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Construction of a two-stage fuzzy management planning model for a computer integrated production management system

SUHUA HSIEH and SIMON HSU

Abstract. A production management system contains many qualitative descriptions and imprecise natures. The conventional crisp and/or stochastic model constructed in the computer integrated production management system (CIPMS) cannot describe these qualitative descriptions and imprecise natures. Therefore, it is difficult to mimic the way managers think, which is conceptual and comprehensive, and to absorb uncertainties such as order cancelled, unstable material supply etc, in a production system. This frequently accounts for why the CIPMS yields a poor performance. This paper presents a fuzzy approach to the CIPMS in order to model qualitative descriptions and imprecise natures. This approach includes two stages. In stage one, a management strategy can be determined in a way that is similar to the way humans think, in which ideas, pictures, images and value systems are formed. In stage two, a fuzzy linear programming model is developed to absorb these imprecise natures in a production system. In doing this, CIPMS can adapt a variety of non-crisp problems in an actual system, thereby improving the performance of CIPMS.

1. Introduction

To ensure the effectiveness of communication within a production system and its ability to respond quickly to the fluctuating changes of market demand, computer integrated production management systems (CIPMS) have gradually replaced the manual production management method. Although it is a highly effective tool for production management and control, CIPMS requires further development before it will find use in practical applications. Derks (1988) cited ten traps to avoid when planning CIPMS. Willis (1990) established five rules on how to implement CIPMS successfully. A production system is usually in a fuzzy and/or imprecise environment within which confusions occasionally occur. Many qualitative descriptions and imprecise natures need to be modelled in the CIPMS to mimic the way managers think and to absorb uncertainties, such as order cancelled, unstable material supply etc. This frequently accounts for why the CIPMS yields a poor performance despite performing many tasks and devoting material resources and time.

Owing to the ingenious and efficient nature of human thinking, experienced high-level managers normally adopt qualitative and equivocal approaches to determine a corporation's management strategies. Managers can normally reach a decision in a conceptual or comprehensive way when encountering problems with thousands of information bits. Once CIPMS is adopted, crisp, deterministic, and stochastic models are constructed and installed in the computer. Computers, in terms of analysing and simulating all possible events and, ultimately, obtaining the optimal solutions, can replace managers. However, real situations are frequently not crisp, deterministic or stochastic and can also not be described adequately in the same manner. If these models were used to describe an actual system, the nearly complete description would require far more detailed data than a human could recognize. Thus, CIPMS do not work well in general.

The fuzzy theory bridges the gaps between the actual system and the model describing the actual system. Zadeh (1965) proposed the fuzzy subset and membership function. Fuzzy set theory provides a strict mathematical framework in which vague conceptual phenomena can be precisely and rigorously studied. Fuzzy theory can also be considered as a modelling language, which can handle a large fraction of semantic contents or uncertainties of actual life situations. Due to

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its generality, this theory can be applied to different circumstances and contexts. However, in many cases this entails context-dependent modification and specification of the original concepts of the formal fuzzy set theory. Unfortunately, this adaptation has not yet progressed to a satisfactory level.

Once a high-level manager determines the corporation's management strategies, the middle-level executors must implement the aggregate production planning, and then perform the production in the same manner as the planning. In any planning setting, a decision appears to be either a good or poor decision, depending on the market demand after the product is made. A decision is not good or bad in itself, but only relative to the circumstance in which the influence of the decision is felt. Obviously, the future status of the global economy cannot be precisely known, necessitating the decision to be made under imprecise conditions.

Since the late 1950s, the imprecise nature of a production management system has been thoroughly investigated using a stochastic approach for decision models where input data have been given probability distributions (Beale 1955, Dantzig 1955, Charnes *et al.* 1958, and Charnes and Cooper 1959). Zadeh (1978) examined the feasibility of using fuzzy sets as the basis for a theory of possibility. Subsequent research on the possibility theory has been extensive (Yazenin 1987, Buckley 1990, Tanaka and Asai 1984, Rommelfanger *et al.* 1989, Luhandjula 1987, Lai and Hwang 1992, 1993). The notion of a fuzzy set provides the basis for constructing of a conceptual framework that parallels, in many aspects, the framework used in the case of ordinary sets, but is more general than the latter and, potentially, has a much wider scope of applicability. Such a framework provides a natural means of dealing with problems in which impreciseness is largely attributed to the absence of sharply defined criteria of class membership rather than the presence of random variables.

Zadeh pioneered the fuzzy theory in 1965. While commenting on the fuzzy set theory, Goguen (1967, 1969) attempted to describe the fuzzy properties of the real world efficiently. Subsequent observations on the fuzzy theory or related applications were made. In the early period, Bellman and Zadeh (1970) proposed the fuzzy decision model. Tanaka *et al.* (1974, 1976) proposed fuzzy mathematical programming and the fuzzy decision, and then applied them to investment problems. In a related work, Negoita and Sularia (1976) discussed the fuzzy mathematical optimization problem. Dubois and Prade (1978) extended the usual algebraic operations on real numbers to fuzzy numbers by adopting a fuzzification principle. Zimmermann (1976) applied fuzzy theory to describe and optimize the fuzzy system. That same investigator proposed fuzzy linear programming with several objective functions (Zimmermann 1978). Fuzzy-related research has received increasing attention since the 1980s. Kacprzyk and Staniewski (1982) applied the fuzzy decision theory to formulate a long-term inventory policy. Karwowski and Evans (1986) extended fuzzy concepts to production management research. Lehtimaki (1987) used fuzzy set theory to determine a master production schedule. Later, Hintz and Zimmermann (1989) applied the fuzzy theory to control a flexible manufacturing system. Werners (1987) described how to use the membership function to represent a fuzzy goal. Trappey *et al.* (1988) further constructed the fuzzy nonlinear programming model to represent a nonstochastic and very complex manufacturing environment. Fuzzy theory applications in a production system have grown considerably in recent years. Masnata and Settineri (1997) and Gindy and Ratchev (1997) applied the fuzzy theory to effectively manage manufacturing cells. Noto La Diega *et al.* (1996) later applied the fuzzy theory to production planning, while Grabot and Geneste (1994), Stanfield and Joines (1996) and Kuroda and Wang (1996) applied it to production scheduling. Furthermore, Perrone and Noto La Diega (1996) applied the fuzzy theory to flexible manufacturing systems.

While previous investigations have either applied the fuzzy decision theory to certain production problems or have developed fuzzy linear programming models for production planning, the fuzzy theory has seldom been applied to CIPMS. In addition, to our knowledge, an integration fuzzy model has never been developed for the two-stage — high-level management and middle-level execution — problems mentioned above. Therefore, this study presents a novel two-stage fuzzy planning model for CIPMS. The proposed model can effectively handle the qualitative descriptions and imprecise natures of a production system, thereby improving the performance of CIPMS.

For qualitative descriptions in a high-level management environment, fuzzy set and fuzzy decision theories are used to construct a mimic–manager–decision model. By doing so, the optimal corporation management strategies can be determined in a manner similar to human thinking. Since human thinking is subjective and instinctive, experienced managers inevitably make incorrect decisions. The main purpose of stage one in this research is to present a model enabled with human thinking and feeling, but this model is not committed to making the correct decision. Therefore, analysis of the results or the performance of the proposed model will not be discussed in this paper.

For the imprecise nature of the middle-level execution environment, fuzzy linear programming is employed to construct an aggregate production planning model. Fuzzy production planning can therefore be achieved when the actual demand is uncertain. Finally, a case study demonstrates the effectiveness of the proposed model. An analysis and comparison with the crisp plan are given.

2. Two-stage fuzzy model for CIPMS

The production activity consists of a series of organized and correlated activities. The production system acquires customer demand information from the market. The company manager must then determine management strategies. Next, executors implement optimal aggregate production plans, and prepare materials, labour and equipment. Finally, workers produce and deliver products to the customers. Complaints and the fluctuating requirements of customers are fed back to the system and then used to adjust the company management strategy and aggregate production plan. The above steps form a production system cycle. A high-level manager normally determines the management strategy, while the middle-level staff implement the aggregate production plan. In this section, we construct a fuzzy model to resolve the two-stage production problem in a CIPMS environment.

2.1. *Qualitative properties of a high-level environment*

High-level managers pursue the goals of an enterprise while adapting to a changing environment. This is the underlying premise of strategic planning. Planning environments faced by managers contain qualitative properties. The qualitative property is included in nearly all potential impacts. These impacts are generally not clearly related to other impacts. Notable examples of these potential impacts include economic, social, and political problems, technological development as well as current and potential competitors. At this level, adopting a qualitative perspective is more prevalent than a quantitative one. Managers accumulate all kinds of information that could be directly or indirectly related to their company. Managers evaluate problems by assigning a weight to each of the problems using their own unique judgement, and then arranging the problems according to the weight or their preference. Hence, the decision-making strategy contains many qualitative properties.

2.2. *Imprecise properties of a middle-level environment*

Middle-level staff employ concrete methods to implement aggregate production planning in order to achieve a high-level manager's strategic goal. Once the high-level decision is made, the maximum production resources and capability are determined. Therefore, aggregate production planning has the following features: (1) it originates from a quantitative perspective more than from a qualitative one; (2) it seeks to fulfil multiple objectives; and (3) it is short-term planning. The environment is not as complex as the high-level stage. However, the product demand, exact available workforce, reliability of equipment and materials supply conditions change daily. Therefore, this stage contains many imprecise properties.

2.3. *Construction of a high-level fuzzy model*

Construction of a high-level fuzzy model to determine the management strategy can be divided in the following steps.

Step 1. List the entire possible goals and constraints of the corporation. The possible items for the corporation's goals or constraints depend on the type of corporation, and the experience, knowledge, and preference of managers. All common corporations' goals and constraints are listed below.

(1) Goals. G_1: obtaining the maximum profit or lowest production cost,
G_2: owning the corporation's leadership in a product market,
G_3: maintaining the best competition policies such as to ensure the product delivery in time, or to accept changes in the purchase order from customers.

(2) Constraints. C_1: limited to the level of economic prosperity,
C_2: limited to the production capacity,
C_3: limited to the ability of research and development.

Step 2. Establish the membership function for each corporation's goal or constraint. Once the corporation's goals and constraints in step 1 are defined, the measurement standards to achieve the corporation's goals and constraints must be established. The membership functions exactly represent the measurement standards. Figure 1

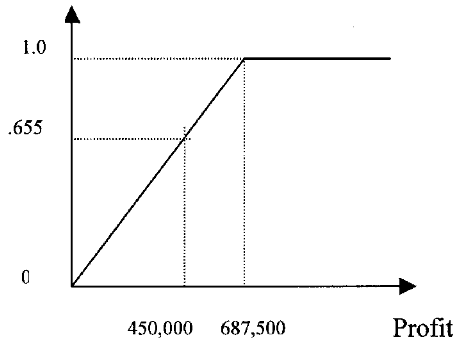

Figure 1. The membership function of maximum profit for a company.

illustrates the membership function of maximum profit for a company. When the company makes a profit equal to, or more than, \$687 500, one degree of membership is reached, indicating that people think that the company makes maximum profit. The company making a profit of \$450 000 indicates that 65.5% people think that the company makes maximum profit.

Step 3. Propose all the possible management strategies for the corporation. In addition, assume that there are k (k is a limited positive integer) possible strategies, $N_1, N_2, \ldots, N_k$, proposed. For example:
N_1: Satisfy the entire customer demands completely, regardless of any problem.
N_2: Never manufacture products when they exceed the maximum production scale.

Step 4. Estimate the degree of membership of each goal or constraint under different strategies. In addition, estimate a group of membership degrees, $\mu_{G_{mn}}(N_i)$ or $\mu_{C_{mn}}(N_i)$, for every strategy N_i.

Step 5. Select the strategy among the strategies of $N_1, N_2, \ldots, N_k$. For simulating the human decision-making process, the rationality of the Max-Min operator is used here. The optimal strategy can be determined by selecting the strategy that has the Max-Min degree of membership, i.e.

Max[Min $\mu_{G_{mn}}(N_i)$ or $\mu_{C_{mn}}(N_i)$ for a particular strategy N_i], $i = 1, \ldots, k$.

2.4. *Construction of a middle-level fuzzy model*

Middle-level staff perform the aggregate production plan by following the corporation's management strategy. Owing to the many imprecise properties in this stage, the fuzzy linear programming model is used here. Steps to construct the fuzzy linear programming model are described in the following.

Step 1. Construct a crisp linear programming model. By assuming that the aggregate production plan includes variables $x_1, x_2, x_3, \ldots, x_n$, a variable vector, $X = [x_1\ x_2\ x_3 \ldots \ldots x_n]^{\mathrm{T}}$, $X \in R^{n\times 1}$, can represent them. In general, the objective function is either to maximize profit or to minimize cost, i.e.

$$\text{Max(or Min)} f_a(X) = C_a^t X, \quad a = 1, 2, 3, \ldots, n$$

where

$f_a \in R^1$,
$C_a \in R^{n\times 1}$: the unit cost vector for each variable,

subject to $DX \leq B$, $X \geq 0$, where

$D \in R^{r\times n}$
$B \in R^{r\times 1}$.

Step 2. Construct the fuzzy constraints. As described above, some resources and demands in the production system are imprecise. Therefore, some elements in constraint vector B could be fuzzy numbers. Then the constraints can be modified as follows.

The fuzzy constraint portion $AX \leq \tilde{B}$
The non-fuzzy constraint portion $NX \leq B'$, $X \geq 0$

$$\left(\frac{A}{N}\right) = D \text{ and } \left(\frac{\tilde{B}}{B'}\right) = B \text{ and } [\tilde{B} = \tilde{b}_1, \tilde{b}_2, \ldots, \tilde{b}_p].$$

Figure 2 depicts the assumption of the membership function of these fuzzy resources $\tilde{b}_j$, $j = 1, 2, \ldots, p$, where $\tilde{b}_j = [{}_a b_j, {}_b b_j, {}_\ell b_j, {}_r b_j]$ is a fuzzy constraint, ${}_\ell b_j$ is the minus tolerance for $\tilde{b}_j$ and ${}_r b_j$ is the plus tolerance for $\tilde{b}_j$.

Correspondingly, the membership function $\mu_j(X)$ of $\tilde{b}_j$ can be derived.

$$\mu_j(x) = \begin{cases} 0 & (AX)_j \leq {}_a b_j - {}_\ell b_j \\ \dfrac{(AX)_j - {}_a b_j + {}_\ell b_j}{{}_\ell b_j} & {}_a b_j - {}_\ell b_j < (AX)_j \leq {}_a b_j \\ 1 & {}_a b_j < (AX)_j \leq {}_b b_j \\ \dfrac{{}_b b_j + {}_r b_j - (AX)_j}{{}_r b_j} & {}_b b_j < (AX)_j \leq {}_b b_j + {}_r b_j \\ 0 & (AX)_j > {}_b b_j + {}_r b \end{cases} \tag{1}$$

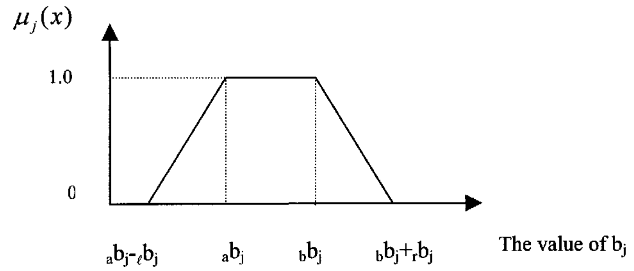

Figure 2. The membership function of the fuzzy constraint.

According to α-cut, a variable λ, $\forall \mu_j(x)$, $\mu_j(x) \geq \lambda = \alpha$ is introduced. Then, the constraint function can be written as

$$\begin{cases} (AX)_j - \lambda\, {}_\ell b_j \geq {}_a b_j - {}_\ell b_j, \\ (AX)_j + \lambda\, {}_r b_j \leq {}_b b_j + {}_r b_j, \end{cases} \quad j = 1, 2, 3, \ldots, p \tag{2}$$

$NX \leq B'$
$X \geq 0$.

Step 3. Construct the fuzzy objective function: since the constraint is fuzzified, the objective function must be fuzzified as well.

$$f_a(X) = C_a^t X, \quad a = 1, 2, 3, \ldots, n$$

where $f_a(X) \in [O_a, P_a]$, O_a is the optimistic value of the purpose function, and P_a is the pessimistic value of the purpose function.

When the objective function is to maximize profit, the membership function, $\mu_a(X)$ can be obtained,

$$\mu_A(X) = \begin{cases} 1 & C_a^t X > O_a \\ \dfrac{C_a^t X - P_a}{O_a - P_a} & P_a \leq C_a^t X \leq O_a \\ 0 & C_a^t X < P_a \end{cases} \tag{3}$$

and when the objective function is to minimize cost, the membership function, $\mu_a(X)$ can be obtained,

$$\mu_a(x) = \begin{cases} 1 & C_a^t X < O_a \\ \dfrac{P_a - C_a^t X}{P_a - O_a} & O_a \leq C_a^t X \leq P_a \\ 0 & C_a^t X > P_a \end{cases} \tag{4}$$

Step 4. Construct the fuzzy linear programming model. When seeking the maximum profit or minimum cost, the model becomes

Maximize λ
Subject to

$C_a^t X - \lambda(O_a - P_a) \geq P_a, \quad a = 1, 2, \ldots, n,$ when seeking the maximum profit,
$C_a^t X + \lambda(P_a - O_a) \leq P_a, \quad a = 1, 2, \ldots, n,$ when seeking the minimum cost,
$(AX)_j - \lambda_\ell b_j \geq {}_a b_j - {}_\ell b_j,$
$(AX)_j + \lambda\, {}_r b_j \leq {}_b b_j - {}_r b_j, \quad j = 1, 2, \ldots, p,$
$NX \leq B',$
$X \geq 0.$ (5)

Step 5. Search for the optimal solution by using LINDO.

3. Illustrative example

This study considers a manufacturer of sunglasses as an illustrative example. As assumed herein, the manufacturer produces only one kind of sunglasses, the optimal production capacity is 25 000 pairs monthly and each pair costs \$500. If the production quantity exceeds the optimal production capacity, then the cost per pair for the exceeded portion is \$650. The monthly fixed production cost is \$860 000. The manufacturer has only one major material supplier. The optimal material supply quantity equals the optimal production quantity {Pi-1}+5%. Notably, market prosperity affects the stability of the material supply. The more prosperous is the market, the less stable is the supply; otherwise the supply is more stable. The major customers are A and B. Customer A is the first major customer, customer B is the second major customer, and others are not important and are not considered here. The manufacturer provides customer A at \$550 each pair and customer B at \$570 each pair. Upon receipt of the purchase order from customers, the manufacturer must deliver the product in time. If not, the manufacturer gives 5% and 2% rebate of the delayed product amount to customers A and B, respectively. Different customers have different levels of tolerance to the product delay. Whenever customers are not tolerant of the delay, they cancel their purchase orders. For a delay of less than or equal to 0.1 month, both customers A and B can 100% accept such a delay. However, the tolerance linearly decreases if the delay lasts longer. For customer A, the tolerance is 100% absent when the delay is equal to, or over, two months. For customer B, the tolerance is 100% absent when the delay is equal to, or over, three months. Market prosperity affects the possibilities that customers fulfil the purchase order. The more prosperous the market, the higher the possibility that the customer fulfils the purchase order; otherwise the possibility is lower. Table 1 lists the next four-month purchase orders that the manufacturer received.

3.1. *Construction of a high-level fuzzy model*

In this section, we sequentially carry out the five steps described in section 3.

Step 1. List all the possible items of the corporation's goals and constraints.

(1) Goals. Assume that the manufacturer has only one goal, G_1, i.e. to reach the maximum profit. In this example, the maximum profit possible is defined as 5% of the gross income at the optimal production quantity, 25 000 pairs per month, and the lowest sale price, \$550 per pair, i.e. maximum profit possible = \$550/pair × 25 000 pairs/month × 5% = \$687 500/month.

(2) Constraints.

(a) The constraint of the optimal production quantity, C_1. As expected, the production quantity is greater than or equal to the minimum production quantity and less than or equal to the maximum production quantity. Due to the monthly fixed cost of \$860 000, the manufacturer's production quantity should exceed a specific quantity so that the manufacturer can pay for the fixed cost. The specific quantity is exactly the minimum production quantity, and is equal to

Minimum production quantity = \$860,000/month/(\$550 − \$500)/pair = \$17,200 pair/month

In addition, the manufacturer never produces products if no profit can be made. The maximum production quantity is defined as the production quantity in which the production cost is equal to the gross income. In this example, the maximum production quantity is equal to

Maximum production quantity = [\$(550 − 500) × 25 000 pairs/month − \$860 000/month)] ÷ \$(650 − 550)/pair + 25 000 pairs = 28 900 pairs/month.

(b) The constraint of the optimal material supply quantity, C_2. The material supply quantity can never exceed the maximum production quantity, 28 900 pairs, and never be less than the minimum production quantity, 17 200 pairs. The company would be satisfied if the material supply quantity is equal to 25 000 pairs ±5%.

(c) The constraint of the stability of the material supply, C_3. The material supplier may or may not supply the material as they promised. Market prosperity affects the stability of the material supply.

(d) The constraint of the customer's tolerance to the product delay, C_4. When no longer tolerating the delay, customers cancel their purchase orders. The tolerance linearly decreases with a longer delay.

(e) The constraint of the fulfilment of the purchase order by customer, C_5. Market prosperity affects the possibility that customers fulfil the purchase orders.

Step 2. Establish the membership function for each goal or constraint.

(1) Goals. Figure 1 illustrates the membership function of the maximum profit.

$\mu_{G_1} = 1$, if profit $\geq 687\,500$,
$\mu_{G_1} =$ profit/687 500, if $0 \leq$ profit $\leq 687\,500$,
$\mu_{G_1} = 0$, if profit ≤ 0.

(2) Constraints.

(a) Figure 3 illustrates the membership function of the optimal production quantity, C_1.

$\mu_{C_1} = 0$, if the production quantity $< 17\,200$,

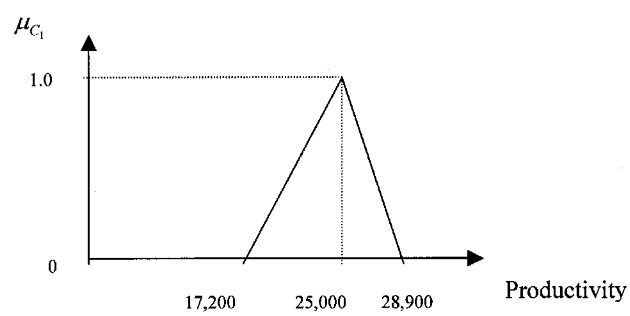

Figure 3. The distribution of the membership function of the optimal production quantity.

μ_{C_1} = (28 000−the production quantity)/(28 900−25 000), if 25 000 ≤ the production quantity ≤ 28 900, and

μ_{C_1} = 0, if the production quantity > 28 900.

(b) Figure 4 illustrates the membership function of the optimal material supply quantity.

μ_{C_2} = 0, if the material supply quantity ≤ 17 200,

μ_{C_2} = (supply quantity−17 200)/(23 750−17 200), if the supply quantity > 17 200 and the supply quantity ≤ 23 750,

μ_{C_2} = 1, if the supply quantity > 23 750 and the supply quantity ≤ 26 250,

μ_{C_2} = (28 900−supply quantity)/(28 900−23 750), if the supply quantity > 23 750 and the supply quantity ≤ 28 900, and

μ_{C_2} = 0, if the supply quantity > 28 900.

(c) Figure 5 illustrates the membership function of the stability of the material supply. The stability of the material supply is linearly decreased when the market prosperity is from fair to good. The stability of the material supply is perfect when the market prosperity is poor.

μ_{C_3} = 1, if market prosperity is poor;
the average degree of the membership, $\bar{\mu}_{C_3}$ = 0.9, if the market prosperity is fair;
the average degree of the membership, $\bar{\mu}_{C_3}$ = 0.7, if the market prosperity is good.

(d) Figure 6 illustrates the membership function of customer's tolerance to product delay. The customer's tolerance linearly decreases with an increase of the delay time.

μ_{C_4} = 1.0, if the delay time is less than or equal to 0.1 month for both A and B,

μ_{C_4} = 0.667 for customer A and μ_{C_4} = 0.8 for customer B, if the delay time is 1.0 month,

μ_{C_4} = 0 for customer A and μ_{C_4} = 0.4 for customer B, if delay time is 2.0 months,

μ_{C_4} = 0 if the delay time is greater than 2 months for customer A and greater than 3 months for customer B.

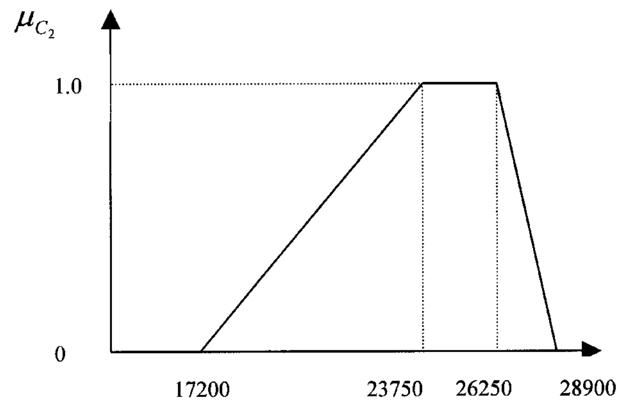

Figure 4. The distribution of the membership function of the optimal material supply quantity.

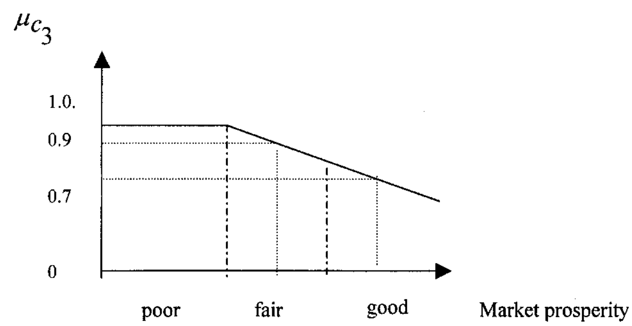

Figure 5. The distribution of the membership function of the stability of the material supply.

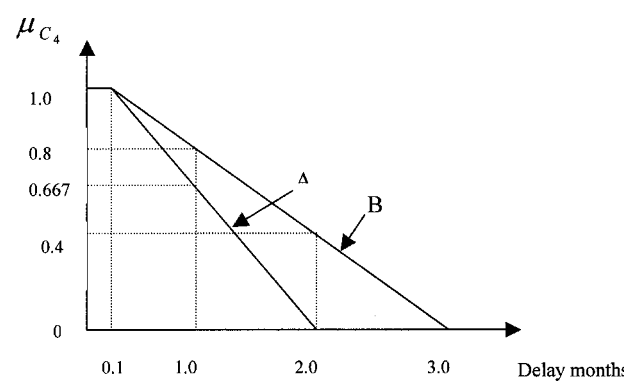

Figure 6. The distribution of the membership function of the customer's tolerance to the product delay.

(e) Figure 7 illustrates the membership function of customers' fulfilment of the purchase order. The possibility of a customer's fulfilment of the purchase linearly increases with an in-

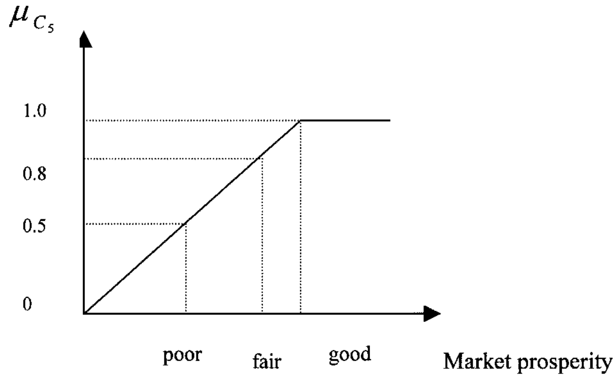

Figure 7. The distribution of the membership function for the customer's fulfilment of the purchase order.

crease of the prosperity of the market:
the average degree of the membership, $\bar{\mu}_{C5} = 0.5$, if the market prosperity is poor;
the average degree of the membership, $\bar{\mu}_{C5} = 0.8$, if the market prosperity is fair;
$\mu_{C_5} = 1.0$, if the market prosperity is good.

Step 3. Propose all possible management strategies. Assume that the company has the following possible strategies, N_i, $i = 1,2,3,4$.

N_1: Satisfy the entire customer demand completely, regardless the profit,

N_2: Treat customers equally and never produce more products if the optimal production capacity is reached. Therefore, when the accumulative quantity exceeds the optimal production capacity, each customer receives part of the order and shares the shortage equally.

N_3: Deal with the customer who has the most shortage and never produces any product once the optimal production capacity is reached. Therefore, the customer with a maximum delayed product quantity has the highest priority to receive the order. When none of the customers have a shortage, customer A would have higher priority than the others.

N_4: Satisfy the maximum profit order prior to others when the accumulative quantity of orders exceeds the optimal production capacity.

Step 4. Estimate the degree of membership of each goal or constraint under different strategies. Based on the membership function of each goal or constraint, the degree of membership of each goal or constraint can be estimated under different strategies. Therefore, the fuzzy evaluation chart for different strategies can be established.

(1) Fuzzy evaluation of the maximum profit goal. According to the order received, table 2 summarizes the company profits for different strategies. The degree of the membership for each strategy is calculated as follows:
N_1: $\mu_{G_1}(N_1) = 0.685$, the average monthly profit is \$471 250;
N_2: $\mu_{G_1}(N_2) = 0.670$, the average monthly profit is \$460 469;
N_3: $\mu_{G_1}(N_3) = 0.660$, the average monthly profit is \$453 000;
N_4: $\mu_{G_1}(N_4) = 0.651$, the average monthly profit is \$447 813.

(2) Fuzzy evaluation of the constraints.

(a) Constraint of the optimal production quantity. Table 3 lists the degrees of the membership of the monthly optimal production quantity for each strategy. The average degree of the membership can thus be estimated.
N_1: the average degree of membership $\mu_{C_1}(N_1) = 0.600$,
N_2: the average degree of membership $\mu_{C_1}(N_2) = 0.920$,
N_3: the average degree of membership $\mu_{C_1}(N_3) = 0.920$,
N_4: the average degree of membership $\mu_{C_1}(N_4) = 0.920$,

(b) Constraint of the optimal quantity of the material supply. Table 4 estimates the degrees of the membership of the optimal material supply quantity per month. Then, the average degree of membership for each strategy is:
N_1: the average degree of membership $\mu_{C_2}(N_1) = 0.716$,
N_2: the average degree of membership $\mu_{C_2}(N_2) = 0.952$,
N_3: the average degree of membership $\mu_{C_2}(N_3) = 0.952$,
N_4: the average degree of membership $\mu_{C_2}(N_4) = 0.952$.

(c) Constraint of the stability of the material supply. As could be known, the market prosperity is fair for the next four months. The degrees of membership of the stability for different strategies for each month are all

0.9. Then the average degree of the membership of each strategy is:

N_1: $\mu_{C_3}(N_1) = 0.900$,
N_2: $\mu_{C_3}(N_2) = 0.900$,
N_3: $\mu_{C_3}(N_3) = 0.900$,
N_4: $\mu_{C_3}(N_4) = 0.900$.

(d) Constraint of customer's tolerance to the product delay. The degrees of the membership for each strategy are:

N_1: the maximum delay time for customers A and B is 0 month, $\mu_{C_4}(N_1) = 1.0$,
N_2: the maximum delay time for customers A and B is 3 months, $\mu_{C_4}(N_2) = 0$,
N_3: the maximum delay time for customers A and B is 1 month, $\mu_{C_4}(N_3) = 0.667$ for A and $\mu_{C_4}(N_3) = 0.800$ for B. The average degree of membership $\mu_{C_4}(N_3) = 0.734$,
N_4: The maximum delay time for customer A is 3 months, and the degree of membership $\mu_{C_4}(N_4) = 0$. The maximum delay time for customer B is 0 month, and the degree of membership $\mu_{C_4}(N_4) = 1.000$. The average degree of membership $\mu_{C_4}(N_4) = 0.500$.

(e) The constraint of the possibility of customers' fulfilment of the purchase order. As could be known, the market prosperity is fair in the next four months. The degrees of membership of the possibility that customers would carry out their orders are all 0.8. Then the average degree of membership of each strategy is:

Table 1. The next four-month purchase orders that the manufacturer received.

	Month			
Customer	Month 1	Month 2	Month 3	Month 4
A	20 000	17 500	12 500	17 500
B	2500	10 000	15 000	7500

Table 2. The estimated company profit according to the customer orders received.

Month		Month 1		Month 2		Month 3		Month 4	
Company		A	B	A	B	A	B	A	B
Ordered quantity		20 000	2500	17 500	10 000	12 500	15 000	17 500	7500
Policy N_1	Production quantity	20 000	2500	17 500	10 000	12 500	15 000	17 500	7500
	Shortage	0	0	0	0	0	0	0	0
	Gross income	11 000 000	1 425 000	9 625 000	5 700 000	6 875 000	8 550 000	9 625 000	4 275 000
	Rebate	0	0	0	0	0	0	0	0
	Cost	12 110 000		14 860 000		14 860 000		13 360 000	
	Profit	315 000		465 000		565 000		540 000	
Policy N_2	Production quantity	20 000	2500	16 250	8750	11 250	13 750	17 500	7500
	Shortage	0	0	1250	1250	2500	2500	2500	2500
	Gross income	11 000 000	1 425 000	8 937 500	4 987 500	6 187 500	7 837 500	9 625 000	4 275 000
	Rebate	0	0	34 375	14 250	68 750	28 500	68 750	28 500
	Cost	12 110 000		13 360 000		13 360 000		13 360 000	
	Profit	315 000		516 375		567 750		442 750	
Policy N_3	Production quantity	20 000	2500	17 500	7500	7500	17 500	22 500	2500
	Shortage	0	0	0	2500	5000	0	0	5000
	Gross income	11 000 000	1 425 000	9 625 000	4 275 000	4 125 000	9 975 000	1 275 000	1 425 000
	Rebate	0	0	0	28 500	137 500	0	0	57 000
	Cost	12 110 000		13 360 000		13 360 000		13 360 000	
	Profit	315 000		511 500		602 500		383 000	
Policy N_4	Production quantity	20 000	2500	15 000	10 000	10 000	15 000	17 500	7500
	Shortage	0	0	2500	0	5000	0	5000	0
	Gross income	11 000 000	1 425 000	8 250 000	5 700 000	5 500 000	8 550 000	9 625 000	4 275 000
	Rebate	0	0	68 750	0	137 500	0	137 500	0
	Cost	12 110 000		13 360 000		13 360 000		13 360 000	
	Profit	315 000		521 250		552 500		402 500	

Table 3. The degree of the membership of the monthly optimal production quantity.

Degree of membership	Month 1	Month 2	Month 3	Month 4
Strategy N_1	0.679	0.360	0.360	1.0
Strategy N_2	0.679	1.0	1.0	1.0
Strategy N_3	0.679	1.0	1.0	1.0
Strategy N_4	0.679	1.0	1.0	1.0

Table 4. The degree of the membership of the stability of the component supply.

Membership degree	Month 1	Month 2	Month 3	Month 4
N_1	0.809	0.528	0.528	1.0
N_2	0.809	1.0	1.0	1.0
N_3	0.809	1.0	1.0	1.0
N_4	0.809	1.0	1.0	1.0

N_1: $\mu_{C_5}(N_1) = 0.800$,
N_2: $\mu_{C_5}(N_2) = 0.800$,
N_3: $\mu_{C_5}(N_3) = 0.800$,
N_4: $\mu_{C_5}(N_4) = 0.800$.

The proposed high-level management strategy evaluation is performed and summarized in table 5.

Step 5. Select the management strategy:
the minimum degree of membership = 0.600 for strategy N_1,
the minimum degree of membership = 0 of strategy N_2,
the minimum degree of membership = 0.660 of strategy N_3, and
the minimum degree of membership = 0.500 of strategy N_4.
The optimal strategy is the one with the largest minimum degree of membership:

$$\underset{i}{\text{Max}} \; \underset{m}{\text{Min}} \; (\mu_{G_1}(N_i), \mu_{C_m}(N_i)) = 0.660$$

the optimal management strategy $D = N_3$.

As mentioned above, this part of the research is intended to present a model enabled with human thinking and feeling. Although it is selected, strategy 3 is not committed to be the best one.

3.2. *Construction of a middle-level fuzzy model*

High-level managers determine the optimal company management strategy, and middle-level staff execute the production in an imprecise/uncertain environment. Since the resources and demands in production system are imprecise, fuzzy numbers can represent some of the elements in the constraints. The given conditions, cost coefficients and variables of the production system are described as follows.

Table 5. The proposed high-level management strategy evaluation results.

Strategy	N_1	N_2	N_3	N_4
$\mu_{G_1}(N_i)$	0.685	0.670	0.660	0.651
$\mu_{C_1}(N_i)$	0.600	0.920	0.920	0.920
$\mu_{C_2}(N_i)$	0.716	0.952	0.952	0.952
$\mu_{C_3}(N_i)$	0.900	0.900	0.900	0.900
$\mu_{C_4}(N_i)$	1.000	0.000	0.734	0.500
$\mu_{C_5}(N_i)$	0.800	0.800	0.800	0.800

(1) Given conditions
  (a) The planning production period is 4 months.
  (b) The planned production quantity, R_i, under strategy N_3 is $R_1 = 22\,500$ pairs, and $R_2 = R_3 = R_4 = 25\,000$ pairs.
  (c) The initial manpower M_0 is 40 men/month.
  (d) The initial inventory stock, I_0, is 0 pairs.
  (e) The initial backorder, B_0, is 0 pairs.
  (f) The working hours per person per day are 8 hours.
  (g) The ratio of overtime and regular working hours cannot be greater than 20%.
  (h) The manufacturing time for a pair of glasses is 0.32 hours/pair.
  (i) There are 25 working days in a month.
(2) Cost coefficient
  C_P: material cost = \$420/pair.
  C_R: regular labour cost = \$15 000/person, month.
  C_H: hiring cost = \$5000/person, month.
  C_L: layoff cost = \$15 000/person, month.
  C_O: overtime labour cost = \$125/hour.
  C_I: inventory cost = \$3/pair, month.
  C_B: backorder cost = \$20/pair, month.
(3) Variables
  M_j: month j manpower,
  H_j: month j hiring work force,
  L_j: month j layoff work force,
  P_{nj}: month j production quantity in regular working hours,
  P_{oj}: month j production quantity in overtime,
  I_j: month j inventory level,
  B_j: month j backorder level.

Step 1. Construct a crisp linear programming model: Assume that the execution achievement goal minimizes the total production cost that includes (1) the material cost, (2) the inventory and backlog cost, (3) the personnel cost. Based on the above information, a crisp linear programming model is established as follows:

(1) The minimum production cost objects:

$$\text{Min} f_C = \sum_{j=1}^{4} C_p(P_{nj} + P_{oj}) + (C_I I_j + C_B B_j) + (C_R M_j + C_H H_j + C_L L_j + C_O(0.32 P_{Oj}))$$

It includes material cost, inventory cost, backlog cost, regular labour cost, hiring cost, layoff cost and overtime cost.

(2) Constraints:

(a) The planned production quantity is equal to the regular production quantity plus overtime production quantity plus the inventory difference between month j and month j–1 minus the backorder difference between month j and month j–1, i.e.

$$R_j = P_{nj} + P_{oj} + I_{j-1} - I_j - (B_{j-1} - B_j), \quad j = 1, 2, 3, 4$$

(b) The manpower level in month j is equal to the manpower level in month j–1 plus the hiring workforce in month j minus the layoff workforce in month j, i.e.

$$M_j = M_{j-1} + H_j - L_j, \quad j = 1, 2, 3, 4$$

(c) The regular working hours in a month must be less than or equal to the working day hours, i.e.

$$0.32 P_{nj} \leq 8 \times 25 M_j, \quad j = 1, 2, 3, 4$$

(d) The overtime work hours must be less than or equal to 20% of the working day hours, i.e.

$$0.32 P_{oj} \leq 0.2 \times 8 \times 25 M_j, j = 1, 2, 3, 4$$

(e) The total planned production quantity must be less than or equal to the total production quantity in the planning production period, i.e.

$$\sum_{j=1}^{N} R_j \leq \sum_{j=1}^{N} P_{nj} + P_{oj}, \quad j = 1, 2, 3, 4$$

(f) All the variables are not less than 0:

$$M_j, H_j, L_j, P_{nj}, P_{oj}, I_j B_j \geq 0, \quad j = 1, 2, 3, 4$$

Table 6. The crisp production plan.

Month	1	2	3	4
Planned regular production quantity	24 375	24 375	24 375	24 375
Planned inventory level	1875	1250	625	0
Planned manpower level	39	39	39	39
Planned layoff workforce	1	0	0	0

(g) Use a linear programming software to search the optimum crisp plan. A crisp plan is shown in table 6.

Step 2. Construct the fuzzy constraints. Owing to imprecise factors such as unstable material supply, customer order cancellation or change, poor products, and machine breakdown, the produced quantity is usually less than the planned one. Therefore, some of the planned resources, workforce or materials are wasted. Here, the planned production quantity is represented by a fuzzy number that gives a minus tolerance of 10% on the original planned production quantity. As expected, the fuzzy planning can provide less waste than the crisp planning. The fuzzy planned production quantity can be represented as follows:

$\tilde{R}_1 = [22500, 22500, 2250, 0]$
$\tilde{R}_2 = [25000, 25000, 2500, 0]$
$\tilde{R}_3 = [25000, 25000, 2500, 0]$
$\tilde{R}_4 = [25000, 25000, 2500, 0]$

According to equations (1) and (2) the original constraint

$$R_j = P_{nj} + P_{oj} + I_{j-1} - I_j - (B_{j-1} - B_j), \quad j = 1, 2, 3, 4$$

can be transferred into

$$P_{n1} + P_{o1} + I_o - I_1 - (B_0 - B_1) - 2250\lambda \geq 20250$$
$$P_{nj} + P_{oj} + I_{j-1} - I_j - (B_{j-1} - B_j) - 2500\lambda \geq 22500, \quad j = 2, 3, 4$$

Step 3. Construct the fuzzy objective function: Since the constraint is fuzzified, the objective function must be fuzzified as well. According to equation (4), we obtain

$$\mu_G(X) = \begin{cases} 1 & f_C(X) \geqslant O_a \\ \dfrac{P_a - f_C(X)}{P_a - O_a} & O_a \leq f_C(X) \leq P_a \\ 0 & f_C(X) > P_a \end{cases}$$

where $O_a = \min f_C(X)$, and $P_a = \max f_C(X)$.

Step 4. Construct the fuzzy linear programming model. According to steps 1 to 3 and equation (5), the result is summarized as follows:

Maximise λ
Subject to

$$f_C(X) + \lambda(P_a - O_a) \leq P_a,$$
$$P_{n1} + P_{o1} + I_0 - I_1 - (B_0 - B_1) - 2250\lambda \geq 20250$$
$$P_{n1} + P_{o1} + I_0 - I_1 - (B_0 - B_1) \leq 22500$$
$$P_{nj} + P_{oj} + I_{j-1} - I_j - (B_{j-1} - B_j) - 2500\lambda \geq 22500, \quad j = 2,3,4$$
$$P_{nj} + P_{oj} + I_{j-1} - I_j - (B_{j-1} - B_j) \leq 25000, \quad j = 2,3,4$$
$$M_j = M_{j-1} + H_j - L_j, \quad j = 1,2,3,4$$
$$0.32P_{nj} \leq 8 \times 25M_j, \quad j = 1,2,3,4$$
$$0.32P_{oj} \leq 0.2 \times 8 \times 25M_j, \quad j = 1,2,3,4$$
$$\sum_{j=1}^{N} R_j \leq \sum_{j=1}^{N} P_{nj} + P_{oj}, \quad j = 1,2,3,4$$
$$M_j, H_j, L_j, P_{nj}, P_{oj}, I_j, B_j \geq 0, \quad j = 1,2,3,4$$

Step 5. Obtain the optimal solution. A linear programming package LINDO (Linear INteractive and Discrete Optimizer; see Schrage 1989) is used to search for O_a, P_a and the optimum solution. After the compromise among objectives and constraints, we obtain:

$\lambda = 0.7446$, and a fuzzy production plan shown in table 7.

Environmental conditions and resources are time-variant. To verify the effectiveness of the proposed fuzzy plan, several numerical experiments are conducted. Perturbing the planned production quantity by 1% to 10% to generate the forecast errors can simulate ten actual production quantities. If the actual production quantity is greater than that planned, the shortage quantity can be produced over time. If the actual production quantity is less than that planned, the excess quantity can be stored in the inventory. The actual production cost can thus be estimated. Figure 8 illustrates the comparison between the fuzzy plan and the crisp plan for the ten cases.

Table 7. The fuzzy production plan.

Month	1	2	3	4
Planned regular production quantity	23 752	23 752	23 752	23 752
Planned inventory level	1827	1218	609	0
Planned manpower level	38	38	38	38
Planned layoff workforce	2	0	0	0

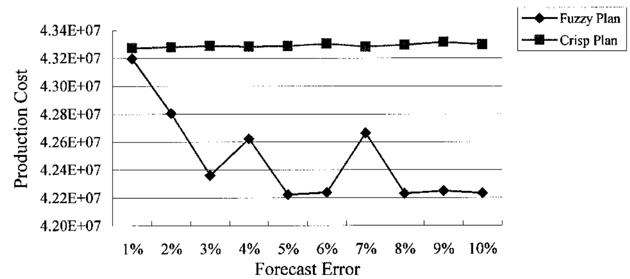

Figure 8. The comparison between the fuzzy plan and the crisp plan on different forecast errors.

3.3. *Summary*

This section has demonstrated the two-stage fuzzy planning model in the CIPMS for the sunglasses manufacturer. This case study confirms that high-level strategic decision and middle-level production planning can be determined in a fuzzy way.

4. Conclusions

CIPMS largely focuses on gaining maximum profits through efficient management/administration. However, the production environment is full of qualitative descriptions and imprecise natures. Thus, using the conventional approach yields an unsatisfactory performance of CIPMS. This study focuses mainly on the qualitative properties in a high-level decision stage. A high-level fuzzy model is also proposed. The management strategy can therefore be determined through the computer in a qualitative way that closely resembles human thinking. Also addressed here are the imprecise properties in the production planning period, in which a middle-level fuzzy model is proposed. The optimal production planning can therefore be obtained at the moment that factors such as the actual demand, resources and manpower are uncertain. The proposed two-stage fuzzy model enables CIPMS to adapt to the complex and changeable environment. Therefore, the model proposed here improves CIPMS.

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