A Differentially Coherent Delay-Locked Loop for Spread-Spectrum Tracking Receivers

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***Abstract*— A novel differentially coherent delay-locked loop (DCDLL) for accurate code tracking is proposed for direct sequence spread spectrum systems. Due to the use of the differential decoder and exactly one correlator, the proposed scheme avoids the problems of gain imbalance. The tracking error variance is derived by linear analysis. When the proposed DCDLL scheme is applied in ranging with additive white Gaussian noise (AWGN) channel, the performance of the proposed DCDLL scheme is about 1.4 dB better than that of one-correlator tau–dither loop (TDL), and near that of noncoherent DLL.**

***Index Terms*— Correlator, delay-locked loop, direct sequence spread spectrum, tracking error variance.**

I. System Description and Signal Model

IN THIS letter, we present a code tracking receiver with less complexity, by employing a differentially coherent technique originally proposed for pseudonoise (PN) acquisition receiver [3]. The proposed differentially coherent delay-locked loop (DCDLL) scheme is shown in Fig. 1. The received signal $r(t)$ is first filtered by front-end band-pass filter (BPF) and the bandwidth of BPF is $2B_c$. B_c is set to be chip rate R_c ($=1/T_c$, where T_c is the chip duration). Then this proposed DCDLL scheme processes the received signal using a differential decoder with a delay of K-chip duration in the delay path. The decoder output is then correlated with the difference of the advanced (early) and retarded (late) versions of the local PN code to produce an error signal. After the error signal is filtered by a low-pass filter (LPF), then it drives the voltage-controlled clock (VCC) through the loop filter and corrects the code phase error of the local PN code generator. In this proposed system, the bandwidth of LPF, denoted as B, is set to be the system data rate $R_b(=1/T_b$, where T_b is the data bit duration). The processing gain A of this direct-sequence spread-spectrum (DS/SS) system is thus given by T_b/T_c or B_c/B. Usually, $A = 1$ if the system is applied in ranging, and $A \gg 1$ if the receiver is used for wireless communication.

As in other DS/SS communication systems, the output of front-end BPF $r_1(t)$ can be expressed as

$$r_1(t) = \sqrt{2P}\, m(t-\tau)c(t-\tau)\cos(\omega_c t + \theta(t)) + n_1(t) \quad (1)$$

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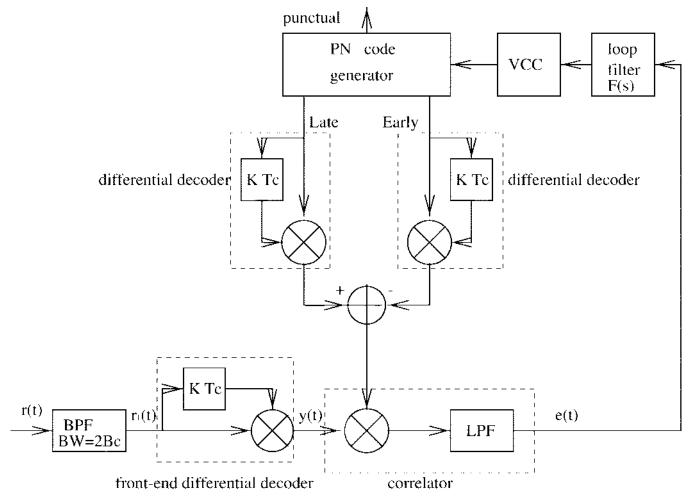

Fig. 1. A DCDLL for DS/SS.

where P is the average signal power, $m(t)$ is a random binary data sequence, $c(t)$ is the transmitted PN sequence, τ is the unknown time delay to be tracked by the code tracking loop and is a function of t, and $\theta(t)$ is a slowly time-varying carrier phase function, which is assumed to be constant over several successive chips. The noise $n_1(t)$ is an additive white Gaussian noise (AWGN) process with two-sided power spectral density of $N_o/2$ W/Hz, and can be expressed in the quadrature form

$$n_1(t) = \sqrt{2}\,(n_{1c}(t)\cos(\omega_c t + \theta(t)) - n_{1s}(t)\sin(\omega_c t + \theta(t))). \quad (2)$$

Note that $n_{1c}(t)$, and $n_{1s}(t)$ are zero mean, of equal variance, and mutually independent Gaussian random processes with the same power spectral density as $n_1(t)$.

Next, one may express $r_1(t-KT_c)$, the K-chip delay form of $r(t)$, as

$$r_1(t-KT_c) = \sqrt{2P}\, m(t-\tau-KT_c)c(t-\tau-KT_c) \cdot \cos(\omega_c t + \theta(t)) + n_2(t) \quad (3)$$

where $\omega_c KT_c$ is assumed to equal to $2\pi n$ and $n_2(t) = n_1(t-KT_c)$. Again, $n_2(t)$ can be written in the following quadrature form:

$$n_2(t) = \sqrt{2}(n_{2c}(t)\cos(\omega_c t + \theta(t)) - n_{2s}(t)\sin(\omega_c t + \theta(t))) \quad (4)$$

where $n_{2c}(t)$ and $n_{2s}(t)$ are processes with stochastic property identical to $n_{1c}(t)$ and $n_{1s}(t)$. Hence, $n_2(t)$ and $n_1(t)$ have

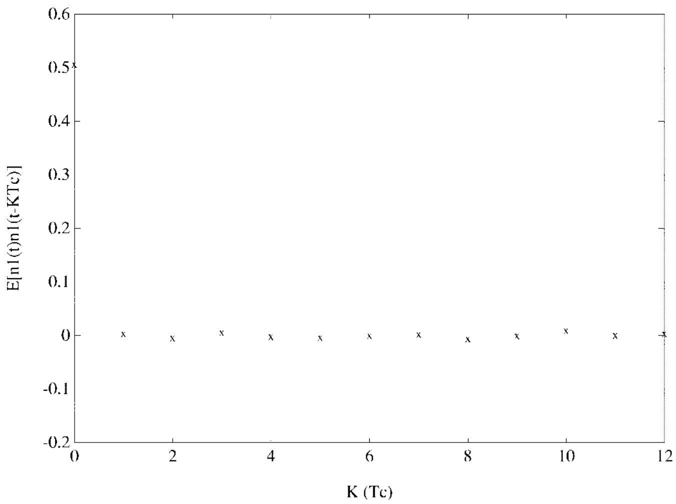

Fig. 2. The simulation for the correlation of $n_1(t)$ and $n_1(t-KT_c)$.

the same property and the correlation of $n_1(t)$ and $n_2(t)$ can be made negligible. By the simulation shown in Fig. 2 and the fact verified in [3], $K=1$ or 2 is enough to generate nearly uncorrelated $n_1(t)$ and $n_2(t)$.

II. Performance Analysis

A. The Derivation of S-Curve

In Fig. 1, the output of the differential decoder $y(t)$ is the product of $r_1(t)$ and $r_1(t-KT_c)$, and the error signal $e(t)$ can be written as

$$\begin{aligned} e(t) =& H_l(s)\{PK_m m(t-\tau)m(t-\tau-KT_c)c(t-\tau-dT_c) \\ &\cdot c_d(t)(1+\cos 2(\omega_c t+\theta(t)))+\sqrt{2P}K_m \\ &\cdot m(t-\tau)c(t-\tau)c_d(t)\cos(\omega_c t+\theta(t))n_2(t) \\ &+\sqrt{2P}\,K_m m(t-\tau-KT_c)c(t-\tau-KT_c)c_d(t) \\ &\cdot \cos(\omega_c t+\theta(t))n_1(t)+K_m c_d(t)n_1(t)n_2(t)\} \end{aligned} \quad (5)$$

where $H_l(s)$ is the Laplace transfer function of the impulse response of LPF, $h_l(t)$, and K_m is phase detector gain. Hence, $H_l(s)\{x(t)\}$ denoted the Heaviside operator $H_l(s)$ operating on the time function $x(t)$ [1], [2]. The form $c(t-\tau-dT_c)$ is resulted from differential decoder, i.e.,

$$c(t-\tau-dT_c)=c(t-\tau)c(t-\tau-KT_c) \quad (6)$$

where d is an integer and $1<d<L$ (L is the period of the PN sequence). Equation (6) holds due to the following shift-and-add property in m-sequence: if an m-sequence is added to a proper phase shift of itself, then the resulting sequence is just another shift of the original m-sequence [2].

For high processing gain and appropriately selected K, the term $H_l(s)\{m(t-\tau)m(t-\tau-KT_c)\}\approx 1$. This indicates that there are almost no degradation effects due to data modulation. Hence, one need not consider the phenomenon of modulation self-noise. By neglecting the code self-noise, (5) can be reduced to

$$e(t)=K_m(P(R_{\mathrm{PN}}(\epsilon-\Delta)-R_{\mathrm{PN}}(\epsilon+\Delta))+N_T(t)) \quad (7)$$

where ϵ is the code phase error normalized w.r.t. T_c. In general, ϵ is a function of t and can be defined by $\epsilon(t)=(\tau(t)-\hat{\tau}(t))/T_c$. $\hat{\tau}$ is the local estimate of the incoming code delay τ and Δ is early–late discriminator offset. $N_T(t)$ is the total noise. The autocorrelation of PN sequence, $R_{\mathrm{PN}}(x)$, is defined by

$$R_{\mathrm{PN}}(x)=\frac{1}{LT_c}\int_0^{LT_c} c(t)c(t+xT_c)\,dt. \quad (8)$$

After some algebra, it can be also shown the so-called S-curve $S(\epsilon,\Delta)$ as

$$S(\epsilon,\Delta)=R_{\mathrm{PN}}(\epsilon-\Delta)-R_{\mathrm{PN}}(\epsilon+\Delta). \quad (9)$$

From (9), we can find the S-curve of the proposed DCDLL scheme is the same as that of the coherent baseband delay-locked loop (BDLL) [2].

B. The Linear Analysis

For high SNR, the normalized tracking phase error ϵ can be kept small most of time. Hence, in the dynamic range there exists a linear region around $\epsilon=0$. Within the linear region, one can assume $S(\epsilon,\Delta)=S'(0,\Delta)\epsilon$, where $S'(0,\Delta)\triangleq dS(\epsilon,\Delta)/d\epsilon|_{\epsilon=0}$ is the slope of the loop S-curve $S(\epsilon,\Delta)$ at $\epsilon=0$ (the origin). The closed-loop transfer function of the linear model $H(s)$ can be shown as [1], [2]

$$H(s)\triangleq\frac{\mathcal{L}\{\hat{\tau}\}}{\mathcal{L}\{\tau\}}=\frac{S'(0,\Delta)K_LPF(s)}{s+S'(0,\Delta)K_LPF(s)}. \quad (10)$$

where $\mathcal{L}\{\cdot\}$ indicates the Laplace transform.

In linear analysis, assuming that the loop bandwidth is narrow compared to the spectrum of total noise $N_T(t)$, the normalized (by T_c) tracking error variance, σ_ϵ^2, can be approximated as [2]

$$\sigma_\epsilon^2\cong\frac{2B_L\psi_{N_T}(0)}{(S'(0,\Delta)P)^2} \quad (11)$$

where B_L is the single-sided bandwidth of $H(s)$, and is defined by

$$B_L=\int_0^\infty |H(j2\pi f)|^2\,df. \quad (12)$$

By definition, $\psi_{N_T}(0)$ satisfies [2]

$$\psi_{N_T}(0)=\int_{-\infty}^{\infty}R_{N_T}(z)\,dz \quad (13)$$

where $R_{N_T}(z)=E\{N_T(t)N_T(t+z)\}$ is the autocorrelation function of the total noise $N_T(t)$. After some algebra [2], it can be shown that

$$\begin{aligned} R_{N_T}(z)=&\ (2\mathrm{PN}_o+2N_o^2B_c)(1-f(\Delta)) \\ &\cdot\int_{-\infty}^{\infty}|H_l(j2\pi f)|^2e^{j2\pi fz}\,df \end{aligned} \quad (14)$$

where

$$f(\Delta)=\begin{cases}1-2\Delta, & 0<\Delta\le 1/2\\ 0, & \dfrac{1}{2}\le\Delta<1.\end{cases} \quad (15)$$

Finally, the tracking error variance for the proposed DCDLL, σ_ϵ^2, can be derived from (11), (13), and (14). Namely, the closed form for σ_ϵ^2 is

$$\sigma_\epsilon^2 = \frac{2B_L}{(S'(0,\Delta)P)^2}(2PN_o + 2N_o^2 B_c)(1 - f(\Delta)). \quad (16)$$

The $S'(0, \Delta)$ is equal to 2 for the proposed DCDLL. For the popular choice of $\Delta = 1/2$, the tracking error variance of the proposed scheme is given by

$$\sigma_\epsilon^2|_{\Delta=1/2} = \frac{N_o B_L}{P} + \frac{N_o^2 B_L B_c}{P^2}. \quad (17)$$

The tracking error formulas of other schemes are available in the literature. For example, the tracking error variance of tau–dither loop (TDL) σ_ϵ^2 for $\Delta = 1/2$ is given by [2]

$$\sigma_\epsilon^2|_{\Delta=1/2} = \frac{N_o B_L}{\alpha P} + \frac{2N_o^2 B_L B}{\alpha^2 P^2} \quad (18)$$

and that of conventional noncoherent delay-locked loop (NDLL) σ_ϵ^2 for $\Delta = 1/2$ is given by [1], [2]

$$\sigma_\epsilon^2|_{\Delta=1/2} = \frac{N_o B_L}{2\alpha P} + \frac{N_o^2 B_L B}{\alpha^2 P^2} \quad (19)$$

where B is the equivalent bandwidth of LPF, and α, the effect of data modulation, satisfies $0 < \alpha \leq 1$. For a system employing nonreturn-to-zero (NRZ) data modulation and ideal LPF's, the value of α is 0.902 [1].

III. Numerical Results and Discussion

In order to present more meaningful numerical results, we define $\gamma_d \triangleq PT_b/N_o$, $\gamma_L \triangleq P/(N_o B_L)$ and let ξ_o be the ratio of loop-SNR(γ_L) to data-SNR(γ_d), or equivalently the ratio of data bandwidth to loop bandwidth. Then it is straightforward to see

$$\xi_o = \frac{R_b}{B_L} = \frac{\gamma_L}{\gamma_d} \quad (20)$$

and the previous (16) can be rewritten as

$$\sigma_\epsilon^2 = \left(\frac{1}{\gamma_L} + \frac{\xi_o A}{\gamma_L^2}\right)(1 - f(\Delta)) \quad (21)$$

and A is set to be 1, i.e., $B_c = B$. Data modulation is not considered here. This situation indicates that we only consider the application in ranging in AWGN channel. This can be shown in Fig. 3. From Fig. 3, we find the standard deviation σ_ϵ of DCDLL is about 1.4 dB better than that of TDL and near that of NDLL with the linear analysis for fixed $\Delta = 1/2$, and different values of ξ_o. Fig. 3 also shows that the standard deviation decreases with the increasing data SNR γ_d.

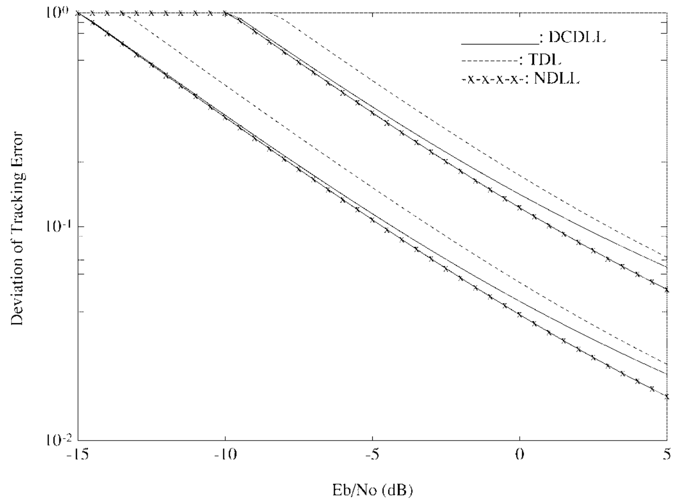

Fig. 3. Deviation σ_ϵ as a function of γ_d dB for DCDLL, TDL, and NDLL in linear analysis, with parameters $\xi_o = 10^2$, 10^3, and $\Delta = 1/2$.

IV. Conclusions

In this paper, we provide a DCDLL code tracking system like TDL with fairly less complexity in structure.

Compared with the conventional NDLL, the proposed DCDLL only employs one-correlator. Hence, it has less complexity than NDLL for implementation, and it also has evolved no difficulties in gain imbalance. When the proposed tracking receiver is applied in ranging, numerical results confirm that the performance of the proposed DCDLL is about 1.4 dB better than that of one-correlator TDL in AWGN channel environment.

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