New Scheme for Calculating Nuclear Radii

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It is pointed out that the nuclear radius is a function not of the mass number A alone, but rather of neutron number N and proton number Z. A convenient interpolation scheme for calculating nuclear radii is then proposed.

The nuclear radii have been traditionally expressed in terms of mass number A. The simplest form is

$$R = r_0 A^{1/3},$$

and the most accurate one up-to-date is the Elton's formula*)

$$R = 1.121A^{1/3} + 2.426A^{-1/3} - 6.614A^{-1}$$

which reproduces all of the data of Stanford electron scattering experiment*). On examining recent experimental data including Stanford electron scattering and the proton and neutron nuclear scattering, we found that the nuclear radii depend rather on both of the neutron number N and the proton number Z just as the binding energy does in the mass formula. This fact may be stated as follows:

(1) The closed shell nuclei of proton number (or of neutron number) 2, 8, 20, 28, 50, 82, 126···have the radii of proton (or neutron) density 2.5, 3.5, 4.5, 5.0, 6.0, 7.0, 8.0. ··fm respectively.

(2) The radii of proton (or neutron) density of other nuclei can be found by an interpolation between the above set of numbers 2.5, 3.5, 4.5, 5.0, 6.0···fm.

Table I illustrates a comparison between the result of Elton's formula and that of the present statement. The nuclides are chosen from the valley of p-stability. For the radii of proton density the comformity between both schemes is surprisingly well, but for the radii of neuton density it is not. Our values of $r_0 (= R/A^{1/3})$ for the proton density agree very well with those determined from the high energy electron scattering experiments by Hofstadter and. his collaborators, and those for neutron density agree with the value $r_0 = 1.35 \pm 0.04$ fm determined from the low energy neutron scattering by Seth[3]. Since the proton density and neutron density together form the matter distribution, we take the average of both r_0 for the matter radius. This also agrees with the value $r_0 = 1.25 \pm 0.05$ fm deduced from the neutron scattering by Fernbach[4]

1) L. R. B. Elton, Nuclear Phys. 5 173 (1958)
2) R. Hofstadter, Revs. Modern Phys. '28 214 (1956)
3) K. K. Seth, Revs. Modern Phys. 30 442 (1958)
4) S. Fernbach, Revs. Modern Phys. 30 414 (1958)

and with that from the proton scattering by Glassgold.[5]

A direct measurement of neutron density distribution still remains very difficult, however, the above values of r_0 for neutron density and for matter density seem to be able to explain the two different trends in the data of nuclear force measures of nuclear size. The failure of Elton's formula to express the radii of neutron density is obvious.

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Table 1.
Comparison between Elton's formula and new scheme. The number in the parentheis represents the value of $r_0 = R/A^{1/3}$.

Elton's Formula		New Scheme				
Mass Number A	Nuclear Radius (fm)	Proton Number Z	Radius of proton density	Neutron Number N	Radius of neutron density	Radius of matter density
5	1.92	2	2.50 (1.46)	3	2.70 (1.58)	2.60 (1.52)
10	2.88	5	3.06 (1.42)	5	3.07 (1.43)	3.07 (1.42)
15	3.30	7	3.35 (1.36)	8	3.50 (1.42)	3.43 (1.39)
20	3.61	9	3.59 (1.32)	11	3.77 (1.39)	3.68 (1.36)
25	3.84	12	3.85 (1.32)	13	3.94 (1.35)	3.90 (1.33)
30	4.04	14	4.02 (1.29)	16	4.19 (1.35)	4.11 (1.32)
35	4.22	16	4.19 (1.26)	19	4.43 (1.35)	4.31 (1.31)
40	4.38	18	4.35 (1.27)	22	4.63 (1.35)	4.49 (1.31)
45	4.52	21	4.56 (1.28)	24	4.75 (1.34)	4.66 (1.31)
50	4.66	23	4.69 (1.27)	27	4.94 (1.34)	4.82 (1.31)
60	4.90	27	4.94 (1.26)	33	5.22 (1.33)	5.08 (1.30)
70	5.11	31	5.14 (1.25)	39	5.50 (1.34)	5.32 (1.29)
80	5.31	35	5.32 (1.24)	45	5.73 (1.33)	5.53 (1.28)
90	5.49	39	5.50 (1.23)	51	6.03 (1.35)	5.77 (1.29)
100	5.66	43	5.68 (1.22)	57	6.22 (1.34)	5.95 (1.28)
110	5.82	47	5.86 (1.22)	63	6.41 (1.34)	6.14 (1.28)
120	5.97	51	6.03 (1.22)	69	6.59 (1.34)	6.31 (1.28)
130	6.11	54	6.13 (1.21)	76	6.81 (1.34)	6.47 (1.28)
140	6.24	58	6.25 (1.20)	82	7.00 (1.35)	6.63 (1.28)
150	6.37	62	6.38 (1.20)	88	7.14 (1.34)	6.76 (1.27)
160	6.49	66	6.50 (1.20)	94	7.27 (1.34)	6.89 (1.27)
170	6.61	69	6.59 (1.19)	101	7.43 (1.34)	7.01 (1.27)
180	6.72	73	6.71 (1.19)	107	7.57 (1.34)	7.14 (1.27)
190	6.83	76	6.81 (1.19)	114	7.73 (1.35)	7.27 (1.27)
200	6.94	80	6.94 (1.19)	120	7.86 (1.34)	7.40 (1.27)
210	7.04	84	7.02 (1.18)	126	8.00 (1.35)	7.51 (1.26)
220	7.14	87	7.11 (1.18)	133	8.12 (1.35)	7.62 (1.26)
230	7.24	91	7.24 (1.18)	139	8.22 (1.34)	7.73 (1.26)
240	7.33	94	7.27 (1.17)	146	8.34 (1.34)	7.81 (1.26)

5) A.E. Glassgold, Revs. Modern Phys. **30** 419 (1958)