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台股指數期貨市場非線性與混沌現象之測試

A Test for Nonlinearity and Chaos in Taiwan Stock Index Futures Market

計畫類別：☒ 個別型計畫 □ 整合型計畫

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摘要

本計劃測試台股指數期貨市場是否顯示非線性與混沌現象。主要是分析此新興期貨市場的時間數列，研究其動態行為是否與成熟發展的金融市場有顯著的差異性。

首先，Variance Ratio 法證實原報酬率存在線性相依。再以 correlation dimension（CD），及 BDS 統計量確定濾過的時間數列中含有非線性現象。將 CD 及 BDS 統計量應用到 EGARCH 模式的標準化的殘差項發現非線性現象並非由 heteroskedasticity 所導致。最後由 Locally Weighted Regression（LWR）法確認 Low Deterministic Chaos 混沌現象之存在。

本研究的結果與之前對成熟發展的金融市場之研究相異。通常學者僅發現微弱的非現性現象，且不存在混沌現象。時間數列中是否存在混沌現象對風險管理及市場效率性具有重要意含。

關鍵詞：**非線性、混沌、期貨市場**

Abstract

This paper investigates the presence of linear and nonlinear trends, and chaos in the Taiwanese futures market. The motivation of the paper is to analyze the time series properties of a new, small, and relatively thin market. It is expected that this young market might behave differently from mature stock markets, such as the American equity market. The results of the variance ratio test indicate linear dependence in the raw returns. The tests based on the correlation dimension, CD, and the more powerful BDS statistic confirm the presence of nonlinearity in the filtered time series. The CD and the BDS statistic applied to the standardized residuals of the EGARCH model reject heteroskedasticity as the cause of nonlinearity in the Taiwanese futures returns. On the other hand, the test of the locally weighted regression, LWR, applied to the filtered time series, indicates the presence of chaos in the Taiwanese futures. These results differ from those reported by previous researchers, who have reported only weak evidence of nonlinear trends and failed to find chaos in the return time series of more mature stock markets. Our findings may have relevant implications for risk management and market efficiency.

Keyword: **chaos, non-linearity, futures markets**

研究報告

METHODOLOGY

A brief description of the several tests employed in this paper follows.

TESTING FOR LINEAR TRENDS WITH THE VARIANCE RATIO TEST

The variance ratio of Lo and MacKinlay (1988) is employed to test for linear dependence in the Taiwanese futures returns time series. According to the variance ratio methodology, if the returns of a time series Y_t are a pure random walk then, the variance of the q-differences grows proportionally with the difference q. Hence, the variance ratio can be defined as:

$$VR(q) = \sigma^2(q) / \sigma^2(1)$$

Where $\sigma^2(q)$ is 1/q the variance of the q-differences and $\sigma^2(1)$ is the variance of the first difference (q = 1).

The formulas for calculating $\sigma^2(q)$ and $\sigma^2(1)$ are the following (Lo and MacKinlay (1988), Liu and He (1991)):

$$\sigma^2(q) = 1 / m \, \Sigma \, (Y_t \quad - Y_{t--q} - q\mu\,)^2$$

from t = q to t = nq

$$\sigma^2(1) = 1 / (nq - 1) \, \Sigma \, (Y_t \quad - Y_{t--1} - \mu\,)^2$$

from t = 1 to t = nq

where:

$$m = q(nq - q + 1)\,(1 - q / nq\,)$$

$$\mu = 1 / nq\,(Y_{nq} \quad - Y_0\,)$$

Y_0 and Y_{nq} are the first and last observations of the time series.

The test statistic z (q), robust to heteroskedasticity, is:

$$Z(q) = [VR(q) - 1] / [\phi\,(q)]^{0.5}$$

Where:

$$\phi\,(q) = \quad \Sigma\,[2(q - j) / q\,]^2\,\delta\,(j)$$

from j = 1 to j = q –1 and

$$\delta\,(j) = \Sigma\,(Y_t \quad - Y_{t--1} - \mu)^2(Y_{t-j} \quad - Y_{t-j--1} - \mu)^2 / [\Sigma\,(Y_t \quad - Y_{t--1} - \mu)^2]^2$$

TESTING FOR NONLINEAR DEPENDENCE: THE CORRELATION DIMENSION AND THE BDS STATISTIC TESTS

This paper employs two tests of non-linearity: the correlation dimension and the BDS statistic.

A) THE CORRELATION DIMENSION OF GRASSBERGER AND PROCACCIA

Grassberger and Procaccia (1983) developed the notion of correlation dimension, CD. The key in detecting nonlinearities with the Grassberger and Procaccia test is to observe the changes in the correlation dimension as the embedding dimension is increased. If the correlation dimension does not explode as the embedding dimension increases, then there is evidence of nonlinearity in the time series.

Grassberger & Procaccia (1983) and Swinney (1985) use a form of correlation integral to define the correlation dimension:

$$C_m(\varepsilon) = \lim_{T\to\infty} \#\{(t,s), 0 < t, s, < T : \| x_t^m - x_s^m \| < \varepsilon\}/\mathrm{T}^2,$$

where : # s = the cardinality of set s; T = sample length; m = embedding dimension; $\| \ \|$ is the sup or max norm. The correlation integral is the fraction of pairs of values of the time series $\left(\mathrm{x}_{\mathrm{s}}^m, x_t^m\right)$, which are close to each other, in the sense that :

$$\max_{i=0,\ldots,m-1} \{| x_{s-i} - x_{t-i} |\} < \varepsilon$$

It is necessary to calculate the slope of the graph of log C_m (ε) versus log ε for small values of ε . More precisely, the correlation dimension is calculated as:

$$V_m = \lim_{\varepsilon\to 0} \log C_m(\varepsilon)/\log\varepsilon$$

If V_m does not increase with m, the data are consistent with chaotic behavior.

In this paper we use the program "Chaos data Analyzer," developed by Sprott and Rowlands (1995) to compute the CD and the BDS. The program automatically chooses the optimal ε and leaves two parameters under user control: the embedding dimension m, and the parameter n, which is the number of sample intervals over which each pair of points is followed before a new pair is chosen. If n is too large, the trajectories get too far apart, and if n is too small, the calculation becomes too slow. In this paper we report the CD and BDS for embedding dimensions 2 through 10 and n of 1, 2, 4, and 8.

B) THE BROCK, DECHERT, AND SCHEINKMAN BDS STATISTIC TEST.

The BDS statistic was developed by Brock, Dechert, and Scheinkman (1987) and, unlike the correlation dimension, it is a statistical test with higher power for testing for nonlinearity than other statistical techniques. In this test the null hypothesis is that the data are independently and identically distributed, IID, or random walk.

Brock, Dechert, and Scheinkman (1987) demonstrate that under the null hypothesis (x_t) is IID with a nondegenerate density F (see also Hsieh (1989)), $C_m(\varepsilon, T) \to C_1(\varepsilon, T)^m$ with probability equal to unity, as $T \to \infty$, for any fixed m and ε. In addition, they show that $T^{1/2}[C_m(\varepsilon, T) - C_1(\varepsilon, T)^m]$ has a normal limiting distribution with zero mean and variance equal to

$$\sigma_m^2(\varepsilon) = 4\left[K^m + 2\sum_{j=1}^{m-1} K^{m-j}C^{2j} + (m-1)^2 C^{2m} - m^2 K C^{2m-2}\right],$$

Where:

$$C = C(\varepsilon) = \int [F(z+\varepsilon) - F(z-\varepsilon)]\, dF(z),$$

$$K(\varepsilon, t) = \frac{6}{T_m(T_m - 1)(T_{m-2})} \sum_{t<s<r} I_\varepsilon(x_t, x_s) I_\varepsilon(x_s, x_r)$$

$C_1(\varepsilon, T)$ is a consistent estimate of *$C(\varepsilon)$*, and

$$K = K(\varepsilon) = \iint [F(z+\varepsilon) - F(z-\varepsilon)]^2 \,\mathrm{dF(z)}.$$

is a consistent estimate of $K(\varepsilon)$. Therefore, $\sigma_m(\varepsilon)$ can be estimated by $\sigma_m(\varepsilon, T)$, which uses $C_1(\varepsilon, T)$ and $K(\varepsilon, T)$ instead of $C(\varepsilon)$ and $K(\varepsilon)$. The BDS statistic has

a standard normal limiting distribution and is calculated by

$$W_m(\varepsilon, T) = T^{1/2}\left[C_m(\varepsilon, T) - C_1(\varepsilon, T)^m\right] / \sigma_m(\varepsilon, T).$$

Brock, Dechert, and Scheinkman (1987) show that, under the null of IID, $W_m \rightarrow N(0, 1)$, as $T \rightarrow \infty$. If the residuals from the estimated linear (nonlinear) model are actually IID, the BDS statistic should be asymptotically N (0,1). On the other hand, large values of the BDS statistic would indicate evidence for non-linearity in the data.

TESTING FOR CONDITIONAL HETEROSKEDASTICITY WITH THE EGARCH MODEL

The finding of nonlinearity in a time series does not necessarily imply the series exhibit low deterministic chaotic behavior. Indeed, nonlinearity in a time series can be due to heteroskedasticity or chaos. Therefore, it is necessary to examine the cause of nonlinearity. In this paper we use the EGARCH model to investigate if nonlinearity in the Taiwanese futures returns are due to the presence of heteroskedasticity.

Following Hsieh (1991), the equation $X_t = g(x_{t-1}, \ldots)\, \varepsilon_t$, is a general model of conditional heteroskedasticity, which includes ARCH-type models as special cases. Let us take the absolute values of the above equation:

$$| x_t | \quad = \quad | g(x_{t-1}, \ldots) | \quad | \varepsilon_t | .$$

If g ( ) is differentiable, a Taylor series expansion would yield the result that $| x_t |$ depends on $| x_{t-1} |$. Thus, if the autocorrelation of the absolute value of the data is computed, the finding of correlation of $| x_t |$ with $| x_{t-1} |$ is evidence of conditional heteroskedasticity. This is important because there is growing evidence that stock return volatility is not only time-varying (French, Schwert, and Stambaugh (1987)), but it is also predictable (Schwert and Seguin (1990)).

Our objective here is to investigate whether the conditional heteroskedasticity captured by ARCH-type models is responsible for the nonlinearity in the Taiwanese futures returns. In order to investigate this issue, we fit an EGARCH model to the data (Hseih (1991)) as follows:

$$x_t \sim \mathrm{N}(0, \sigma_t^2),$$

$$\log\sigma_t^2 = \phi_0 + \phi\,|\,x_{t-1}/\sigma_{t-1}| + \Psi\log\sigma_{t-1}^2 + \gamma\, x_{t-1/}\sigma_{t-1}.$$

We have chosen the EGARCH model because it has several advantages over other models. In effect, ARCH and GARCH impose restrictions on the signs of the parameters to guarantee that estimated variances are positive, which creates numerical problems associated with constrained optimization. The EGARCH model does not impose these restrictions and, in addition, it can accommodate conditional skewness.

If the EGARCH model is correctly specified, then the standardized residuals

$Z_t = x_t / \sigma_t$, should be IID. In this formula, σ_t is the fitted value of the standard deviation from the variance equation. In other words, the BDS statistic can be applied to the standardized residuals to test if heteroskedasticity captures the nonlinearities present in the time series.

TESTING FOR CHAOS WITH THE LOCALLY WEIGHTED REGRESSION

If the EGARCH model rejects heteroskedasticity as the cause of nonlinearity in the Taiwanese futures returns, we proceed to test for the presence of chaos. The test of low deterministic chaos employed in this paper is one type of nonparametric regression known as the locally weighted regression, LWR (see, e.g., Diebold and Nason (1990); LeBaron (1988)). The LWR can be briefly described as follows: suppose the time series data are generated according to:

$X_{t+1} = f(x_t)$. Based on the observed x_t, x_{t+1}, ..., to forecast x_{t+1}, LWR makes use of

the k nearest neighbors of x_t, and a scheme, which gives more weight to closer observations and less weight to farther observations. Several parameters must be selected: (a) the number of nearest neighbors to use. This study tries 10%, up to 90%, increasing in steps of 10%; (b) the number of lags of x_t to include as arguments of the unknown function f (x). We use lags 1 through 5; (c) the weighting scheme. We use the "tricubic" scheme proposed by Cleveland and Devlin (1988). If returns are governed by low complexity chaos, we should be able to use LWR to forecast returns better than with simple methods, such as the random walk model (Hsieh (1991, 1993)).

DATA

The data correspond to daily prices for the Taiwan Index Futures, TAIFEX, traded in Taipei, Taiwan, for the July 21, 1998 through June 13, 2001. The Taiwanese futures index has five delivery months: the current spot month, the next calendar month, and the next three consecutive quarter months of March, June, September, and December. We use data for the nearby futures contract and we do not roll over to the next contract until the current nearby contract has expired. Since the last trading day in a delivery month is the third Wednesday, the price used on the next trading day (Thursday) is of the new nearby contract. For example, if the last trading date for the November contract is 11/20, we use data for the November contract until 11/20, and roll over to the December contract on 11/21. We do not roll over to the next contract a week before the current nearby contract expires (the usual procedure) because in Taiwan most investors are daily traders, or very short-term investors, and they trade mainly the nearby contract almost until expiration day. Thus, the trading volume of the nearby contract does not drop until the contract expires.

Daily returns are computed as:

$$R_t = 100 * Ln\ (R_t / R_{t-1})$$

The tests of the CD, BDS statistics, and chaos require large samples. Our data sample is small, but this is a perennial problem faced by most researchers when testing for nonlinearity and chaos in finance and economics time series. Scheinkman and LeBaron (1989) use 1,040 weekly returns and embedding dimension of as high as 25. Hsieh works with less than 1,300 weekly observations for several foreign currency futures (1989) and weekly stock returns (1991). Brock and Sayers (1988) employ less than 150 quarterly observations for employment, unemployment, and industrial production. Willey (1992) works with 945 observations of the NASDAQ index, and LeBaron (1992) uses around 720 weekly observations for foreign currency exchange rates. Data limitation in finance is a fact of life. Some argue that the problem can be avoided by using tick by tick data. However tick by tick data captures bid-ask bounces and other micromarket artificial dependencies which will be picked up by the tests of nonlinear dynamics (Hsieh, 1991). When long histories of data are available, the researcher can increase the number of observations by increasing the sample time period, but by extending a data-set further and further back in time, we face the problem of nonstationarity in the data, which is also detected by tests of nonlinear dynamics (Hsieh, 1991). In other words, the requirements of long sampling intervals (to avoid micromarket structure dependencies) and short stories (to avoid nonstationarity) impose severe data limitations in finance.

ANALYSIS OF EMPIRICAL RESULTS

The descriptive statistics of the Taiwanese futures returns are shown in Table 1. The data exhibit little skewness but the kurtosis number shows fat tails. In addition, the Jarque-Bera statistic strongly rejects the null hypothesis of normality. Therefore, we conclude that the sample of daily returns of the Taiwanese futures market does not come from a normal distribution.

The results of the variance ratio test, performed in the time series of raw returns, are presented in Table 2. Variance ratios are reported for the intervals q = 2, 5, 10, 20, 30, 40, and 60 days. It can be observed that the null hypothesis of random walk is generally rejected in favor of linear dependence. In effect, most of the variance ratios are statistically significantly larger than one. Variance ratios larger than one implies that the variances grow more than proportionally with time. Since the Z-statistic is robust to heteroskedasticity, it can safely be concluded that the rejection of the random walk is due to auto-correlation in the returns.

Since the time series of raw returns exhibits linear dependence, we need to filter the data before to proceed with the tests of nonlinearity. We do that by fitting an autoregressive integrated moving average, ARIMA, model to the raw data, and run the tests of nonlinearity: the correlation dimension, CD, and the BDS statistics, in the residuals of the ARIMA model. Table 3 presents the results of the tests of nonlinearity for the filtered returns. The CDs of Grassberger and Procaccia are shown in panel A, and the BDS statistic of Brock, Dechert and Scheinkman are reported in panel B. The CDs increase but do not explode with the embedding dimension suggesting the presence of nonlinearity in the returns. In addition, the BDS, which is a statistical test that detects deviations from IID returns, rejects the null of IID. Thus, the BDS statistic also indicates the presence of non-linearity in the time series of Taiwanese futures returns.

However, nonlinearity in a return time series can be stochastic or chaotic. In other words, our finding of nonlinearity can be due to the presence of heteroskedasticity or chaos. Therefore, we need to run additional tests to find out if nonlinearity in the Taiwanese futures returns is due to heteroskedasticity or chaos.

In order to investigate whether the nonlinearity in Taiwanese futures returns is due to heteroskedasticity we fit an EGARCH model to the data and compute the CDs and the BDS statistics in the standardized residuals of the EGARCH model. The results are presented in Table 4. The CDs do not explode as the embedding dimension increases, and the BDS statistic strongly rejects the null of IID. These results imply that nonlinearity is still present even after controlling for heteroskedasticity. In other words, the results of the EGARCH model indicate that heteroskedasticity is not the cause of nonlinearity in the Taiwanese futures returns.

Since heteroskedasticity is not the cause of nonlinearity in the data, we proceed to test for the presence of chaos in the Taiwanese futures returns. Following Hsieh (1991, 1993) if a return time series is governed by low complexity chaos we should be able to forecast returns better with the locally weighted regression, LWR, than with the random walk model. The results of the locally weighted regression applied to the filtered data are reported in Table 5. They show that the mean squared errors obtained with the LWR are smaller than those obtained with the random walk, what is an indication that the time series of Taiwanese futures returns exhibit low deterministic chaotic behavior. That is, the tests of the locally weighted regression confirm our hypothesis that the nonlinearity detected in the Taiwanese futures returns is due to the presence of chaos in the data.

CONCLUSION

The objective of this paper is to examine the time series properties of the young Taiwanese futures market. Specifically, the presence of linearity, nonlinearity, and chaos are empirically tested. Our results confirm the presence of linear and nonlinear dependence in the data. In effect, the variance ratio test rejects the null of random walk., and the tests of the correlation dimension, CD, and the more powerful BDS statistic, applied to the filtered data, strongly indicates the presence of non-linearity in the time series of Taiwanese futures returns.

The CD test and the BDS statistic applied to the standardized residuals of the EGARCH model reject the null of IID, indicating that conditional heteroskedasticity is not the cause of the presence of nonlinearity in the Taiwanese futures market. On the other hand, the mean squared errors obtained from the locally weighted regression, LWR, are smaller than those of the random walk, suggesting the presence of chaos in the data. Thus, the test of LWR supports our hypothesis that the existence of nonlinearity in the Taiwanese futures returns is due to the presence of low deterministic chaos. This is an important result since most of previous research has failed to report empirical evidence that the time series of financial asset returns exhibit chaotic behavior.

The detected chaos may come from within the structure of the Taiwanese economy. Under constant market interventions by the Taiwanese government, adaptive-learning market traders help to drive the evolutionary price dynamics to nonlinear chaos. If nonlinear dynamics are at work, market efficiency becomes questionable, since profitable, non-linearity-based trading rules might be possible. It also implies that all hedging strategies based on linear dynamics need to be modified.

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