_{30})$	$(x_6-x_{22})-(x_{14}-x_{30})$ $(x_6-x_{22})+(x_{14}-x_{30})$	$(x_6+x_{22})-(x_{14}+x_{30})$ $(x_6+x_{22})-(x_{14}+x_{30})$	...
CORDIC 6	NOP	NOP	CORDIC(0,6)+Y	CORDIC(1,6)+Y	CORDIC(2,6)+Y	CORDIC(3,6)+Y	CORDIC(4,6)+Y	CORDIC(0,6)+Y	CORDIC(1,6)+Y	...
DDF 5 A_n = B_n =	NOP	$(h_5+h_{21})+(h_{13}+h_{29})$ $(h_5+h_{21})+(h_{13}+h_{29})$	$(h_5-h_{21})-(h_{13}-h_{29})$ $(h_5-h_{21})+(h_{13}-h_{29})$	$(h_5+h_{21})-(h_{13}+h_{29})$ $(h_5+h_{21})-(h_{13}+h_{29})$	$(h_5-h_{21})+(h_{13}-h_{29})$ $(h_5-h_{21})-(h_{13}-h_{29})$	$(h_5+h_{21})+(h_{13}+h_{29})$ $(h_5+h_{21})+(h_{13}+h_{29})$	$(x_5+x_{21})+(x_{13}+x_{29})$ $(x_5+x_{21})+(x_{13}+x_{29})$	$(x_5-x_{21})-(x_{13}-x_{29})$ $(x_5-x_{21})+(x_{13}-x_{29})$	$(x_5+x_{21})-(x_{13}+x_{29})$ $(x_5+x_{21})-(x_{13}+x_{29})$	...
CORDIC 5	NOP	NOP	CORDIC(0,5)+Y	CORDIC(1,5)+Y	CORDIC(2,5)+Y	CORDIC(3,5)+Y	CORDIC(4,5)+Y	CORDIC(0,5)+Y	CORDIC(1,5)+Y	...
DDF 7 A_n = B_n =	NOP	$(h_7+h_{23})+(h_{15}+h_{31})$ $(h_7+h_{23})+(h_{15}+h_{31})$	$(h_7-h_{23})-(h_{15}-h_{31})$ $(h_7-h_{23})+(h_{15}-h_{31})$	$(h_7+h_{23})-(h_{15}+h_{31})$ $(h_7+h_{23})-(h_{15}+h_{31})$	$(h_7-h_{23})+(h_{15}-h_{31})$ $(h_7-h_{23})-(h_{15}-h_{31})$	$(h_7+h_{23})+(h_{15}+h_{31})$ $(h_7+h_{23})+(h_{15}+h_{31})$	$(x_7+x_{23})+(x_{15}+x_{31})$ $(x_7+x_{23})+(x_{15}+x_{31})$	$(x_7-x_{23})-(x_{15}-x_{31})$ $(x_7-x_{23})+(x_{15}-x_{31})$	$(x_7+x_{23})-(x_{15}+x_{31})$ $(x_7+x_{23})-(x_{15}+x_{31})$	...
CORDIC 7	NOP	NOP	CORDIC(0,7)+Y	CORDIC(1,7)+Y	CORDIC(2,7)+Y	CORDIC(3,7)+Y	CORDIC(4,7)+Y	CORDIC(0,7)+Y	CORDIC(1,7)+Y	...
CONS Cell-1	NOP	NOP	NOP	$H_0H_8H_{16}H_{24}$	$H_1H_9H_{17}H_{25}$	$H_2H_{10}H_{18}H_{26}$	$H_3H_{11}H_{19}H_{27}$	$H_4H_{12}H_{20}H_{28}$	$X_0X_8X_{16}X_{24}$	...
CONS Cell-2	NOP	NOP	NOP	$H_0H_{24}H_{16}H_8$	$H_{31}H_{23}H_{15}H_7$	$H_{30}H_{22}H_{14}H_6$	$H_{29}H_{21}H_{13}H_5$	$H_{28}H_{20}H_{12}H_4$	$X_0X_{24}X_{16}X_8$	...

**$CORDIC\_Y(m,n)=A_n\sin(2\pi/N(mn))+B_n\cos(2\pi/N(mn))$ **$CORDIC\_X(m,n)=A_n\cos(2\pi/N(mn))-B_n\sin(2\pi/N(mn))$

Fig. 8 The data flow of the proposed systolic array for the DHT with N=32. Double data folding are used in this design. It shows that the throughput is eight DHT transforms per clock and the latency is 4, (N/4*4)+2.