Toward a CPFLOW-Based Algorithm to Compute all the Type-1 Load-Flow Solutions in Electric Power Systems

Chih-Wen Liu, *Senior Member, IEEE*, Chen-Sung Chang, Joe-Air Jiang, *Member, IEEE*, and Guey-Haw Yeh

***Abstract*—This paper presents a potential algorithm based on continuation power flow (CPFLOW) to compute all the Type-1 load-flow solutions. Type-1 solutions are of a single positive real-part eigenvalue associated with Jacobian of load-flow equations and are used in conjunction with techniques such as energy methods and the voltage instability proximity index (VIPI) for assessing system voltage stability. The benefits of the proposed algorithm are the following. The algorithm has the potential to find all the Type-1 load-flow solutions by tracing a small number of homotopy curves. Traditional methods, which can locate some of the Type-1 solutions, suffer from the uncertainty that there might be another Type-1 solution that might be more suitable for voltage stability assessment. This uncertainty is eliminated if all the Type-1 solutions are located. The proposed algorithm has been tested for two example systems, and encouraging results have been obtained.**

***Index Terms*—Continuation power flow (CPFLOW), stable equilibrium point (SEP), voltage instability proximity index (VIPI), voltage stability.**

I. Introduction

In RECENT years, an instability, usually termed a voltage instability, has been observed and been responsible for several major network collapses in many countries [1], [2]. Significant research efforts have been devoted to understanding of voltage phenomena. It reveals that the phenomena were not always in response to a contingency such as the loss of an important transmission line or a generator, but rather in response to an unexpected raise in the load level. Indeed, numerous authors have computed voltage stability limits based upon some type of load-flow analysis [3]–[16]. In fact, one class of methods for determination of the voltage stability limits utilizes the distance between the operable load-flow solution and an appropriate low-voltage load-flow solution [3]–[8].

A number of methods have been proposed for determining some (or all) of the load-flow solutions [17], [19], [20], [23], [24]. In [17], an algorithm that attempts to find all of the load-flow solutions by Newton–Raphson load-flow calculations from initial $2^N - 1$ voltage guesses is presented, where N is the number of buses. It should be noted that the method can not be guaranteed to find all the load-flow solutions, because recent findings of some fractal patterns produced by Newton–Raphson load-flow calculations imply the scheme may not necessarily converge to a solution one think it should [18]. In [19], a globally convergent homotopy method with probability-one is successfully applied to the load-flow equation of a 5-bus and a 7-bus system and obtains all the load-flow solutions to these systems. The load-flow solutions are obtained by tracing the homotopy curves starting from the known solutions of the load-flow equation. The number of homotopy curves that need to be traced is $\binom{2N}{N}$, where N is the number of buses excluding the slack bus, although the method is guaranteed to find all low-voltage load-flow solutions. The computation burden prohibits the practical use of the method even for a moderately sized power system. More recently, Ma and Thorp [20] have derived an efficient algorithm to solve for all the load-flow solutions. The number of homotopy curves that need to be traced is at most $(N \times S)/2$, where S is the number of the load-flow solutions of the system actually have. Although [19] and [20] both have developed the method which can locate all the load-flow solutions, [21] and [22] theoretically indicate that only Type-1 load-flow solutions are closely associated with voltage instability phenomenon. Based on this fact, it seems that reasonable computation burden can be alleviated for voltage stability assessment by locating only Type-1 load-flow solutions. Recently, [23] and [24] have presented algorithms to compute Type-1 load-flow solutions, but the algorithm can not be guaranteed to find all the Type-1 load-flow solutions.

In this paper, we propose an efficient algorithm that is on the road toward computing all the Type-1 load-flow solutions. The method is based on continuation power flow (CPFLOW) method [25]. The algorithm only traces $2(N-1)$ solution curves, where N is the total number of buses. This paper is organized as follows. Section II reviews the CPFLOW method. Section III describes the organization of the algorithm. Numerical examples and results are presented in Section IV. Concluding remarks are drawn in Section V.

II. Continuation Power Flow

In this section, we review an established numerical procedure called the CPFLOW method [25] which is used to trace

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the smooth curves (manifolds), and is able to remain well-conditioned at and around the saddle-node bifurcation point. It is based on a predictor–corrector scheme to trace a solution path of a set of the parameterized load-flow equations. To formulate the parameterized load-flow equation, we first consider the conventional load-flow equation defined as

$$\begin{aligned} P_i(X) - P_i &= 0, \qquad i = 2, \cdots, N \\ Q_i(X) - Q_i &= 0, \qquad i = m+2, \cdots, N \end{aligned}$$

where N is the total number of buses including slack bus, m is the total number of PV buses excluding slack bus. The state variables X is represented as vector $[\theta\ V]^T$. And the parameterized load-flow equation can be written in matrix form

$$F(X,\alpha) = \begin{bmatrix} P_2(X) - P_2 + \alpha_{p2} \\ \vdots \\ P_n(X) - P_n + \alpha_{pn} \\ Q_{m+2}(X) - Q_{m+2} + \alpha_{q(m+2)} \\ \vdots \\ Q_n(X) - Q_n + \alpha_{qn} \end{bmatrix} = 0 \qquad (1)$$

where load parameters α_{pi} indicate the variations of the active power of PV buses and PQ buses, and load parameters α_{qi} indicate the variations of the reactive power of PQ buses.

The CPFLOW method is based on the following three elements to find solution as one parameter in the (1) varies each time.

Predictor: Once a base stable solution point has been found by flat start, taking an appropriate step size in the tangent direction of the manifold can make the prediction of the next solution. Therefore, the task in the predictor process is to calculate the tangent vector. The tangent vector calculation is derived by taking the derivative of both sides of the parameterized load-flow equation and presented in matrix form as

$$\left[\frac{\partial F}{\partial \theta} \quad \frac{\partial F}{\partial V} \quad \frac{\partial F}{\partial \alpha}\right] \begin{bmatrix} d\theta \\ dV \\ d\alpha \end{bmatrix} = 0 \qquad (2)$$

where $[\partial F/\partial\theta\ \partial F/\partial V\ \partial F/\partial\alpha]$ is the conventional load-flow Jacobian augmented by one column $\partial F/\partial\alpha$ and the number of equations remains unchanged. Thus, one more equation is needed. This problem can be solved by choosing a nonzero magnitude (say one) for one of the components of the tangent vector, and the tangent vector is defined as $t = [d\theta, dV, d\alpha]^T$, and t_k is equal to $+1$ or -1 depending on how the k-th state variable variations as the solution curve is being traced. If it increases, then the $+1$ should be used, otherwise the -1 is used. Therefore, one equation is added to (2) and the equation can be modified as

$$\begin{bmatrix} \frac{\partial F}{\partial \theta} & \frac{\partial F}{\partial V} & \frac{\partial F}{\partial \alpha} \\ & E_k & \end{bmatrix} t = \begin{bmatrix} 0 \\ \pm 1 \end{bmatrix} \qquad (3)$$

where E_k is a row vector with all elements equal to zero except the k-th element that equals one. Vector t has a nonzero norm such that the augmented Jacobian will be nonsingular at the saddle-node bifurcation point.

If the tangent vector has been found by (3), then the predicted point could be written as

$$\begin{bmatrix} \theta^p \\ V^p \\ \alpha^p \end{bmatrix} = \begin{bmatrix} \theta \\ V \\ \alpha \end{bmatrix} + h \begin{bmatrix} d\theta \\ dV \\ d\alpha \end{bmatrix} \qquad (4)$$

where p denotes the predicted solution and h is the designed step size. h is one key element of affecting the computational efficiency of continuation methods. Choosing a constant, for example, let h be a small step length in the computation that it would lead to inefficient computation at the "flat" part of the manifolds. Similarly, an inadequately large step length can cast the predicted point to be laid far away the true solution point, and as a result, the corrector needs much iteration to converge, or lead to diverge. So in this paper, we adopt a voltage security index, the minimum singular value (S_n) which is able to measure how "close" a specific operating point is to the point of voltage collapse, as the step size length control index. The use of this index, obtained from a singular value decomposition of the load-flow Jacobian matrix, has been proposed and verified by Tiranuchit and Thomas in [14]. The minimum singular value S_n is an indicator of the proximity to the voltage security limit; it is as a measure of the distance between the operating point and the static bifurcation point which is associated with voltage collapse phenomena [14]. Therefore, the step size is controlled in order that the magnitude of the step size (h) decreases as the solution point gets closer to the voltage collapse point.

Corrector: Once a predicted solution of the manifolds has been found, the error must be corrected. Thus a good predictor gives an approximate solution, $[\theta^p, V^p, \alpha^p]^T$, which is in the neighborhood of the next solution, $[\theta^c, V^c, \alpha^c]^T$. So, a little iteration is used in a corrector to achieve the accurate solution. The Newton–Raphson method is chosen as the corrector. This choice has an advantage that the existing load-flow computer package based on the Newton–Raphson method can be utilized. The accurate point can be expressed as

$$\begin{bmatrix} \theta^c \\ V^c \\ \alpha^c \end{bmatrix} = \begin{bmatrix} \theta^p \\ V^p \\ \alpha^p \end{bmatrix} + \begin{bmatrix} \Delta\theta \\ \Delta V \\ \Delta\alpha \end{bmatrix} \qquad (5)$$

where $[\Delta\theta, \Delta V, \Delta\alpha]^T$ is found by Newton–Raphson iterations.

Choosing the Continuation Parameter: Every continuation method has a particular parameterization scheme. The parameterization offers a way of identifying each solution in the manifold so that the "previous" solution or "next" solution can be quantified. The scheme used in this paper is local parameterization. That is the original set equation is augmented by one equation that specifies the value of one of state variables and this state variable is referred to as continuation parameter. In load-flow equations, this means a bus voltage magnitude, a bus angle, or the load parameter, α. How to know which variable should be used as the continuation parameter? Mathematically, it should be the state variable that has the largest tangent vector component.

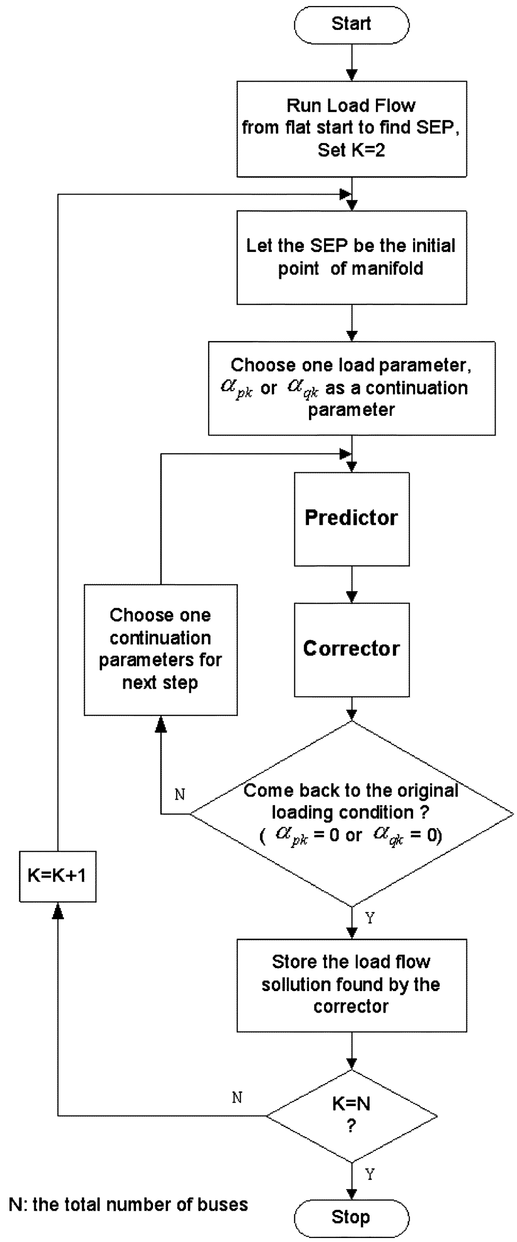


Fig. 1. Flowchart of the proposed CPFLOW-based algorithm.

III. CPFLOW-Based Algorithm for Calculating Type-1 Load-Flow Solutions

The principle of the proposed algorithm is described in the following. Assume that the operating point is the stable equilibrium point (SEP), which is the solution of load-flow equations with negative real-part eigenvalues under normal conditions. But it will slowly move as the changing of load demand, α. However, the SEP is stable. One fact is that, if system has a stable equilibrium point denoted as $X_s(\alpha)$ changing slowly on manifold in response to variation of parameter α, then, the only way in which the $X_s(\alpha)$ can disappear is by coalescing with a unstable equilibrium point $X_1(\alpha)$ in a saddle-node bifurcation point [21]. Just before the bifurcation point $X_1(\alpha)$ is on the stability boundary of $X_s(\alpha)$ and $X_1(\alpha)$ is the closest unstable equilibrium point to $X_s(\alpha)$. And the $X_1(\alpha)$ must be Type-1. The Jacobian of Type-1 has a single positive real-part eigenvalue, and the others are negative real-part eigenvalues. At the bifurcation point, $\alpha = \alpha^*$, $X_s(\alpha)$ and $X_1(\alpha)$ coalesce to an equilibrium point $X^* = X_s(\alpha^*) = X_1(\alpha^*)$. The Jacobian at X^* has a zero eigenvalue associated with an eigenvector in the direction in which $X_s(\alpha)$ and $X_1(\alpha)$ coalesced. The other eigenvalues of the Jacobian of X^* remain negative. The system cannot operate at the equilibrium point X^*, because the X^* is unstable, and any small perturbation will cause a voltage collapse [21]. Thus, it is an important task to calculate the Type-1 solutions for assessing system voltage stability.

Based on the above facts, the proposed algorithm uses CPFLOW to search for all Type-1 load-flow solutions by varying one load parameter, α_{pi} or α_{qi}, for each trace and going through all PV buses and PQ buses. The flowchart of the proposed CPFLOW-based algorithm is shown in Fig. 1.

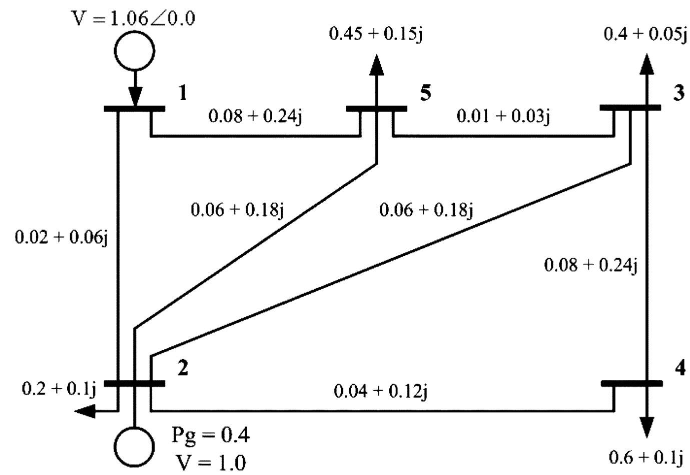


Fig. 2. The 5-bus system.

IV. Simulation Results

In this section, we take two systems to demonstrate the potential of the proposed CPFLOW-based algorithm. These two numerical examples (5-bus and 7-bus systems) are chosen because all load-flow solutions have been calculated in [19] and [20]. Therefore, they can serve as good examples for evaluation of the proposed algorithm.

Example 1: The one-line diagram of the 5-bus system with line and bus parameters is shown in Fig. 2.

In this first test example, we select the variations of active power of bus 2, and the variations of reactive power of bus 3, 4, and bus 5, as continuation parameters at the beginning of each trace. We take the flat point as an initial state to find the stable equilibrium point. To follow, it is based on the predictor–corrector scheme to search for the Type-1 load-flow solutions in the direction of increasing load parameters and the direction of decreasing load parameters. Four Type-1 load-flow solutions found by the proposed method for 5-bus system is shown in Table I. The Jacobian eigenvalues of Type-1 solutions for the 5-bus system are listed in Table II. In Table II, the eigenvalues of the power flow Jacobian for the Type-1 load-flow solutions are the ones with a single positive real-part eigenvalue, and the others are negative real-part eigenvalues.

TABLE I
ALL TYPE-1 LOAD-FLOW SOLUTIONS FOUND BY PROPOSED ALGORITHM FOR 5-BUS SYSTEM UNITS FOR MAGNETIC PROPERTIES

Variables	Solutions			
	1	2	3	4
θ_1	0.000000	0.000000	0.000000	0.000000
θ_2	–138.967855	–16.503994	–12.146784	–16.908807
θ_3	–134.863861	–81.865310	–13.879429	–37.786156
θ_4	–141.660453	–23.451893	–71.501631	–23.872760
θ_5	–129.850916	–26.042191	–12.679258	–69.041445
V_1	1.060000	1.060000	1.060000	1.060000
V_2	1.000000	1.000000	1.000000	1.000000
V_3	0.587894	0.030137	0.740259	0.184600
V_4	0.831660	0.628869	0.057975	0.686517
V_5	0.501169	0.197185	0.793300	0.034177

TABLE II
JACOBIAN EIGENVALUES OF TYPE-1 SOLUTIONS FOR 5-BUS SYSTEM (EIGENVALUES WITH POSITIVE REAL PARTS ARE CIRCLED)

Solution number	eigenvalue	Solution number	eigenvalue
1	–29.1332+4.2405j	3	–46.6051+14.1799j
	–29.1332-4.2405j		–46.6051-14.1799j
	7.9327		–23.7529
	–9.9385+3.1723j		3.1191
	–9.9385-3.1723j		–6.2482
	–4.0537+1.5817j		–4.5908
	–4.0537-1.5817j		–1.8625
2	–20.7505	4	–21.3485
	2.7585		4.7062
	–1.1522		–8.3728
	–3.7614		–4.4286
	–7.1770		–0.9235
	–6.7345		–1.6543
	–2.5251		–6.4539

Comparing our results with that of [19] and [20], it is clearly observed that all searched Type-1 load-flow solutions in the 5-bus system by the proposed CPFLOW-based algorithm are complete. The searching process of the proposed algorithm for the 5-bus system is shown in Table III. The sign "+" corresponds to the searching for the Type-1 load-flow solutions in the direction of increasing load parameters at the beginning of manifold; and the sign "−" corresponds to the searching for the Type-1 load-flow solutions in the direction of decreasing load parameters at the beginning of manifold. The number of solutions indicates the sequence in which the continuation traces performed, i.e. the first traced Type-1 load-flow solution is marked 1, the second traced solution is marked 2, etc. For example, the P2+ represents a load parameter for variations of active power of the bus 2, and search for the Type-1 load-flow solution in the increasing direction of load parameter. Similarly, the P2- represents searching for the Type-1 load-flow solution in the direction of decreasing load parameter, and the searched Type-1 load-flow solution is the solution number 1.

The manifolds of the first two traces for solutions number 1 for the 5-bus system as plotted in $\alpha_{P2} - V_5$ plane are shown in Fig. 3(a) and (b). Due to space limit, the other manifolds are not shown in this paper.

Example2: The one-line diagram of the 7-bus system with line and bus parameters is shown in Fig. 4.

TABLE III
SEARCHING PROCESS OF PROPOSED ALGORITHM

Load Parameter	P2+	P2−	Q3+	Q3−	Q4+	Q4−	Q5+	Q5−
Solution Number	1	1	2	1	3	1	4	1

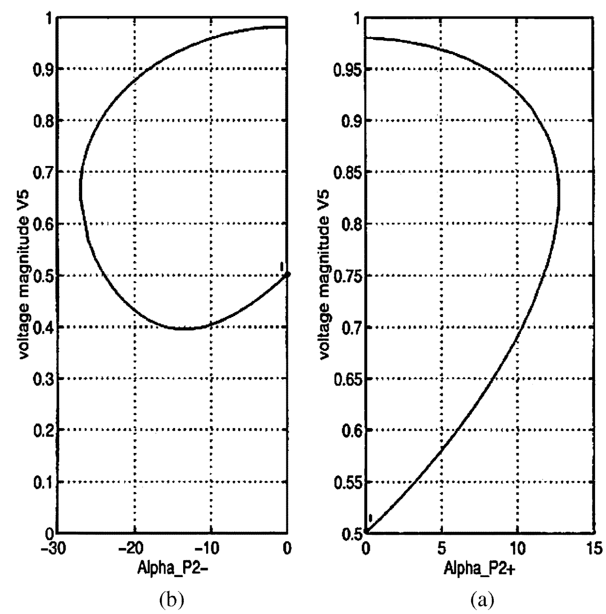


Fig. 3. Manifold of the first trace in $\alpha_{P2} - V_5$ plane. (a) $\alpha_{P2+} - V_5$ plane. (b) $\alpha_{P2-} - V_5$ plane.

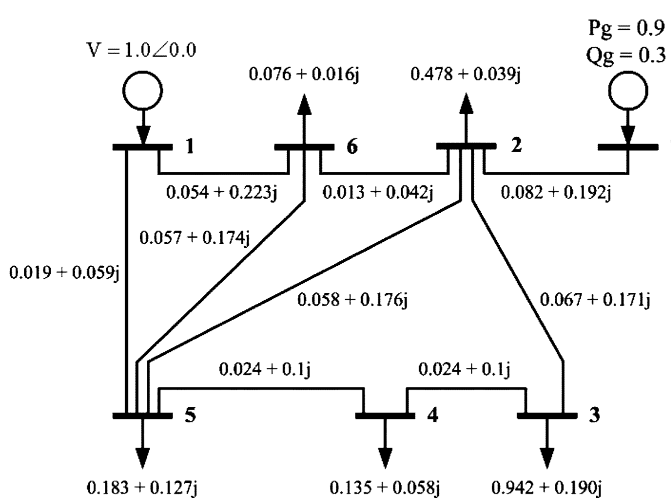
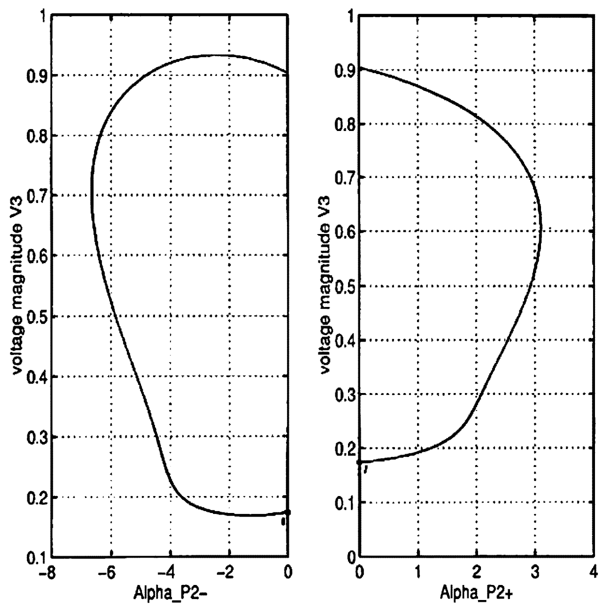


Fig. 4. The 7-bus system.

In the second test example, we select the variations of active power of bus 2, 3, 4, 5, 6 and bus 7 at the beginning of each trace. The process of searching for the Type-1 load-flow solutions by the proposed CPFLOW-based algorithm for 7-bus system is illustrated in Table IV. P2, P3, P4, P5, and P6 are load parameters which are selected to search for the Type-1 load-flow solutions of the 7-bus system. The sign "+" and "−" respectively correspond to the searching for the Type-1 load-flow solutions in the direction of increasing and decreasing load parameters at the beginning of manifold. The manifolds of the first two traces for solutions number 1 of the 7-bus system that are plotted in

TABLE IV
SEARCHING PROCESS OF PROPOSED ALGORITHM FOR 7-BUS SYSTEM

Load Parameter	P2+	P2−	P3+	P3−	P4+	P4−
Solution Number	1	1	1	1	1	1
Load Parameter	P5+	P5−	P6+	P6−	P7+	P7−
Solution Number	1	1	1	1	2	2



(b) (a)

Fig. 5. Manifold of the first trace in $\alpha_{P2} - V_3$ plane. (a) $\alpha_{P2+} - V_3$ plane. (b) $\alpha_{P2-} - V_3$ plane.

TABLE V
ALL TYPE-1 LOAD-FLOW SOLUTIONS FOUND BY PROPOSED ALGORITHM FOR 7-BUS SYSTEM

Variables	Solutions		Variables	Solutions	
	1	2		1	2
θ_1	0.000000	0.000000	V_1	1.000000	1.000000
θ_2	−5.221185	−6.293491	V_2	0.587562	0.541465
θ_3	−52.677524	−19.809837	V_3	0.174508	0.543027
θ_4	−14.205551	−11.246364	V_4	0.412175	0.645773
θ_5	−3.205593	−3.861835	V_5	0.722940	0.775000
θ_6	−4.303086	−5.016062	V_6	0.663768	0.640149
θ_7	14.957620	101.82011	V_7	0.731202	0.287970

$\alpha_{P2} - V_3$ plane are shown in Fig. 5(a) and (b). Due to space limit, the other manifolds are not shown in this paper.

Two Type-1 load-flow solutions of the 7-bus system are listed in Table V. And the Jacobian eigenvalues of Type-1 solutions for the 7-bus system are listed in Table VI. The Jacobian eigenvalue characteristics of the Type-1 load-flow solutions are the ones with a single positive real-part eigenvalue, and the others are negative real-part eigenvalues.

Comparing our results with [19] and [20], it is clearly seen that all searched Type-1 load-flow solutions in the 7-bus system by the proposed CPFLOW-based algorithm are complete.

From the simulation results, one has the following observations.

- The proposed algorithm has the potential to find all the Type-1 load-flow solutions by tracing a smaller number of manifolds, compared to [19] and [20].

TABLE VI
JACOBIAN EIGENVALUES OF TYPE-1 SOLUTIONS FOR 7-BUS SYSTEM
(EIGENVALUES WITH POSITIVE REAL PARTS ARE CIRCLED)

Solution number	eigenvalue	Solution number	eigenvalue
1	−28.2369+5.8497j −28.2369−5.8497j −23.7162+5.7252j −23.7162-5.7252j −6.1334+1.5566j −6.1334−1.5566j −6.7628 1.8125 −2.4340 −1.0813 −1.8035 −4.2080	2	−26.0601+4.3956j −26.0601−4.3956j −27.0596+6.9031j −27.0596-6.9031j −11.2898+1.3657j −11.2898−1.3657j 0.9611 −5.9566+1.3536j −5.9566−1.3536j −0.9947 −1.8087 −3.1520

- The proposed algorithm can avoid the fractal phenomenon.
- The algorithm only needs to traces $2(N-1)$ manifolds for computing all of the Type-1 load-flow solutions.

V. CONCLUSION

It has been demonstrated that a novel and potential algorithm for computing all the Type-1 load-flow solutions of power systems is developed. The algorithm based on CPFLOW algorithm is that its computational complexity is proportional to $2(N-1)$, instead of $\binom{2N}{N}$ for [19] and $(N \times S)/2$ for [20]. This potential algorithm provides more complete information for voltage stability assessment, instead of just one pair of closely located load-flow solutions [11] or partially located Type-1 load-flow solutions [23]. Further investigation such as theoretical justification of the proposed algorithm needs to be conducted.

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