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Insurer's insolvency risk and tax deductions for the individual's net losses

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Abstract Using the representative agent approach as in Kaplow (Am Econ Rev 82:1013–1017, 1992b), this paper shows that providing tax deductions for the individual's net losses is socially optimal when the insurer faces the risk of insolvency. We further show that the government should adopt a higher tax deduction rate for net losses when the insurer is insolvent than when the insurer is solvent. Thus, tax deductions for net losses could be used to provide an insurance for individuals against the insurer's risk of insolvency. These findings could also be used to explain why a government provides supplementary public insurance or government relief. Finally, we discuss that, if the individuals are heterogeneous in terms of loss severity, loss probability, or income level, providing a tax deduction for the individual's net losses may not always achieve a Pareto improvement, and cross subsidization should be taken into consideration.

Keywords Tax deduction · Insolvency risk · Public insurance · Government relief · Cross subsidization

JEL Classification G22 · H24 · D50

1 Introduction

Kaplow (1992b) documented that a tax deduction for net losses plays a role as social insurance and decreases the insurance coverage that individuals purchase in the

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private market. Moreover, he showed that a positive tax deduction rate for net losses is Pareto-inferior to the zero tax deduction rate when the private insurance market is available. In addition, he demonstrated that his finding is robust even after taking into consideration moral hazard, administration costs, and certain imperfections in the private insurance market. However, this type of income tax deduction for net losses can still be observed in many countries.[1] Why does a government provide tax deductions for net losses? This paper proposes that the government could have incentives for providing tax deductions for individuals' net losses when the insurer faces the risk of insolvency.

Recently, the issue of insolvency risk has received much attention from insurance researchers and regulators. Cummins and Sommer (1996) reported that the number of insolvencies in the property-liability insurance industry had dramatically increased from 10 per year during 1969–1983 to more than 30 per year during 1984–1992. Even since the year 2000, insolvency risk has continued to trouble the insurance industry. For example, Conseco Inc. filed for Chapter 11 in 2002, thereby constituting the third largest bankruptcy in the United States over the 1980–2005 period.

In the United States, the state guaranty associations will pay the insured parts of the coverage which are not paid by the insolvent insurer. During the period 1969–2000, the state guaranty associations paid $6.9 billion for property-liability insurance (NCIFG 2000). Hall (2000) and Grace et al. (2003) report that the average net cost to the guaranty associations for property-liability insurance was $1.22 and $1.10 for each $1 of assets that the insurer possessed to bankruptcy over the periods 1986–1994 and 1986–1999, respectively. Given that the insolvency of insurance companies cannot be ignored in the real world,[2] this paper intends to demonstrate that a tax deduction for net losses could serve as an insurance for insolvency risk and will increase social welfare.

Kaplow (1992b) assumed that there was no default risk for either the insurer or the government. Although the government may still bear insolvency risk, it is reasonable to assume that the benefits from tax deductions for net losses are subjected to almost no default risk. However, insurance purchased from a private insurer may give rise to the risk of insolvency. Doherty and Schlesinger (1990) showed that an individual purchases less insurance coverage after considering the insurer's insolvency risk. Since tax deductions for net losses as government-provided insurance are better insurance from the point of view of the insolvency risk, an insurer's insolvency risk could be one of the imperfections in the private insurance market that make tax deductions for net losses Pareto-improving.

In this paper, we employ two kinds of tax deductions for net losses. In the first model, we assume that the government offers the same tax deduction rate in both the solvency and insolvency states. We find that as long as the insurer has a positive

[1] For example, in the United States, individuals can deduct their medical expenses and casualty and theft net losses from their taxable income.

[2] Even under strict regulations to control insolvency risk, we have still observed insurance companies filing for Chapter 11 in recent years. Some examples from the United States include Fortune Financial, Inc. and Trenwick Group, Ltd. in 2003, Metropolitan Mortgage & Securities, Co. Inc. and MIIX Group, Inc. in 2004, and Acceptance Insurance Companies, Inc. and Frontier Insurance Group, Inc. in 2005. The above data are taken from http://www.bankruptcydata.com

insolvency probability, the government should then provide a positive tax deduction rate for net losses. In the second model, we relax the above assumption, and allow for different tax deduction rates under the insolvency and solvency states. We show that, if the insurance is an inferior good for individuals, the optimal tax deduction rate under the insolvency state should be greater than that under the solvency state. Thus, a tax deduction for net losses could be used to provide insurance for individuals against the insurer's insolvency risk.

To sum up, our paper contributes to the literature in three ways in the case where the insurer faces the risk of insolvency. First, we find that an increase in the tax deduction rate in relation to the individual's net losses may not always reduce the individual's demand for private insurance. Second, we find that providing a tax deduction for the individual's net losses is socially optimal. Both of the above findings are different from and complement those of Kaplow (1992b). Third, we find that, when an increase in wealth makes an individual purchase less insurance, the government should set up a larger tax deduction rate for individuals when the insurer is insolvent than that when the insurer is solvent.

The remainder of this paper is organized as follows: Sect. 2 focuses on a model with the same tax deduction rates for both the solvency and insolvency states. Section 3 analyzes the case where the tax deduction rates for the solvency and insolvency states are allowed to differ. Section 4 then discusses the applications of this paper for both supplementary social insurance and government relief. Section 5 considers the welfare improvement under the cases where the individuals may be heterogeneous in loss severity, loss probability or income level rather than homogeneous as assumed in Sects. 2 and 3. Section 6 concludes the paper.

2 Model I: the same tax deduction rates

We consider a compulsory tax transfer system for individuals who could purchase insurance from the private market. The model's structure is designed as a two-stage process as shown in Fig. 1. In the first stage, the government chooses an optimal tax deduction rate to maximize social welfare, and a self-financing tax deduction system is employed, as in Kaplow (1992b). The government provides a proportional tax deduction rate t for an individual's "net losses", which is equal to the loss amount minus private insurance coverage received. The deduction is financed by a lump-sum

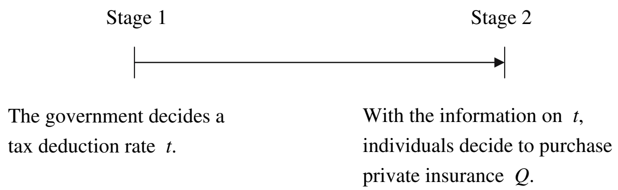

Fig. 1 The structure of the model

tax[3] τ, which means that all individuals have to pay a fixed tax amount τ. A self-financing tax deduction system is designed such that the total lump-sum tax amount the government receives is equal to the total expected tax deductions the government pays.

In the second stage, given the tax deduction, the representative insured[4] and the insurer sign the optimal coverage in the competitive private insurance market. Assume that the insured suffers a fixed loss amount L with a probability π. Following Doherty and Schlesinger (1990), the insurer's insolvency problem is employed in the private insurance market. Conditional upon the occurrence of a loss, the risk-neutral insurer will be solvent or insolvent with probabilities ρ or $1-\rho$, respectively. Assume that $0<\rho<1$. With complete information, the insurance premium P is $\lambda\rho\pi Q$, where λ is a loading factor and Q denotes the amount of coverage the insured chooses.[5] Assume that $\lambda \geq 1$. The insured is strictly risk-averse with a von Neumann-Morgenstern utility of wealth, represented by a twice differentiable function u with $u' > 0$, $u''<0$. Let w be the insured's initial wealth; thus the insured's final wealth is

$$\begin{cases} w_N = w - \lambda\pi\rho Q - \tau & \text{if no loss occurs;} \\ w_S = w - (L-Q)(1-t) - \lambda\pi\rho Q - \tau & \text{if a loss occurs and the insurer is solvent;} \\ w_I = w - L(1-t) - \lambda\pi\rho Q - \tau & \text{if a loss occurs and the insurer is insolvent.} \end{cases}$$

Specifically, we evaluate whether the optimal tax deduction for net losses is positive while the insurer may be insolvent.

The optimal tax deduction rate could be obtained by means of a backward induction. Thus, we begin our discussion from the second stage: what is the optimal coverage for individuals under an arbitrary tax deduction rate t? In the private insurance market, the insured's optimization problem will be

$$\max_{Q} \quad Eu = (1-\pi)u(w_N) + \pi\rho u(w_S) + \pi(1-\rho)u(w_I), \tag{1}$$

i.e., the individual will choose an optimal level Q, taking t and τ as given, to maximize his/her expected utility. The first-order condition for an interior solution,[6] Q^*, is

$$\Gamma = -\lambda\pi(1-\pi)\rho u'(w_N^*) + \pi\rho(1-t-\lambda\pi\rho)u'(w_S^*) - \lambda\pi^2\rho(1-\rho)u'(w_I^*) = 0, \tag{2}$$

where the w_i^*, $i = N, S, I$, denote the final wealth under Q^* for different states.

Doherty and Schlesinger (1990) establish that, in the absence of tax deductibility but with insolvency resulting in total default, less than full insurance coverage is

3 In reality, taxes may be raised according to income. We will relax the lump-sum tax assumption in Sect. 5.

4 Here we implicitly assume that all the individuals are homogeneous as in Kaplow (1992b).

5 $\rho > 0$ implies $Q < L$ in our model.

6 If $t \geq 1 - \lambda\pi\rho$, then $\Gamma < 0$ and therefore $Q = 0$. Thus, implicitly, we assume that $t < 1 - \lambda\pi\rho$. We also assume that the second-order condition holds.

always optimal for the individual even at a fair price ($\lambda = 1$). The intuition for their results is that the marginal utility of the insured is not equated across states with full coverage, and is higher in the loss and insolvency state. Reducing coverage marginally could redistribute toward that state.

As in Kaplow (1992b), the relationship between t and the private insurance coverage choice in the second stage of the game is examined. Unlike Kaplow (1992b), we find that an increase in t may not always decrease private insurance in the second stage.

Assume that the loading of the tax deduction is the same as the loading of the insurance, λ. Thus, the government's budget constraint is

$$\tau = \lambda\pi(\rho(L - Q^*) + (1 - \rho)L)t = \lambda\pi(L - \rho Q^*)t. \quad (3)$$

The following Lemma demonstrates the sufficient conditions for an increase in t to decrease Q^*.

Lemma 1 *If* $L - Q^* - \lambda\ \pi\ (L - \rho\ Q^*) > 0$ *and* $R(w_I^*) < 1/(1 - t - \lambda\pi\rho)(L - \lambda\pi(L - \rho Q^*))$, *then* $dQ^*/dt < 0$,, *where* $R(w_I^*)$ *denotes the absolute risk aversion index of the insured when he or she obtains wealth level* w_I^*.

Proof Please see Appendix A.

Lemma 1 shows that an increase in the tax deduction rate will decrease private insurance if two conditions hold. The first one is related to the direct effect of t on the net loss and the lump-sum tax holding Q^* constant, and implies that the marginal benefit of the tax deduction, $L - Q^*$, should be greater than the marginal cost of it, $\lambda\pi(L - \rho Q^*)$. When considering the optimal coverage level, individuals know that an increase in t will not only increase their total coverage but also increase the lump-sum tax, since that the tax deduction system is self-financed. The second condition requires that individuals cannot exhibit a large degree of risk aversion while facing the default risk of the insurance.

In the first stage, the government chooses the optimal tax deduction rate to maximize the expected utility of the representative consumer given that the consumer will choose insurance coverage $Q^*(t)$. It is worth noting that $Q^*(t)$ is the optimal coverage in the second stage of the game. It is a function of the tax deduction rate t, which will be determined in the first stage. Thus, the model of the optimal tax deduction can be written as:

$$\max_t \quad (1 - \pi)u(w_N^*) + \pi\rho u(w_S^*) + \pi(1 - \rho)u(w_I^*), \quad (4)$$

$$s.t. \qquad \tau = \lambda\pi(L - \rho Q^*(t))t. \quad (5)$$

The following Proposition is obtained:

Proposition 1 *Given that the insurer faces an insolvency risk, a positive tax deduction rate for net losses socially dominates a zero tax deduction rate for net losses.*

Proof Please see Appendix B.

Proposition 1 indicates that the tax deduction for net losses will be a Pareto improvement when the insurer bears an insolvency risk. Doherty and Schlesinger (1990) proved that partial coverage is optimal in the private insurance market under the considerations of an insolvency risk. The insurer's insolvency risk reduces the optimal coverage level, compared with that under no insolvency risk. A reduction in insurance coverage due to the insurer's default risk leaves room for the government to improve social welfare.

It is important to recognize that Proposition 1 still holds even if the loading factor $\lambda = 1$. Administration costs also lead the insured to purchase partial coverage. However, Kaplow (1992b) showed that a non-zero tax deduction rate is never socially optimal as long as the government is as inefficient as the insurance company. Since we assume that the loading of the tax deduction is the same as the loading of the insurance, administration costs are not a reason for concluding a non-zero tax deduction in our model.

It is also worth noting that our model includes Kaplow (1992b) as a special case since, on the basis of Eq. B5 in Appendix B, the optimal tax deduction rate is zero when $\rho = 1$, i.e., the insurer is under no default risk, as assumed by Kaplow (1992b).

3 Model II: different tax deduction rates

In this section, we allow the government to provide different tax deduction rates in the insolvency and solvency states. Assume that the tax deduction rate t is for the solvency state only, and that t_I is that for the insolvency state. Thus, the individual's final wealth will become:

$$\begin{cases} \hat{w}_N = w - \lambda\pi\rho Q - \tau & \text{if no loss occurs;} \\ \hat{w}_S = w - \lambda\pi\rho Q - \tau - (L - Q)(1 - t) & \text{if a loss occurs and the insurer is solvent;} \\ \hat{w}_I = w - \lambda\pi\rho Q - \tau - L(1 - t_I) & \text{if a loss occurs and the insurer is insolvent.} \end{cases}$$

In the second stage of the game, for any arbitrary t and t_I, the individual's optimal level $\hat{Q}^*$ now satisfies the following equation:

$$\hat{\Gamma} = -\lambda\pi(1-\pi)\rho u'(\hat{w}_N^*) + \pi\rho(1 - t - \lambda\pi\rho)u'(\hat{w}_S^*) - \lambda\pi^2\rho(1-\rho)u'(\hat{w}_I^*) = 0. \tag{6}$$

The following Lemma concludes the sign of $\partial\hat{Q}^*/\partial t$ and $\partial\hat{Q}^*/\partial t_I$ given that the individuals anticipate that a change in t or t_I will also influence the lump-sum tax, since the government's budget constraint is now

$$\tau = \lambda\pi\big(\rho(L - \hat{Q}^*))t + (1-\rho)Lt_I\big). \tag{7}$$

Lemma 2

$$(a) \quad \frac{\partial \hat{Q}^*}{\partial t} < 0,$$

$$(b) \quad \frac{\partial \hat{Q}^*}{\partial t_I} > \; if \; \frac{\partial \hat{Q}^*}{\partial w} < 0.$$

Proof Please see Appendix C.

Unlike Kaplow (1992b), Lemma 2 demonstrates that a tax deduction for net losses does not necessarily crowd out private insurance. If the government allows different tax deduction rates for both the solvency and insolvency states, then an increase in the tax deduction rate for the insolvency state will increase the purchase of private insurance if the wealth effect on the insurance decision is negative. Doherty and Schlesinger (1990) suggest that the insured will purchase less insurance because of insolvency risk. When the sufficient condition holds, an increase in t_I gives rise to a decrease in the individual's wealth and therefore leads to an increase in private insurance since $\partial \hat{Q}^*/\partial w < 0$. On the other hand, if the government increases t_I, then it will lower an individual's loss in the insolvency state and lower the welfare reduction of individuals due to the insurer's insolvency. The severity of insolvency risk is lightened. Thus, the insured will also demand more private insurance.

The optimal tax deduction rates t and t_I can be obtained according to the following optimization problem:

$$\max_{t, t_I} \quad (1-\pi)u(\hat{w}_N^*) + \pi\rho u(\hat{w}_S^*) + \pi(1-\rho)u(\hat{w}_I^*), \tag{8}$$

$$s.t. \qquad \tau = \lambda\pi\big(\rho(L - \hat{Q}^*(t, t_I))t + (1-\rho)Lt_I\big). \tag{9}$$

From the first-order conditions of the above model, we can generate Proposition 2.

Proposition 2 *If $\hat{Q}^*(t, t_I) \geq 0$ and $\partial \hat{Q}^*/\partial w < 0$, then $t_I^* \geq t^*$, where t^* and t_I^* denote the solution of the optimization problem (8) and (9).*

Proof Please see Appendix D.

Proposition 2 indicates that if the insurance is an inferior good and the government could set up different tax deduction rates when the insurer is solvent or insolvent, then the optimal tax deduction rate for the insolvency state should be greater than that for the solvency state. This result is intuitive. In the insolvency state, the individuals are less insured. In order to increase the social welfare, the government should provide a higher degree of coverage (tax deduction) to the individuals. Indeed, a tax deduction for net losses plays the role of an insurance in relation to the insurer's insolvency risk.

4 Applications of the models

Propositions 1 and 2 can be used to answer, at least in part, the other two questions: (1) “Why does a government provide supplementary public insurance?” and (2) “Why does a government provide government relief?”

In the literature, several papers have analyzed whether a government should provide supplementary public insurance. Specifically, Besley (1989) argues that public insurance for severe illnesses could interact with private coverage for less severe illnesses. An increase in public coverage for highly severe illnesses could reduce the inefficiency in the private insurance provided for less severe illnesses, and at the same time increase welfare. Selden (1993) modifies Besley’s (1989) model into a two-stage decision model but finds no such interaction between public coverage and private coverage. Thus Selden (1993) concludes that public insurance results in no welfare gains. Consistent with Selden’s (1993) findings, Blomqvist and Johansson (1997) modify Besley’s (1989) and Selden’s (1993) model, and demonstrate that a mixed insurance system is strictly less efficient than a purely private insurance system. Petretto (1999) set up a three-stage decision model instead of a simultaneous decision model for the insured that were faced with both public and private insurance. He demonstrated the optimal conditions for public and private insurance, and concluded that, under certain conditions, public insurance could improve social welfare. Although this line of research has provided many insights that have been either for or against supplementary public insurance, none of them has addressed the insurer’s default risk. On the other hand, Kaplow (1991, 1992a) found that it is not socially optimal for a government to provide government relief. However, as in the case of most of the literature, Kaplow (1991, 1992a) assumed that the insurance company is always solvent.

In this section, we intend to demonstrate that the models encompassing supplementary public insurance or government relief could be analyzed using models that generate Propositions 1 and 2. Let us recall the tax deduction for the net losses of individuals in Model I. A self-financing tax deduction system offers a proportional tax deduction rate t for an individual’s net losses and provides an indemnity tax deduction $t(L-Q)$. Since the deduction is financed by a lump-sum tax, the individual pays τ in exchange for a contingent indemnity $t(L-Q)$.

Now, consider the case where there is supplementary public insurance. Assume that supplementary public insurance is designed to provide coverage $\theta(L-Q)$, where θ represents the proportional coverage of the social insurance. Further assume that the individual pays the premium K to receive the social insurance coverage. Since the roles of θ and K in supplementary public insurance are similar to those of t and τ in tax deductions for net losses, it could also be concluded that the government has an incentive to provide supplementary social insurance when the insurer faces the risk of insolvency.

On the other hand, in the case where there is government relief, assume that the government provides government relief M after a loss occurs. Further assume that the government relief is self-financed by a lump-sum tax Π. By the same token, since the roles of M and Π in the government relief are similar to those of $t(L-Q)$ and τ in the tax deductions for net losses, it could also be concluded that the government has an incentive to provide government relief when the insurer faces the risk of insolvency.

5 Heterogeneous individuals

In the previous sections,[7] we have assumed that there is a representative agent in the market and find that a tax deduction for net losses, which is financed by a lump-sum tax system, could make the representative agent better off when the insurer faces insolvency risk. However, to implement a tax deduction system in reality, two issues need to be re-considered. The first one is that individuals in the insurance market could be heterogeneous[8] in wealth, loss severity or risk probability. The second one is that taxes are usually raised according to income level when the incomes of individuals vary. Thus, in this section, we will discuss whether the results in our previous section hold if the individuals in the society are not homogeneous. We show that, even after considering the insurer's insolvency risk, a tax deduction for net losses may not always Pareto improve all individuals' welfare when individuals are heterogeneous.

Let us first focus on the case where individuals are heterogeneous in loss severity. For simplicity,[9] we adopt all the assumptions in Model I except that some individuals do not suffer any loss. Thus the utility of these no-loss individuals will equal $u(w)$ when the tax deduction system does not exist, whereas it will equal $u(w - tax\ payment)$ when the tax deduction system exists, regardless of the kinds of tax transfers that there are. It is obvious that these no-loss individuals are worse off under a tax deduction system.

Now let us discuss another case where individuals are heterogeneous in loss probability. For simplicity, assume that some individuals have a zero probability of suffering a fixed loss L. This case is equal to the one where some individuals have a zero loss amount. From the above discussion, we can conclude that these individuals with zero loss probability are worse off under a tax deduction system.

Finally, let us assume that individuals are heterogeneous in initial wealth. Assume that the individuals could be divided into two groups: high and low income groups. A proportion θ of the individuals with initial wealth w_1 are in the low income group, and $1-\theta$ of them with initial wealth w_2 are in the high income group, where $w_2 > w_1$. For simplicity, let us further assume that w_2 is large enough so that the individuals in the high income group behave like risk neutral agents. Doherty and Schlesinger's (1990) paper predicts that these individuals in the high income group will not purchase any private insurance when the insurer's insolvency risk exists. Thus, whether these high income individuals will become better off under a

[7] We keep our previous models as close as possible to Kaplow's (1992b) except that the insurer may default, so that we can compare our results with his findings.

[8] It should be noted that our paper assumes away asymmetric information problems in order to maintain our focus.

[9] In this section, we employ special cases to demonstrate that tax deductions for individuals' net losses may not always result in a Pareto improvement. Our results can be shown to hold generally.

tax deduction system will depend on whether the net tax transfer is positive or negative.

If the tax deduction system is financed by a lump-sum tax as assumed by Kaplow (1992b), then the net transfer from the government will be

$$\pi Lt - \lambda\pi\big(\rho\big(\theta(L - Q_1^*) + (1-\theta)L\big) + (1-\rho)L\big)t, \tag{10}$$

where Q_1^* is the optimal coverage level that is purchased by the individuals with initial wealth w_1. Thus, if λ is large enough, that is:

$$\lambda > \frac{1}{1 - (Q_1^*/L)\rho\theta}, \tag{11}$$

then the net tax transfer to the individuals in the high income group from the government is negative. In other words, the individuals in this group are worse off.

Since we assume that the wealth of individuals varies, we would like to further discuss whether our results hold, if the tax deduction is financed by a tax on the basis of the individual's wealth which is similar to the income tax system in reality. We find that the individuals in the high income group may still be worse off under a tax deduction system.

Assume that the income tax rate is k for all individuals. Thus, the tax payment will be kw_i, $i = 1, 2$ for the individual with w_i. The government's budget constraint will be

$$\theta k w_1 + (1-\theta)k w_2 = \lambda\pi\big(\rho\big(\theta(L - Q_1^*) + (1-\theta)L\big) + (1-\rho)L\big)t. \tag{12}$$

Solving k from Eq. 12 yields

$$k = \frac{\lambda\pi\big(\rho\big(\theta(L - Q_1^*) + (1-\theta)L\big) + (1-\rho)L\big)t}{\theta w_1 + (1-\theta)w_2}. \tag{13}$$

Thus, the net tax transfer to the individuals in the high income group will be

$$\pi Lt - \lambda\pi\big(\rho\big(\theta(L - Q_1^*) + (1-\theta)L\big) + (1-\rho)L\big)t\left(\frac{w_2}{\theta w_1 + (1-\theta)w_2}\right). \tag{14}$$

If w_2 is huge enough, Eq. 14 will approach to

$$\pi Lt - \frac{\lambda}{1-\theta}\pi\big(\rho\big(\theta(L - Q_1^*) + (1-\theta)L\big) + (1-\rho)L\big)t. \tag{15}$$

Thus, the net tax transfer to the individual will become negative if

$$\lambda > \frac{1-\theta}{1 - (Q_1^*/L)\rho\theta}. \tag{16}$$

In other words, if λ is large enough, then the utility of the individuals in the high income group will decrease under a tax deduction system.

In the above cases, we demonstrate that, if individuals are not homogeneous, then providing a tax deduction for individuals' net losses may not be Pareto optimal, even though the tax deduction for net losses can be considered to be a better insurance than the private insurance from the point of view of default risk. When the loss severity, the loss probability, and the wealth of individuals vary, the government should take cross subsidization into consideration when designing a tax deduction system for individuals' net losses, since the system cannot always be Pareto-superior for all individuals.

6 Conclusion

This paper provides one possible reason for the existence of tax deductions for individuals' net losses, namely, that the government could improve social welfare from the tax deduction system when the insurer faces an insolvency risk. The insured purchases less coverage because of the insurer's default risk. This thus leaves room for the government to provide social insurance in the form of tax deductions for net losses to improve an individual's welfare.

Moreover, we find that the government could set up different tax deduction rates that are conditional upon whether the insurer is solvent or insolvent. The optimal tax deduction rate when the insurer is solvent would then be less than it would be when the insurer is insolvent. We demonstrate that the tax deduction for net losses could be used as insurance against the insurer's insolvency risk. Furthermore, we apply our results to analyze other types of government intervention, such as supplementary social insurance and government relief. We demonstrate that the government should consider the insurer's insolvency risk while establishing a social insurance or government relief.

By relaxing the assumption of homogeneous individuals, we further find that a tax deduction for a net loss system may not always improve the welfare of all individuals when individuals are heterogeneous in loss severity, loss probability or income level. We show that individuals with less loss severity, a smaller loss probability or higher income may be worse off because they will subsidize others under a tax deduction system. We suspect that, in reality, cross-subsidization may be one of the reasons why a government should consider setting up a tax deduction system for the individual's net loss, since the insured is hardly homogeneous in the real world.

In our models, we only discuss tax deduction in relation to the individual's net loss. However, the insurance premium may be also deductible. The effect of the premium being deductible could have both an income effect and a substitution effect. Although the results of the premium tax deduction may be mixed, this issue deserves further study. On the other hand, while both Kaplow (1992b) and our paper focus on the individual's insurance, tax deductions for net losses are also applied to corporations. Tax, limited liability, and principal-agent problems are shown in the literature to be key reasons why there is a demand for corporate insurance. A tax deduction for corporate net losses should have an integrated impact on the demand for corporate insurance, and future studies on this issue could prove fruitful.

Appendix A

The proof of Lemma 1: According to the implicit function theory, from Eq. 2,

$$\frac{dQ^*}{dt} = -\frac{\partial\Gamma/\partial t}{\partial\Gamma/\partial Q}. \tag{A1}$$

Since the second-order condition ensures that $\partial\Gamma/\partial Q < 0$, we have

$$sign\left\{\frac{dQ^*}{dt}\right\} = sign\left\{\frac{\partial\Gamma}{\partial t}\right\}. \tag{A2}$$

Taking the partial derivative of Γ with respect to t yields

$$\begin{aligned}\frac{\partial\Gamma}{\partial t} =& -\lambda\pi(1-\pi)\rho u''(w_N^*)\left(-\frac{\partial\tau}{\partial t}\right) - \pi\rho u'(w_S^*) + \pi\rho(1-t-\lambda\pi\rho)u''(w_S^*)\\ &\times\left(L - Q^* - \frac{\partial\tau}{\partial t}\right) - \lambda\pi^2\rho(1-\rho)u''(w_I^*)\left(L - \frac{\partial\tau}{\partial t}\right).\end{aligned} \tag{A3}$$

From Eqs. 2 and 3, we have

$$\pi\rho u'(w_S^*) = \frac{\lambda\pi(1-\pi)\rho}{1-t-\lambda\pi\rho}u'(w_N^*) + \frac{\lambda\pi^2\rho(1-\rho)}{1-t-\lambda\pi\rho}u'(w_I^*), \tag{A4}$$

and

$$\frac{\partial\tau}{\partial t} = \lambda\pi(L - \rho Q^*), \tag{A5}$$

respectively. Substituting Eqs. A4 and A5 into Eq. A3 yields

$$\begin{aligned}\frac{\partial\Gamma}{\partial t} =& -\lambda\pi(1-\pi)\rho u'(w_N^*)\left[\lambda\pi(L-\rho Q^*)R(w_N^*) + \frac{1}{1-t-\lambda\pi\rho}\right]\\ &+ \pi\rho(1-t-\lambda\pi\rho)u''(w_S^*)[L - Q^* - \lambda\pi(L-\rho Q^*)]\\ &- \lambda\pi^2\rho(1-\rho)u'(w_I^*)\left[\frac{1}{1-t-\lambda\pi\rho} - (L - \lambda\pi(L-\rho Q^*))R(w_I^*)\right],\end{aligned} \tag{A6}$$

where R denotes the insured's absolute risk aversion index. Equation A6 will be negative if

$$L - Q^* - \lambda\pi(L - \rho Q^*) > 0, \text{ and} \tag{A7}$$

$$R(w_I^*) < \frac{1}{(1-t-\lambda\pi\rho)(L - \lambda\pi(L-\rho Q^*))}. \tag{A8}$$

Q.E.D.

Appendix B

The proof of Proposition 1: The first derivative of the consumer's expected utility with respect to t is

$$\Lambda = \frac{dQ^*(t)}{dt}\left[-\lambda\pi(1-\pi)\rho u'(w_N^*) + \pi\rho(1-t-\lambda\pi\rho)u'(w_S^*) - \lambda\pi^2\rho(1-\rho)u'(w_I^*)\right]$$
$$-\frac{d\tau}{dt}\left[(1-\pi)u'(w_N^*) + \pi\rho u'(w_S^*) + \pi(1-\rho)u'(w_I^*)\right]$$
$$+\pi\rho(L-Q^*(t))u'(w_S^*) + \pi(1-\rho)Lu'(w_I^*), \tag{B1}$$

where

$$\frac{d\tau}{dt} = \lambda\pi\left(L - \rho Q^*(t) - \rho\frac{dQ^*(t)}{dt}t\right). \tag{B2}$$

According to Eq. 2, the first term in Eq. B1 becomes zero, and the second term will be

$$-\frac{d\tau}{dt}\frac{1-t}{\lambda}u'(w_S^*). \tag{B3}$$

Substituting Eqs. B2 and B3 into Eq. B1 yields

$$\Lambda = -\pi(1-t)\left(L - \rho Q^*(t) - \rho\frac{dQ^*(t)}{dt}t\right)u'(w_S^*)$$
$$+\pi\rho(L-Q^*(t))u'(w_S^*) + \pi(1-\rho)Lu'(w_I^*). \tag{B4}$$

Evaluating Eq. B4 at $t = 0$ yields

$$\Lambda|_{t=0} = \pi(1-\rho)L(u'(w-L-\lambda\pi\rho Q^*(0)) - u'(w-(L-Q^*(0))-\lambda\pi\rho Q^*(0))). \tag{B5}$$

Given that $0 < \rho < 1$ and $u'' < 0$, therefore

$$\Lambda|_{t=0} > 0. \tag{B6}$$

The optimal tax deduction rate is greater than zero immediately following Eq. B6. Q.E.D.

Appendix C

The proof of Lemma 2: This proof is similar to the proof of Lemma 1. The implicit function theory and the second-order condition for the choice of private insurance coverage ensure that

$$sign\left\{\frac{\partial\hat{Q}^*}{\partial x}\right\} = sign\left\{\frac{\partial\hat{\Gamma}}{\partial x}\right\}, \tag{C1}$$

where $x = t$ and t_I.

Taking the partial derivatives of Eq. (6) with respect to t and t_I yields

$$\begin{aligned}\frac{\partial\hat{\Gamma}}{\partial t} =& -\lambda\pi(1-\pi)\rho u''(\hat{w}_N^*)\left(-\frac{\partial\tau}{\partial t}\right) - \pi\rho u'(\hat{w}_S^*) + \pi\rho(1-t-\lambda\pi\rho)u''(\hat{w}_S^*)\\ &\times\left(L - Q^* - \frac{\partial\tau}{\partial t}\right) - \lambda\pi^2\rho(1-\rho)u''(\hat{w}_I^*)\left(-\frac{\partial\tau}{\partial t}\right), \text{ and}\end{aligned} \tag{C2}$$

$$\begin{aligned}\frac{\partial\hat{\Gamma}}{\partial t_I} =& -\lambda\pi(1-\pi)\rho u''(\hat{w}_N^*)\left(-\frac{\partial\tau}{\partial t_I}\right) + \pi\rho(1-t-\lambda\pi\rho)u''(\hat{w}_S^*)\left(-\frac{\partial\tau}{\partial t_I}\right)\\ &- \lambda\pi^2\rho(1-\rho)u''(\hat{w}_I^*)\left(L - \frac{\partial\tau}{\partial t_I}\right),\end{aligned} \tag{C3}$$

where

$$\frac{\partial\tau}{\partial t} = \lambda\pi\rho\left(L - \hat{Q}^*\right), \text{ and} \tag{C4}$$

$$\frac{\partial\tau}{\partial t_I} = \lambda\pi(1-\rho)L \tag{C5}$$

from Eq. 7. It is obvious that Eq. C4 is negative. Thus, $\partial\hat{Q}^*/\partial t < 0$.

Rearrange Eq. C5 as follows:

$$\begin{aligned}\frac{\partial\hat{\Gamma}}{\partial t_I} =& \left(\lambda\pi(1-\pi)\rho u''(\hat{w}_N^*) - \pi\rho(1-t-\lambda\pi\rho)u''(\hat{w}_S^*) + \lambda\pi^2\rho(1-\rho)u''(\hat{w}_I^*)\right)\frac{\partial\tau}{\partial t_I}\\ &- \lambda\pi^2\rho(1-\rho)Lu''(\hat{w}_I^*).\end{aligned} \tag{C6}$$

Since

$$\frac{\partial\hat{\Gamma}}{\partial\tau} = \lambda\pi(1-\pi)\rho u''(\hat{w}_N^*) - \pi\rho(1-t-\lambda\pi\rho)u''(\hat{w}_S^*) + \lambda\pi^2\rho(1-\rho)u''(\hat{w}_I^*), \tag{C7}$$

we have

$$\frac{\partial\hat{\Gamma}}{\partial t_I} = \frac{\partial\hat{\Gamma}}{\partial\tau}\frac{\partial\tau}{\partial t_I} - \lambda\pi^2\rho(1-\rho)Lu''(\hat{w}_I^*). \tag{C8}$$

It is obvious that $\partial\hat{\Gamma}/\partial t_I > 0$ if $\partial\hat{\Gamma}/\partial\tau > 0$, since $\partial\tau/\partial t_I > 0$ and $u''(\hat{w}_I^*) < 0$. Moreover, by implicit function theory and the second-order condition for the choice of private insurance coverage,

$$sign\left\{\frac{\partial\hat{Q}}{\partial w}\right\} = sign\left\{\frac{\partial\hat{\Gamma}}{\partial w}\right\}, \tag{C9}$$

where

$$\frac{\partial\hat{\Gamma}}{\partial w} = -\lambda\pi(1-\pi)\rho u''(\hat{w}_N^*) + \pi\rho(1-t-\lambda\pi\rho)u''(\hat{w}_S^*) - \lambda\pi^2\rho(1-\rho)u''(\hat{w}_I^*)$$
$$= -\frac{\partial\hat{\Gamma}}{\partial\tau}. \tag{C10}$$

Thus, $\partial\hat{\Gamma}/\partial t_I > 0$ if $\partial\hat{Q}/\partial w < 0$.
Q.E.D.

Appendix D

The proof of Proposition 2: The first-order conditions of the optimization problem will be

$$\hat{\Lambda}_t = \frac{\partial\hat{Q}^*(t,t_I)}{dt}\left[-\lambda\pi(1-\pi)\rho u'(\hat{w}_N^*) + \pi\rho(1-t-\lambda\pi\rho)u'(\hat{w}_S^*) - \lambda\pi^2\rho(1-\rho)u'(\hat{w}_I^*)\right]$$
$$-\frac{\partial\tau}{\partial t}\left[(1-\pi)u'(\hat{w}_N^*) + \pi\rho u'(\hat{w}_S^*) + \pi(1-\rho)u'(\hat{w}_I^*)\right]$$
$$+\pi\rho(L-\hat{Q}^*(t,t_I))u'(\hat{w}_S^*), \tag{D1}$$

and

$$\hat{\Lambda}_{t_I} = \frac{\partial\hat{Q}^*(t,t_I)}{dt_I}\left[-\lambda\pi(1-\pi)\rho u'(\hat{w}_N^*) + \pi\rho(1-t-\lambda\pi\rho)u'(\hat{w}_S^*) - \lambda\pi^2\rho(1-\rho)u'(\hat{w}_I^*)\right]$$
$$-\frac{\partial\tau}{\partial t}\left[(1-\pi)u'(\hat{w}_N^*) + \pi\rho u'(\hat{w}_S^*) + \pi(1-\rho)u'(\hat{w}_I^*)\right] + \pi\rho L u'(\hat{w}_I^*), \tag{D2}$$

where

$$\frac{\partial\tau}{\partial t} = \lambda\pi\left[\rho(L-\hat{Q}^*(t,t_I)) - \rho t\frac{\partial\hat{Q}^*(t,t_I)}{\partial t}\right], \tag{D3}$$

and

$$\frac{\partial\tau}{\partial t_I} = \lambda\pi\left[(1-\rho)L - \rho t\frac{\partial\hat{Q}^*(t,t_I)}{\partial t_I}\right] \tag{D4}$$

from Eq. 7. Equation 2 ensures that the first terms in both Eqs. D1 and D2 are zero. Thus, by rearranging Eqs. D1 and D2, and dividing the former by the latter yields

$$\frac{\pi\rho\left(L-\hat{Q}^{*}(t,t_I)\right)u'(\hat{w}_S^{*})}{\pi(1-\rho)Lu'(\hat{w}_I^{*})}=\frac{\partial\tau/\partial t}{\partial\tau/\partial t_I}. \tag{D5}$$

Substituting Eqs. D3 and D4 into Eq. D5 yields

$$\frac{\pi\rho\left(L-\hat{Q}^{*}(t,t_I)\right)u'(\hat{w}_S^{*})}{\pi(1-\rho)Lu'(\hat{w}_I^{*})}=\frac{\rho(L-\hat{Q}^{*}(t,t_I))-\rho t(\partial\hat{Q}^{*}(t,t_I)/\partial t)}{(1-\rho)L-\rho t(\partial\hat{Q}^{*}(t,t_I)/\partial t_I)}. \tag{D6}$$

Under the condition $\partial\hat{Q}^{*}/\partial w<0$, Lemma 2 predicts that $(\partial\hat{Q}^{*}(t,t_I))/\partial t<0$ and $(\partial\hat{Q}^{*}(t,t_I))/\partial t_I>0$. Thus, we have

$$\frac{\pi\rho\left(L-\hat{Q}^{*}(t,t_I)\right)u'(\hat{w}_S^{*})}{\pi(1-\rho)Lu'(\hat{w}_I^{*})}\geq\frac{\rho(L-\hat{Q}^{*}(t,t_I))}{(1-\rho)L}. \tag{D7}$$

or,

$$u'(\hat{w}_S^{*})\geq u'(\hat{w}_I^{*}). \tag{D8}$$

Since the individual is risk averse, this means that in equilibrium

$$\begin{aligned} w-&(L-\hat{Q}^{*}(t^{*},t_I^{*}))(1-t^{*})-\lambda\pi\rho\hat{Q}^{*}(t^{*},t_I^{*})-\tau \\ &\leq w-L(1-t_I^{*})-\lambda\pi\rho\hat{Q}^{*}(t^{*},t_I^{*})-\tau, \end{aligned} \tag{D9}$$

or

$$(L-\hat{Q}^{*}(t^{*},t_I^{*}))(1-t^{*})\geq L(1-t_I^{*}). \tag{D10}$$

Thus, we have $t_I^{*}\geq t^{*}$ if $\hat{Q}^{*}(t^{*},t_I^{*})\geq 0$.
Q.E.D.

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