# 行政院國家科學委員會專題研究計畫　成果報告

## 流動性限制之倒帳與不確定性

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# 摘要

關鍵詞：流動性限制之倒帳；不確定性；貨幣；信用

貨幣搜尋模型已被廣泛用來研究貨幣的價值，以及貨幣如何促進交易、增進福利等議題。由於信用也是現實生活中重要的支付工具，學者們已著手修改貨幣搜尋模型的一些假設以討論貨幣與信用的相關問題，然而這些研究卻都忽略了倒帳風險。由於信用交易得以進行的最重要條件是人們對於債權得以確保的信心，因此研究倒帳的誘因與相關問題便很重要。除了惡意倒帳之外，人們倒帳也可能與能力有關，例如因生產面的衝擊造成流動性限制而無法償還債務，這是本文所謂的「流動性限制之倒帳」。我們考慮一個具有金融中介的一般均衡搜尋模型，金融中介促進信用交易的進行，但它所具有的執行力是有限的。如果我們將信用額度視爲對不確定性的保險程度的高低，則在此架構下可以討論倒帳誘因、總體信用風險以及信用市場所提供的保險程度之高低等因素的關聯。我們初步發現如下：如果技術許可的話，讓不同倒帳記錄的人有不同的信用額度可能增進福利，也就是說，容許流動性限制之倒帳者比惡意倒帳者在未來進行信用交易時有較低的限制以及較高的信用額度，將更能發揮信用交易的保險功能，然而，在某些情況下卻可能提高人們倒帳的誘因，如果前者的效果夠大，那麼，容許流動性限制之倒帳者較高的信用額度可以增進福利。此外，貨幣交易相對於信用市場的利得越高則越可能提高人們倒帳的誘因。

**Abstract**

Keywords: default; production uncertainty; credit; money

Search-theoretic models have successfully been used to demonstrate how decentralized allocations can be expanded by engaging in money-based bilateral trades. An obvious next step is to relax some of the assumptions to study allocations in similar environments when agents can engage in intertemporal trading arrangements. A common feature in previous studies is that default is either ignored or inconsistent with equilibrium. However, essential for credit is a belief in repayment by the borrower and the incentive to repay is key to understand the credit arrangement. Incentive to repay aside, debtors may encounter production shock, which affects the ability to repay. This is considered as liquidity constrained default. I construct a general equilibrium random-matching model with limited commitment and limited enforcement, where an intermediary can facilitate intertemporal trade with limited ability in enforcement and record-keeping. If we interpret credit limit as the level of insurance provided by the credit market against production uncertainty, then we can study the link between the level of insurance, the incentive to repay and aggregate credit risk. Some preliminary findings are as follows. If the enforcement and commitment technology allows, it may be preferable to treat differently the intentional defaulters and liquidity constrained defaulters, say, by giving them different credit limits. Allowing liquidity constrained agents but not intentional defaulters to accumulate debt enhance the insurance function provided by the credit market, but may reduce borrowers' incentives to repay. If the former effect dominates, granting the liquidity constrained agents a higher credit limit would improve welfare. If the gains of trade from monetary exchange increase relative to that from intermediated credit trade, then we may observe a higher default risk.

# 1 Introduction

Search-theoretic models have successfully been used to demonstrate how decentralized allocations can be expanded by engaging in money-based bilateral trades. An obvious next step is to relax some of the assumptions to study allocations in similar environments when agents can engage in intertemporal trading arrangements. Examples include Diamond (1990), Shi (1996), Li (2001), Calvacanti and Wallace (1999) and Williamson (1999). A common feature in those studies is that default is either ignored or inconsistent with equilibrium. However, essential for credit is a belief in repayment by the borrower and the incentive to repay is key to understand the credit arrangement. One wants to know to what extent do difficulties in enforcing debt contracts undermine the effectiveness of trading arrangement based on private credit? Incentive to repay aside, debtors may encounter production shock, which affects the ability to repay. This is considered as liquidity constrained default. If the enforcement and commitment technology allows, would it be preferable to treat differently the intentional defaulters and liquidity constrained defaulters, say, by giving them different credit limits? How will the differential treatment affect the function of credit arrangements? Since accumulation of debt is commonly observed real-world phenomenon, we want to know whether allowing liquidity constrained agents but not intentional defaulters to accumulate debt affect borrowers' incentive to repay.

To study the above issues, I construct a general equilibrium random-matching model with limited commitment and limited enforcement, where an intermediary can facilitate intertemporal trade with limited ability in enforcement and record-keeping. Thus, borrowers may have incentive to default the debt. The intermediary can punish intentional defaulters by excluding them from utilizing credit instrument for a period. Incentive to repay aside, agents in this economy are subject to a production shock that affects their ability to repay. The intermediary makes collection of bad loans and stands as a guarantor to make repayment to the creditors whose debt are defaulted.

To be more specific, there are two sectors in the economy, one is called search sector in which trade is conducted in the spot market; the other is called intermediated sector with an intermediary to facilitate intertemporal trade. In the search sector, trade histories

are unobservable, agents are unable to commit to future actions, and proposed transfers cannot be enforced. It follows that spot trade must be facilitated by a tangible medium of exchange. A location shock determines in which sector an agent conducts trade. After the location shock, a production shock arrives to each agent which determines whether or not an agent can produce in a period. Credit trade thus provides an opportunity to insure against production uncertainty.

I first consider the case where liquidity constrained debtors are not forgiven in the sense that repayment is required for receiving credit in the future. Next I consider that liquidity constrained default is forgiven and so the credit market provides a higher level of insurance against production uncertainty. I assume that, in order to have their debt forgiven and be able to accumulate debt, agents must submit all related information to the intermediary. This implies that the intermediary has complete enforcement on repayment by the liquidity constrained debtors. That is, higher ability in commitment is accompanied with higher credit limit.

If we interpret credit limit as the level of insurance provided by the credit market against production uncertainty, then we can study the links between the level of insurance, the incentive to repay and aggregate credit risk, in this framework. Some preliminary findings are as follows. If the enforcement and commitment technology allows, it may be preferable to treat differently the intentional defaulters and liquidity constrained defaulters, say, by giving them different credit limits. Allowing liquidity constrained agents but not intentional defaulters to accumulate debt enhance the insurance provided by the credit market, but may reduce borrowers' incentives to repay. If the former effect dominates, allowing the liquidity constrained agents a higher credit limit would improve welfare. If the gains of trade from monetary exchange increase relative to that from intermediated credit trade, then we may observe a higher default risk.

This paper is closely related to Camera and Li (2003) in that we study intentional default only. Previous studies on how the availability of credit affects allocations and the role of money include, for example, Shi (1996), Li (2001), Cavalcanti et al. (1999), Calvacanti and Wallace (1999), Kocherlakota and Wallace (1998), Williamson (1999), Azariadis et al. (2001) and Jafarey and Rupert (2001). Default is not an issue in those papers, except Jafarey and Rupert (2001), who study default in an economy with limited

commitment and adverse selection. The major difference from previous studies is here I consider production uncertainty which affects the ability to repay, and agents who have defaulted the debts due to liquidity shortage may accumulate debt. We can study the link between the level of insurance against production uncertainty, the incentive to repay and aggregate default risk.

# 2 The Basic model

**Environment**

There is a continuum of infinitely-lived individuals, and of non-storable goods, normalized to one. There is one consumption good that potentially can be produced by everyone. Agents do not consume what they produce so trade is necessary An Agent's period utility from consuming $q > 0$ units of good is $u(q)$, while production of $q$ units generates disutility $q$. The function $u(q)$ is strictly increasing, concave, and twice differentiable, $u(0) = 0$, $u'(0) = \infty$, $u(\hat{q}) \geq \hat{q} > 0$ for $q \leq \hat{q}$, and the future is discounted at rate $r > 0$. Initially, a fraction $m \in (0, 1)$ of agents is randomly endowed with one unit of indivisible fiat money that can be discarded.

There are two sectors in this economy, one is a *search sector* in which trade is conducted in the spot market. The other is an *intermediated sector* in which intertemporal trade is possible. The matching process is identical across sectors. It provides each agent a random pairing for the period. For simplicity, assume that matching technology is such that in the search sector a producer never meets another producers so barter is ruled out. Trade histories are unobservable, agents are unable to commit to future actions, and proposed transfers cannot be enforced. It follows that spot trade must be facilitated by a medium of exchange, and that intertemporal trade must be supported by a technological innovation (see below).

An i.i.d. location shock arrives to agents at the beginning of each period determine which sectors an agent will conduct trade – an agent locates in the intermediated sector with probability $\sigma$, and in the search sector with probability $1 - \sigma$. If an agent is involved in a credit contract, he receives a location shock after the life-span of the contract has elapsed. After the location shock, an i.i.d. production shock arrives to each agent without

money. An agent receives a production opportunity with probability $\lambda$, which allows him to produce goods once. The production opportunity become totally obsolete after the use. With probability $1-\lambda$ an agent does not receive the production opportunity and is not able to produce.

Intertemporal trade is a two-period unilateral transfer sequence $\{q_c, q_d\}$, facilitated by a technological innovation, the 'intermediary'. If an agent is matched to a producer, the latter produces $q_c$ goods, the consumer becomes a 'debtor', and the producer a 'creditor'. In the following period the intermediary matches debtors to creditors, who should transfer $q_d$ goods. A debtor who receives an unfavorable productivity shock is unable to repay the debt and is treated as 'liquidity constrained debtor.' A debtor who receives a favorable productivity shock but intentionally defaults the debt is treated as 'defaulter.'

**Intermediary and financial contracts**

The intermediary keeps records of each agent's *last* credit transaction, checking it when the agent enters the credit market. However, the intermediary has distinct information and enforcement technology regarding intentional defaulters and liquidity constrained debtors. A prior intentional default is detected by the intermediary with probability $\theta \in (0,1)$, and the defaulter need produce $q_p \geq 0$ goods if he can produce; otherwise, he will be forced to stay in the intermediated sector till he is able to produce to repay the debt.[1] After repayment he would be excluded from credit trade in a period, and his record reset as good credit.

A debtor who receives an unfavorable productivity shock is perfectly observed by the intermediary and the default is forgiven in the sense that he would be allowed to conduct credit trade one more time in the future. Assume that in order to have their debt forgiven and be able to accumulate debt, agents must submit all relevant information to the intermediary. This implies that the intermediary has complete enforcement on repayment by the liquidity constrained debtors. A prior liquidity constrained debtor is perfectly known to the intermediary when he enters the credit market. If he becomes a debtor in the new debt contract, he will be forced to repay the debt if he is able to

[1]This is an innocuous assumption. If the intermediary is able to collect the bed loans when the debtors has *some* liquidity, then this assumption simply implies that it can make the collection just at the time when debtors get the liquidity.

produce. If he is unable to produce, the intermediary can enforce him to stay in the credit market until he receives a production opportunity to repay the debt, by producing $q_l$ units of goods. After production, his record is reset to good credit.

The intermediary's ability to make collections allows him to provide guaranty on transaction in the credit market. The creditors who suffer a default receive $q_r$ goods from the intermediary as a compensation.

# 3 Symmetric Stationary Equilibria

We focus on symmetric equilibria where strategies and distributions are time invariant, and agents in identical state choose identical actions. An agent's state depends on his inventory, the market in which he is trading, and his credit record $j = g, b, l$, that denotes good credit, bad credit and liquidity constrained defaulter, respectively. If he has no asset he can be (i) a *producer* with credit record $j$, trading either in the spot or credit markets, or (ii) a *creditor* or a *debtor* who is in the credit market for a second consecutive period, or (iii) a defaulter who are waiting for the opportunity to repay the debt. If the agent owns an asset she is in the spot market as a money holder with record $j$.

Let $j_p$ and $j_m, j = G, B, L$, denote the stationary population proportion of agents who at the beginning of $t$ are, producers or money holders, with good credit, bad credit, and liquidity constrained default. Let $P_c$ and $P_d$ ($L_c$ and $L_d$) denote the stationary proportion of agents who at the beginning of $t$ are in the credit market as, respectively, creditors and debtors with good credit (liquidity constrained default) record. Finally, let $D_d$ ($B_d$) denote agents who default their second debt (default first debt intentionally and is detected by the intermediary) and stay in the credit market waiting for the favorable productivity shock.

Steady state distributions satisfy

$$\begin{aligned} m &= G_m + B_m + L_m, \\ 1 - m &= G_p + B_p + L_p + P_c + P_d + L_c + L_d + D_d + B_d. \end{aligned}$$

Note that

$$P_c + L_c = P_d + L_d = \sigma[G_p + B_p(1 - \theta) + L_p]/2.$$

The stationary mass of producers in the spot market is

$$P_p \equiv (1-\sigma)(G_p + B_p + L_p).$$

A debtor who is able to produce choose whether or not to default the debt. Recall that liquidity constrained defaulter has no such option. Let $\alpha$ denote the probability of repayment by debtors who are able to produce. An agent choose strategy $\alpha'$ to maximize his life-time expected utility. A producer chooses whether or not to accept money in a trade. We look for pure monetary equilibrium where fiat money is universally acceptable. Of course we need to check the conditions under which the incentive constraints are satisfied. The best response condition satisfies

$$\alpha' = \begin{cases} 1 & \text{if } V_{gp} - q_d > V_{bp} \\ [0,1] & \text{if } V_{gp} - q_d = V_{bp} \\ 0 & \text{if } V_{gp} - q_d < V_{bp} \end{cases}$$

Let $x = 1/2$. Note that an agent trade in the intermediated sector has equal ex ante probability to be a producer or a consumer, even though the productivity shock is denoted by$\lambda$. The Bellman's equations satisfy:

$$\begin{aligned}
rV_{gp} &= \sigma\{x\left[u(q_c) + V_{gd}\right] + x(-q_c + V_{gc}) - V_{gp}\} \\
&\quad +(1-\sigma)\lambda m \max\left\{-q_m + V_{gm} - V_{gp}, 0\right\} \\
rV_{bp} &= \sigma\{(1-\theta)\left\{x\left[u(q_c) + V_{gd}\right] + x(-q_c + V_{gc})\right\} + \theta[\lambda(-q_p + V_{gp}) \\
&\quad +(1-\lambda)V_{bd}] - V_{bp}\} + (1-\sigma)\lambda m \max\left\{-q_m + V_{bm} - V_{bp}, 0\right\} \\
rV_{lp} &= \sigma\{x\left[u(q_c) + V_{ld}\right] + x(-q_c + V_{lc}) - V_{lp}\} \\
&\quad +(1-\sigma)\lambda m \max\left\{-q_m + V_{lm} - V_{lp}, 0\right\} \\
rV_{pc} &= \alpha\lambda\left[u(q_d) + V_{gp}\right] + (1-\alpha\lambda)[u(q_r) + V_{gp}] - V_{pc} \\
rV_{lc} &= \alpha\lambda\left[u(q_d) + V_{lp}\right] + (1-\alpha\lambda)[u(q_r) + V_{lp}] - V_{lc} \\
rV_{pd} &= \lambda \max\alpha\left\{V_{gp} - q_d, V_{bp}\right\} + (1-\lambda)V_{lp} - V_{pd} \\
rV_{ld} &= \lambda(V_{lp} - q_d) + (1-\lambda)V_{dd} - V_{ld} \\
rV_{dd} &= \lambda(-q_l + V_{gp} - V_{dd}) \\
rV_{bd} &= \lambda(-q_p + V_{gp} - V_{bd}) \\
rV_{jm} &= P_p\lambda\left[u(q_m) + V_{jp} - V_{jm}\right]
\end{aligned} \tag{1}$$

**Exchange and Prices**

Consider first the spot market. Since trading histories are private information the offers $q_m$ cannot depend on the producer's credit record, unless the producers' distribution across markets is degenerate. Hence, the equilibrium offer may leave different surplus to producers with different records. To see why, note that a buyer with money chooses $q_m \in \{q_{gm}, q_{bm}, q_{lm} | \ q_{jm} = V_{jm} - V_{jp}, \ j = g, b, l\}$, i.e. her optimal offer must leave no surplus to at least some producers. No other offer can increase the probability of a purchase, without decreasing the buyer's expected gain. Since all buyers face the same matching probabilities, $q_m$ is independent of the buyer's record.

The optimal offer $q_m$ of a buyer with credit record $j'$ is unique and must maximize her expected surplus, contingent on a random match with a producer:

$$\begin{aligned} q_m = \underset{q_m \in \{q_{gm}, q_{bm}, q_{lm}\}}{\arg\max} & [V_{j'p} + u(q_k) - V_{j'k}](B_p \mathbf{1}_b + G_p \mathbf{1}_g + L_p \mathbf{1}_l) \\ \text{s.t.} \ & q_{jm} = V_{jm} - V_{jp}. \end{aligned} \tag{2}$$

where $\mathbf{1}_j = 1$, if $V_{jm} - q_m - V_{jp} \geq 0$, and 0 otherwise.

Now consider intertemporal trades. Everyone in the credit market is recognized as having a good record. Hence,

$$\begin{aligned} q_c &= V_c - V_{gp} \\ q_d &= V_{gp} - V_d. \end{aligned} \tag{3}$$

Thus, undetected defaulters may earn surplus from lending, if $V_{gp} > V_{bp}$.

The quantity $q_p$ and $q_l$ satisfy

$$q_p = V_{gp} - V_{bd} \tag{4}$$

$$q_l = V_{gp} - V_{dd} \tag{5}$$

Equation (4) and (5) imply $V_{bd} = V_{dd} = 0$. Since creditors who suffer a default receive $q_r$ goods from the intermediary as a compensation, and that is financed via collections, $q_r$ must satisfy

$$(1 - \alpha\lambda)(P_c + L_c)q_r = (B_p \sigma\theta + B_d)\lambda q_p + D_d \lambda q_l. \tag{6}$$

## 3.1 Steady-state distribution

Consider any period $t$ and let $\Delta z(t) = z(t) - z(t-1)$, for any variable $z$. The laws of motion satisfy

$$\begin{aligned}
\Delta G_p(t) &= B_p\sigma\theta\lambda + \alpha\lambda\left(P_c + P_d\right) + (D_d + B_d)\lambda + G_m P_p\lambda \\
&\quad -G_p\sigma - G_p(1-\sigma)\lambda m \\
\Delta B_p(t) &= (1-\alpha)\, P_d + B_m P_p\lambda - B_p\sigma - B_p(1-\sigma)\lambda m \\
\Delta L_p(t) &= L_c + \lambda L_d + (1-\lambda)P_d + L_m P_p\lambda - L_p\sigma - L_p(1-\sigma)\lambda m \\
\Delta D_d(t) &= L_d(1-\lambda) - D_d\lambda \\
\Delta B_d(t) &= B_p\sigma\theta(1-\lambda) - B_d\lambda \\
\Delta G_m(t) &= G_p(1-\sigma)\lambda m - G_m P_p\lambda \\
\Delta B_m(t) &= B_p(1-\sigma)\lambda m - B_m P_p\lambda \\
\Delta L_m(t) &= L_p(1-\sigma)\lambda m - L_m P_p\lambda
\end{aligned} \tag{7}$$

In a steady state

$$\Delta j_k(t) = 0, \text{ where } j = G, B, L \text{ and } k = p, m, \text{ and } \Delta D_d(t) = \Delta B_d(t) = 0 \tag{8}$$

**Definition.** *Let $j = b, g, l$ and $k = m, p$, a symmetric steady-state monetary equilibrium is the strategy $\alpha'$, quantities $\{q_c, q_d, q_m, q_r, q_p, q_l\}$, value functions $\{V_{gc}, V_{gd}, V_{lc}, V_{ld}, V_{dd}, V_{bd}, V_{jm}, V_{jp}\}_{\forall j}$, and distribution of agents $\{G_k, B_k, L_k, D_d, B_d\}_{\forall k}$ that satisfy (1)-(8) and $\alpha' = \alpha$.*

**Welfare**

Define welfare $W$ as the ex-ante expected utility to an agent, when money is valued.

$$\begin{aligned}
W &= G_p V_{gp} + B_p V_{bp} + L_p V_{lp} \\
&\quad +G_m V_{gm} + B_m V_{bm} + L_m V_{lm} \\
&\quad +P\left(V_{pc} + V_{pd} + V_{lc} + V_{ld}\right).
\end{aligned}$$

# 4 Preliminary results

If the enforcement and commitment technology allows, it may be preferable to treat differently the intentional defaulters and liquidity constrained defaulters, say, by giving them different credit limits. Allowing liquidity constrained agents but not intentional defaulters to accumulate debt enhance the insurance provided by the credit market, but may reduce borrowers' incentives to repay. If the former effect dominates, allowing the liquidity constrained agents a higher credit limit would improve welfare. If the gains of trade from monetary exchange increase relative to that from intermediated credit trade, then we may observe a higher default risk.

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