行政院國家科學委員會補助專題研究計畫執行進度報告

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※　　　　　　貝氏統計在結構系統模態　　　　　　※
※　　　　　　　與動態識別之應用(1/2)　　　　　　※
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計畫主持人： 劉 佩 玲

共同主持人：

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一、中文摘要

本計畫之主旨在發展一以貝氏統計為基礎之結構識別方法。首先，蒐集結構資料，包括幾何性質及起始的桿件性質分佈，並據以建立結構之有限元素模型。接著，量測結構之模態或動態反應，再根據結構反應，以貝氏統計法修正桿件性質分佈。此識別法可用以修正結構之數學模型，也可用於偵測結構之受損桿件，及評估結構之可靠度。

本計畫第一、二年度將分別發展以模態反應及動態反應作為觀察數據的貝氏識別法。除了提出識別方法外，本研究也將討論模態或動態反應量測誤差對識別結果的影響。最後，將所發展之反算方法編寫為電腦程式，以數值算例驗証其可行性。

關鍵詞：非破壞檢測、貝氏統計、可靠度

二、計畫緣由與目的

臺灣四面環海，又地處颱風與地震帶，土木工程構造物經常受強風及地震侵襲，再加上環境腐蝕及材料老化的作用，這都會引致材料強度降低，繼而產生無預警的破壞。例如公路橋樑及高架陸橋長期受載重車疲勞荷重，會引致局部裂縫延伸及鋼筋的鏽蝕，橋樑的基礎結構經過一段年限的使用後亦會有局部腐蝕和強度降低等問題。這些結構一旦在無預警狀況下發生破壞，不但會產生嚴重的人員傷亡，所造成的經濟損失也十分慘重。因此，如何利用非破壞評估方法預測工程結構的可靠度，是目前急待探討之問題。值得一提的是，若能有效地評估結構物的安全性，不但可以避免災害發生，而且可對結構物做及時適當的維修，這更可以延長結構物的使用年限，相對的使該結構物的經濟效益提高。例如原本設計四十年使用壽命的核電廠，經由非破壞評估的方法判定可延長十年使用期限，則不但可節省巨額的成本，更可大幅減少日益嚴重的環保問題。因此，重要的結構物在平時及災後都應該進行檢測，以評估結構物可靠度的高低，並進一步決定該結構是否必須維修，以及適當的維修步驟。

早在 1970 年代，就有學者專家提出以自然頻率及振態進行結構系統識別。這個領域早期的研究重點大多是作航太結構的系統識別，主要的目的是經由模態試驗，修正數值模擬的勁度矩陣，以便準確的預測航太結構的動態反應。進行系統識別的第一個步驟，就是進行結構試驗。視所採用的方法，量測結構的模態、靜態或動態反應。接著，必須選定一個結構的數學模型，一般而言，都是以有限元素模型來模擬結構行為。然後，解下列的最佳化問題：

$$\text{minimize} \quad E(\mathbf{p})$$

其中 E 為誤差函數，p 為結構的未知參數，如桿件的剛度、阻尼等等。

除了一些特殊方法，系統識別通別要以迭代求解上述最佳化問題，因此相當耗時，也常面臨落入局部最小值的困難。

本計畫發展一套利用結構模態或動態反應偵測桿件性質之方法，此方法是以貝氏統計修正桿件性質，而不採用反算程序，因此不需要解最佳化問題，當然計算較為快速，也避開局部最小值的問題。

本研究係一二年期計畫，本年度係發展模態識別法，以下各節將詳述目前之進度。

三、可靠度分析

The structural reliability analysis is formulated based on two fundamental assumptions: (1) the state of the structure is defined in the outcome space of a vector of basic random variables, **V**; (2) the structure can be in one of two states, the safe state or the failure state. The state of the structure is determined by the value of a limit-state function $g(\mathbf{v})$, which is formulated such that when $g(\mathbf{v}) > 0$, the structure is safe, and when $g(\mathbf{v}) \le 0$, the structure fails. The failure probability of the structure associated with the specific failure criterion, then, can be obtained by integrating the joint probability density function of **V** in the failure domain. That is

$$P_f = \int_{g(\mathbf{v})\le 0} f_{\mathbf{V}}(\mathbf{v})\, d\mathbf{v}$$

where $f_{\mathbf{V}}(\mathbf{v})$ is the joint probability density function of **V**. It is usually difficult to perform the multifold integral directly. Hence, the first-order reliability method (FORM) is often adopted to estimate the failure probability.

Apparently, the reliability analysis gives accurate results only when the joint distribution $f_{\mathbf{V}}(\mathbf{v})$ is valid. However, $f_{\mathbf{V}}(\mathbf{v})$ changes when the structure is damaged. Such change is reflected in the structural response. Therefore, $f_{\mathbf{V}}(\mathbf{v})$ can be updated based on the response of the damaged structure.

四、貝氏統計

Usually, the statistics of the structural properties are available at the construction stage. Such data may be obtained by experiments, theoretical derivations, and/or engineering judgment. Therefore, some prior information is known about the structural properties.

Suppose structural test is carried out on the target structure. Hence, new information is collected from the test. Apparently, such information should be incorporated to give a new estimate of the structural properties.

The Bayesian approach can incorporate new experimental outcomes to update the distribution of a random variable. Hence, it can be applied to meet our needs.

Let **V** denote the element properties of the target structure with prior distribution $f_{\mathbf{v}}'(\mathbf{v})$, and **t** denote the modal data of the structure, e.g., natural frequencies and mode shapes. According to the Bayesian formula, the updated probability density of **V** is

$$f_{\mathbf{V}}''(\mathbf{v}) = k f_{\mathbf{t}|\mathbf{V}}(\mathbf{t}\,|\,\mathbf{v}) f_{\mathbf{V}}'(\mathbf{v})$$

where $f_{\mathbf{t}|\mathbf{v}}(\mathbf{t}\,|\,\mathbf{v})$ is the conditional probability density of observing **t** as **V** = **v**, and $k = \int\int_{-\infty}^{\infty} f_{\mathbf{t}|\mathbf{V}}(\mathbf{t}\,|\,\mathbf{v}) f_{\mathbf{V}}'(\mathbf{v})\, d\mathbf{v}$ is a normalizing constant.

Firstly, consider **t**=**Λ** = $\left[\omega_1^2 \;\; \omega_2^2 \;\; \cdots \;\; \omega_m^2\right]$ = measured natural frequencies. For a given set of element properties, assume **Λ** = $N\left(\overline{\Lambda}, \sigma_\lambda^2 \mathbf{I}\right)$, where $\overline{\Lambda}$, the mean vector of **Λ**, is obtained by solving $\mathbf{K}\phi = \lambda \mathbf{M}\phi$. Then,

$$f_{\Lambda|\mathbf{V}}(\Lambda\,|\,\mathbf{v}) = \mathrm{N}\left(\overline{\Lambda}, \sigma_\lambda^2 \mathbf{I}\right)$$

$$= \frac{1}{(2\pi)^{m/2}\sigma_\lambda^m} \exp\left[-\frac{1}{2\sigma_\lambda^2}\left(\Lambda - \overline{\Lambda}\right)^T \left(\Lambda - \overline{\Lambda}\right)\right]$$

In the above expression, $\overline{\Lambda}$ is an implicit function of **V**. That makes the Bayesian update difficult to apply. To overcome this problem, $\overline{\Lambda}$ is expanded into a Taylor's series:

$$\Lambda(\mathbf{v}) = \Lambda_0 + \left.\frac{\partial \Lambda}{\partial \mathbf{v}}\right|_{\mathbf{v}_0} \Delta\mathbf{v} + \cdots$$

where $\Lambda_0 = \Lambda(\mathbf{v}_0)$, $\mathbf{v}_0$ is the vector of the original properties, and $\Delta\mathbf{v} = \mathbf{v} - \mathbf{v}_0$. Suppose that the eigenvectors are normalized such that $\phi^T \mathbf{M}\phi = 1$, and $\phi^T \mathbf{K}\phi = \lambda$. By taking derivative of $\mathbf{K}\phi = \lambda\mathbf{M}\phi$ with respect to **v**, one can show that

$$\frac{\partial \Lambda}{\partial \mathbf{v}} = \phi^T \frac{\partial \mathbf{K}}{\partial \mathbf{v}} \phi$$

For linear structures, **K** is a linear function of **v**. Therefore, the derivative of **K** with respect to **v** can be easily derived.

Substitute the above equation into the conditional probability, one gets

$$f_{\Lambda|\mathbf{V}}(\mathbf{\Lambda}\,|\,\mathbf{v}) = \mathrm{N}\left(\boldsymbol{\mu}_\lambda, \mathbf{C}_\lambda\right)$$

in which $\boldsymbol{\mu}_\lambda$ = $\sigma_\lambda^{-2}\mathbf{C}_\lambda \mathbf{H}^T\left(\tilde{\Lambda}_0 - \Lambda\right)$, and $\mathbf{C}_\lambda = \sigma_\lambda^2\left(\mathbf{H}^T\mathbf{H}\right)^{-1}$, where $\mathbf{H} = -\phi^T \left.\frac{\partial \mathbf{K}}{\partial \mathbf{v}}\right|_{\mathbf{v}_0} \phi$, and $\tilde{\mathbf{\Lambda}}_0 = \mathbf{\Lambda}_0 - \mathbf{H}\mathbf{v}_0$.

Suppose the prior distribution of **V** is

$f'_{\mathrm{V}}(\mathbf{v}) = \mathrm{N}(\boldsymbol{\mu}'_{\mathrm{V}}, \mathbf{C}'_{\mathrm{V}})$, where $\boldsymbol{\mu}'_{\mathrm{V}}$ is the prior mean vector, and $\mathbf{C}'_{\mathrm{V}}$ is the prior covariance matrix. Then, the posterior distribution of **V** is

$$f''_{\mathrm{V}}(\mathbf{v}) = \mathrm{N}(\boldsymbol{\mu}''_{\mathrm{V}}, \mathbf{C}''_{\mathrm{V}})$$

where

$$\boldsymbol{\mu}''_{\mathrm{V}} = \left(\mathbf{C}'^{-1}_{\mathrm{V}} + \mathbf{C}^{-1}_{\lambda}\right)^{-1}\left(\mathbf{C}'^{-1}_{\mathrm{V}}\boldsymbol{\mu}'_{\mathrm{V}} + \mathbf{C}^{-1}_{\lambda}\boldsymbol{\mu}_{\lambda}\right)$$

$$\mathbf{C}''_{\mathrm{V}} = \left(\mathbf{C}'^{-1}_{\mathrm{V}} + \mathbf{C}^{-1}_{\lambda}\right)^{-1}$$

are the posterior mean vector and covariance matrix of **V**, respectively.

Next, consider $\mathbf{t} = \boldsymbol{\phi}$ = measured modal shape. For a given set of element properties, suppose $\boldsymbol{\phi} = N\left(\bar{\phi}, \sigma^2_{\phi}\mathbf{I}\right)$, where $\bar{\phi}$, the mean vector of $\boldsymbol{\phi}$, is obtained by solving $\mathbf{K}\phi = \lambda\mathbf{M}\phi$. Then,

$$f_{\phi|\mathrm{V}}(\phi \mid \mathbf{v}) = \mathrm{N}\left(\bar{\phi}, \sigma^2_{\phi}\mathbf{I}\right)$$

$$= \frac{1}{(2\pi)^{m/2}\sigma^m_{\phi}} \exp\left[-\frac{1}{2\sigma^2_{\phi}}\left(\phi - \bar{\phi}\right)^T\left(\phi - \bar{\phi}\right)\right]$$

Again, expand $\bar{\Lambda}$ into a Taylor's series:

$$\phi(\mathbf{v}) = \phi_0 + \left.\frac{\partial\phi}{\partial\mathbf{v}}\right|_{\mathbf{v}_0} \Delta\mathbf{v} + \cdots$$

where $\frac{\partial\phi}{\partial\mathbf{v}}$ can be obtained by taking derivative of $\mathbf{K}\phi = \lambda\mathbf{M}\phi$ with respect to **v.**

Substitute the above equation into the conditional probability, one gets

$$f_{\phi|\mathrm{V}}(\phi \mid \mathbf{v}) = \mathrm{N}\left(\boldsymbol{\mu}_{\phi}, \mathbf{C}_{\phi}\right)$$

in which $\boldsymbol{\mu}_{\phi} = \sigma^{-2}_{\phi}\mathbf{C}_{\phi}\mathbf{R}^T\left(\tilde{\phi}_0 - \phi\right)$, and $\mathbf{C}_{\phi} = \sigma^2_{\phi}\left(\mathbf{R}^T\mathbf{R}\right)^{-1}$, where $\mathbf{R} = -\left.\frac{\partial\phi}{\partial\mathbf{v}}\right|_{\mathbf{v}_0}$, and $\tilde{\phi}_0 = \phi_0 - \mathbf{R}\mathbf{v}_0$.

Once the Bayesian modification is applied, one obtains the modified distributions of the structural properties. Then, reliability analysis can be carried out using the updated distributions.

六、與系統識別修正法之比較

The main difference between the proposed method and the conventional identification methods is that the Bayesian modification is adopted in the identification process. No optimization problem is solved in the proposed method. Obviously, the proposed method has the advantage that no iterations are required, and thus is more efficient. It also avoid the problem of local minima.

七、結論

This project develops a method to identify the element properties of a structure of using the modal data. Several conclusions can be made from this study:

1. The Bayesian method provides a systematic way of incorporating the structural response to update the distributions of the structural properties.
2. The proposed identification method does not require solution of an optimization problem.

七、計畫成果自評

本計畫預計完成的工作項目如下：

1. 建立以結構模態反應直接修正桿件性質的方法。
2. 發展電腦程式
3. 比較貝氏模態識別法及一般模態識別法。
4. 以數值算例驗証貝氏模態識別法。

在本年度內，本研究已達成前三項預定目標，數值算例正在分析中。